An Intuitionistic Fuzzy Approach for Solving Generalized Trapezoidal Travelling Salesman Problem

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Abstract- *In this paper the Generalized trapezoidal intuitionistic fuzzy numbers and ranking method has been adopted and for finding an optimal solutionfor travelling salesman problem (TSP) Hungarian method is used. This method requires least iteration to obtain the optimality.The method is illustrated by a numerical example.*

Keywords- *Travelling salesman problem, generalized trapezoidal intuitionistic fuzzy numbers, fuzzy Hungarian method, optimal solution.*

I. INTRODUCTION

The travelling salesman problem is one of the most intensively studied problems in computational mathematics. Its aim is to select the sequence in which the cities are visited in such a way that his total travelling time (cost) is minimized. Cost can be distance, time, money, energy.The intuitionistic fuzzy sets were first introduced by Atanassov[1,2] which is generalization of the concept of fuzzy sets. An efficient method for ordering the fuzzy numbers is by the use of a ranking function, which maps each fuzzy number into the real line, where a natural order exists. NagoorGani and Mohamed [4] introduced methods for ordering two generalized trapezoidal intuitionistic fuzzy numbers. Thamaraiselvi and Santhi[7], has discussed Solving Fuzzy Transportation problem with Generalized Hexagonal Fuzzy Numbers,Jat.R.N,et.al;[3] has discussed about Fuzzy approach for solving mixed intuitionistic fuzzy travelling salesman problem .

In this paper, a new method is introduced to solve the generalized intuitionistic fuzzy travelling salesman problem.

II. PRELIMINARIES

A.Fuzzy number[11]

A fuzzy set A of real line R with membership function μ_A(x):R→[0,1] is called fuzzy number if

i)A is normal and convex fuzzy set

ii)support of $\tilde{A}$ must be bounded

iii)$\alpha_A$must be closed interval for every αϵ [0,1]

B.Triangular fuzzy number[9]

A fuzzy number $\tilde{A}$is a TFN denoted by$\tilde{A}$=(a_1, a_2, a_3),where a_1, a_2, a_3 are real numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-a_1}{a_2-a_1} & if\ a_1 \le x \le a_2\\ 1 & if\ x = a_2\\ \frac{a_3-x}{a_3-a_2} & if\ a_2 \le x \le a_3\end{cases}$$

C.Trapezoidal fuzzy number[9]

A fuzzy number $\tilde{A}=(a_1, a_2, a_3, a_4)$ is a TrFN where $a_1, a_2, a_3, a_4$are real numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-a_1}{a_2-a_1} & if\ a_1 \le x \le a_2\\ 1 & if\ a_2 \le x \le a_3\\ \frac{a_4-x}{a_4-a_3} & if\ a_3 \le x \le a_4\end{cases}$$

D.Generalized trapezoidal fuzzy number[4]

A generalized fuzzy number $\tilde{A}=(a_1, a_2, a_3, a_4, w)$ is said to be generalized trapezoidal fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}}(x)=\begin{cases}w(\frac{x-a_1}{a_2-a_1}) & if\ a_1 \le x \le a_2\\ w & if\ a_2 \le x \le a_3\\ w(\frac{x-a_4}{a_3-a_4}) & if\ a_3 \le x \le a_4\end{cases}$$

E.Intuitionistic fuzzy number[6]

An intuitionistic fuzzy set $\tilde{A}^I$ is called as intuitionistic fuzzy number if it satisfies the following condition

i)$\tilde{A}^I$is normal.(ie)there exist atleast two points x_0, x_1 such that $\mu_{\tilde{A}}(x_0)$=1 and $\nu_{\tilde{A}}(x_1) = 1$

ii)$\tilde{A}^I$is convex.(ie)membership function is fuzzy convex and non membership function is concave

iii)Its membership function is upper semicontinuous and its non membership function lower semi continuous and the set $\tilde{A}^I$ is bounded

F.Trapezoidal intuitionistic fuzzy number[10]

