Confidence Intervals for the Difference and the Ratio of Coefficients of Variation of Normal Distribution with a Known Ratio of Variances

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Abstract-- *In this paper, confidence intervals for the difference and the ratio of coefficients of variation of normal distribution with a known ratio of variances are proposed. The modified confidence intervals perform well both for the coverage probability and the average length. We show these results via Monte Carlo simulation.*

Keywords-- *Coverage probability, expected length, Monte Carlo simulation*

I. INTRODUCTION

It is well known that the coefficient of variance (CV) has been one of the most widely used statistical measures of the relative dispersion. This is due to its important property such that it is a dimensionless (unit-free) measure of variation and also its ability that can be used to compare several variables or populations with different units of measurement. As a result, it has been frequently used in numerous fields of knowledge in science, engineering, medical, biology and economics The ratio of two independent coefficients of variation (CVs) is a particular problem in the estimation of two independent normally distributed random variables. This problem has previously appeared in the literature of Verrill and Johnson [6]. Their simulation results indicated that the performance of Verrill and Johnson confidence interval for the ratio of coefficients of variation by using the normal approximation worked quite well for small samples and the coverage probability of Verrill and Johnson confidence interval is not at least the nominal confidence levels. Recently, Wongkho et al. [8] have shown that the new confidence intervals based on the the generalized confidence interval (GCI) of Weerahandi [7] and the new confidence interval based on the method of variance estimates recovered from the confidence limits (MOVER) described by Donner and Zou [1] are better than Verrill and Johnson's confidence interval. As a result, they recommended both method for constructing the confidence interval for the ratio of coefficients of variation. In this paper, we consider not only the ratio of the CVs of normal distribution but also the difference between CVs when we known a ratio of variances, Schechtman and Sherman [5]. Schechtman and Sherman [5] described 'a situation of a known ratio of variances arises in practice when two instruments report (averaged) response of the same object based on a difference number of replicates. If the two instruments have the same precision for a single measurement, then the ratio of the variance of the responses is known, and it is simply the ratio of the number of replicates going into each response." They proposed a *t*-test statistic, which has an exact *t*-distribution with *n*+*m*-2 degrees of freedom, compared to the Satterthwaite's t-test statistic. They found that their proposed test has more power than an existing Satterthwaite's test. However, they did not investigate the coverage probability and the expected length of the confidence intervals for the difference and the ratio of CVs when the ratio of variances is known.

II. CONFIDENCE INTERVALS FOR THE DIFFERENCE AND THE RATIO OF COEFFICIENTES OF VARIATION OF NORMAL DISTRIBUTION WITH A KNOWN RATIO OF VARIANCES

Suppose $X_i \sim N(\mu_1, \sigma_1^2), i = 1, 2, ..., n$, $Y_j \sim N(\mu_2, \sigma_2^2), j = 1, 2, ..., m$ and $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