A trapezoidal intuitionistic fuzzy number is denoted by $\tilde{A}^I=(a_1,a_2,a_3,a_4)(a'_1,a_2,a_3,a'_4)$where $a'_1\le a_1\le a_2\le a_3\le a_4\le a'_4$ with membership and non membership function defined as follows,

$$\mu_{\tilde{A}^I}(x)=\begin{cases}\frac{x-a_1}{a_2-a_1} & if\ a_1\le x\le a_2\\ 1 & if\ a_2\le x\le a_3\\ \frac{a_4-x}{a_4-a_3} & if\ a_3\le x\le a_4\end{cases}\quad \text{and}$$

$$\nu_{\tilde{A}^I}(x)=\begin{cases}\frac{x-a'_1}{a_2-a'_1} & if\ a'_1\le x\le a_2\\ 1 & if\ a_2\le x\le a_3\\ \frac{a'_4-x}{a'_4-a_3} & if\ a_3\le x\le a'_4\end{cases}$$

G.Generalized trapezoidal intuitionistic fuzzy number[4]

An intuitionistic fuzzy number $\tilde{A}^I$ is said to be generalized trapezoidal intuitionistic fuzzy number with parameters $b_1\le a_1\le b_2\le a_2\le a_3\le b_3\le a_4\le b_4$ and is denoted by

$\tilde{A}^I=((a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4);w_A,u_A)$ if its membership and nonmembership function is defined as follows,

$$\mu_{\tilde{A}^I}(x)=\begin{cases}w_A\left(\frac{x-a_1}{a_2-a_1}\right) & if\ a_1\le x\le a_2\\ w_A & if\ a_2\le x\le a_3\\ w_A\left(\frac{a_4-x}{a_4-a_3}\right) & if\ a_3\le x\le a_4\end{cases}\quad \text{and}$$

$$\gamma_{\tilde{A}^I}(x)=\begin{cases}1 & if\ x<b_1\\ \frac{(b_2-x)+u_A(x-b_1)}{b_2-b_1} & if\ b_1\le x\le b_2\\ u_A & if\ b_2\le x\le b_3\\ \frac{(x-b_3)+u_A(b_4-x)}{b_4-b_3} & if\ b_3\le x\le b_4\\ 1 & if\ x>b_4\end{cases}$$

H.Properties of generalized trapezoidal intuitionistic fuzzy numbers[5]

Let A=$((a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4);w_A,u_A)$ and B=$((c_1,c_2,c_3,c_4),(d_1,d_2,d_3,d_4);w_B,u_B)$ be two generalized trapezoidal intuitionistic fuzzy numbers and λ be a real number. Then,

1. A+B= $((a_1+c_1,a_2+c_2,a_3+c_3,a_4+c_4),(b_1+d_1,b_2+d_2,b_3+d_3,b_4+d_4);w,u)$where,w=min$(w_A,w_B)$ and u =max(u_A,u_B)
2. A-B= $((a_1-c_4,a_2-c_3,a_3-c_2,a_4-c_1),(b_1-d_4,b_2-d_3,b_3-d_2,b_4-d_1);w,u)$ where ,w=min(w_A,w_B) and u =max(u_A,u_B)

3.λA=$((\lambda a_1,\lambda a_2,\lambda a_3,\lambda a_4),(\lambda b_1,\lambda b_2,\lambda b_3,\lambda b_4);w_A,u_A)$if λ>0

=$((\lambda a_4,\lambda a_3,\lambda a_2,\lambda a_1),(\lambda b_4,\lambda b_3,\lambda b_2,\lambda b_1);w_A,u_A)$ if λ<0

4.A*B=$((a_1c_1,a_2c_2,a_3c_3,a_4c_4),(b_1d_1,b_2d_2,b_3d_3,b_4d_4);w,u)$if A>0,B>0

=$((a_1c_4,a_2c_3,a_3c_2,a_4c_1),(b_1d_4,b_2d_3,b_3d_2,b_4d_1);w,u)$ if A<0,B>0

=$((a_4c_4,a_3c_3,a_2c_2,a_1c_1),(b_4d_4,b_3d_3,b_2d_2,b_1d_1);w,u)$ if A<0,B<0

where , w=min(w_A,w_B) and u =max(u_A,u_B)

5.A÷B=$\left(\left(\frac{a_1}{c_4},\frac{a_2}{c_3},\frac{a_3}{c_2},\frac{a_4}{c_1}\right),\left(\frac{b_1}{d_4},\frac{b_2}{d_3},\frac{b_3}{d_2},\frac{b_4}{d_1}\right)\right)$where , w=min$(w_A,w_B)$ and u =max(u_A,u_B)if d_1>0

III. RANKING OF GENERALIZED TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS FOR THE TRAVELLING SALESMAN PROBLEM

In the travelling salesman problem, the salesman has to visit 'n' cities. He needs to start from a particular city visit each city once and then return to his starting point.The problem whose solution will yield the minimum travelling time for the variable x_{ij}be defined as

$$x_{ij}=\begin{cases}1 & from\ i\ to\ j\\ 0 & otherwise\end{cases}$$

Ranking of generalized trapezoidal intuitionistic fuzzy number A=$((a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4);w_A,u_A)$ is defined by R(A)=$\frac{W_AS(\mu_A)+U_AS(\nu_A)}{W_A+U_A}$ where, $S(\mu_A)=\left(\frac{2(a_1)+7(a_2)+7(a_3)+2(a_4)}{18}\right)\left(\frac{7(w_A)}{18}\right)$ and

$S(\nu_A)=\left(\frac{2(b_1)+7(b_2)+7(b_3)+2(b_4)}{18}\right)\left(\frac{11+7(u_A)}{18}\right)$

IV. FORMULATION OF TRAVELLING SALESMAN PROBLEM AS AN ASSIGNMENT PROBLEM

Let the cost of travel from i^{th} city to j^{th} city be c_{ij} and x_{ij}=1 if the salesman goes directly from city i to city j and x_{ij}=0 otherwise. No city is visited twice before the tour of all cities are completed. In particular, he cannot go directly from city i to i itself. This possibility may be avoided in the minimization process by adopting the convention $c_{ij}=\infty$ which ensures that x_{ij} can never be unity.

The cost matrix is

$$\begin{pmatrix} \infty & C_{12} & C_{13} & \cdots & C_{1n} \\ C_{21} & \infty & C_{23} & \cdots & C_{2n} \\ C_{31} & C_{32} & \infty & \cdots & C_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ C_{n1} & C_{n2} & C_{n3} & \cdots & \infty \end{pmatrix}$$

Thus ,the above model can be expressed as

$$\text{Minimize } \tilde{z} = \sum_i \sum_j \tilde{c}_{ij}\, x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j=1,2,\ldots n \quad i\neq j, i\neq m$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i=1,2,\ldots m \quad i\neq j, i\neq n$$

x_{mk}=1

x_{ij}=0 or 1

V. ALGORITHMFOR THE PROPOSED METHOD

Step-1: The travelling salesman problem in which cost must be of generalized intuitionistic fuzzy numbers is constructed.

Step-2: The ranking index for each generalized intuitionistic fuzzy number is calculated

Step-3: The generalized intuitionistic fuzzy numbers is replaced by their respective ranking indices.

Step-4: The resulting problem is solved by using existing Hungarian method to find the optimal solution.

VI. NUMERICAL EXAMPLE

A company has four territories. The salesman working in the company has to deliver the products from each territory to other territories. The cost of travel from territories I,II,III,IV to I,II,III,IV is given below,

$$\begin{pmatrix} \infty & ((2,5,7,11),(1,6,12,15);0.5,0.1) & ((1,5,6,10)(2,5,7,11);0.3,0.5) & ((1,2,5,1),(1,5,3,2);0.2,0.5) \\ ((3,5,12,15),(2,1,1,5);0.1,0.2) & \infty & ((1,5,5,1),(1,3,9,11);0.1,0.1) & ((2,7,11,15),(2,5,6,2);0.5,0.6) \\ ((1,5,5,1),(2,5,6,2);0.1,0.6) & ((1,2,5,1),(2,3,5,6);0.2,0.2) & \infty & ((2,5,8,10),(2,1,15);0.6,0.2) \\ ((2,3,5,6),(2,3,5,6);0.5,0.2) & ((2,5,8,10),(1,3,9,11);0.6,0.1) & ((2,5,7,11),(1,6,12,16);0.5,0.3) & \infty \end{pmatrix}$$

Which route will the salesman choose to minimize the travelling cost.

Step:1 Consider ,

$$\begin{pmatrix} \infty & ((2,5,7,11),(1,6,12,15);0.5,0.1) & ((1,5,6,10)(2,5,7,11);0.3,0.5) & ((1,2,5,1),(1,5,3,2);0.2,0.5) \\ ((3,5,12,15),(2,1,1,5);0.1,0.2) & \infty & ((1,5,5,1),(1,3,9,11);0.1,0.1) & ((2,7,11,15),(2,5,6,2);0.5,0.6) \\ ((1,5,5,1),(2,5,6,2);0.1,0.6) & ((1,2,5,1),(2,3,5,6);0.2,0.2) & \infty & ((2,5,8,10),(2,1,15);0.6,0.2) \\ ((2,3,5,6),(2,3,5,6);0.5,0.2) & ((2,5,8,10),(1,3,9,11);0.6,0.1) & ((2,5,7,11),(1,6,12,16);0.5,0.3) & \infty \end{pmatrix}$$

Step:2 calculate the rank of ((2,5,7,11),(1,6,12,15);0.5,0.1)(ie)from territoryI→territoryIIusing the ranking formulae given above in section III,

$R(A)=\frac{W_A S(\mu_A)+U_A S(\nu_A)}{W_A+U_A}$ where,

$S(\mu_A)=\left(\frac{2(a_1)+7(a_2)+7(a_3)+2(a_4)}{18}\right)\left(\frac{7(w_A)}{18}\right)$ and

$S(\nu_A)=\left(\frac{2(b_1)+7(b_2)+7(b_3)+2(b_4)}{18}\right)\left(\frac{11+7(u_A)}{18}\right)$

$\therefore S(\mu_A)=\left(\frac{2(2)+7(5)+7(7)+2(11)}{18}\right)\left(\frac{7(0.5)}{18}\right)=\frac{385}{324}=1.1882$

$S(\nu_A)=\left(\frac{2(1)+7(6)+7(12)+2(15)}{18}\right)\left(\frac{11+7(0.1)}{18}\right)=5.7055$

$R(A)=\frac{0.5(1.1882)+0.1(5.7055)}{0.5+0.1}=\frac{1.16465}{0.6}=1.9410$

Proceeding similarly,

R((1,5,6,10),(2,5,7,11);0.3,0.5)=3.3173,

R((1,2,5,1),(1,5,3,2);0.2,0.5)=2.0472 ,

R((3,5,12,15),(2,1,1,5);0.1,0.2)=0.826 ,

R((1,5,5,1),(1,3,9,11);0.1,0.1)=2.0299 ,

R((2,7,11,15),(2,5,6,2);0.5,0.6)=2.9606 ,

R((1,5,5,1),(2,5,6,2);0.1,0.6)=3.4407 ,

R((1,2,5,1),(2,3,5,6);0.2,0.2)=1.4925 ,

R((2,5,8,10),(2,1,1,5);0.6,0.2)=1.3859 ,

R((2,3,5,6),(2,3,5,6);0.5,0.2)=1.3431 ,

R((2,5,8,10),(1,3,9,11);0.6,0.1)=1.8348 ,

R((2,5,7,11),(1,6,12,16);0.5,0.3)=3.1682

Step:3 From the above matrix the following ranking value is obtained

$$\begin{pmatrix} \infty & 1.9410 & 3.3173 & 2.0472 \\ 0.826 & \infty & 2.0299 & 2.9606 \\ 3.4407 & 1.4925 & \infty & 1.3859 \\ 1.3431 & 1.8348 & 3.1682 & \infty \end{pmatrix}$$

Step:4

$$\begin{pmatrix} \infty & \boxed{0} & 0.1724 & 0.1062 \\ 0 & \infty & \boxed{0} & 2.1346 \\ 2.0548 & 0.1066 & \infty & \boxed{0} \\ \boxed{0} & 0.4917 & 0.6212 & \infty \end{pmatrix}$$

Thus the total travel cost is

I→II, II→III, III→IV, IV→I is 6.6999

VII. CONCLUSION

This paper ranks generalized trapezoidal intuitionistic fuzzy numbers to solve travelling salesman problems occurring in real life situation .It is given through a numerical example fora non mixedgeneralised trapezoidal intuitionistic fuzzy numbers.Hence it is applicable for mixed and non mixed fuzzy numbersto obtain minimum cost.

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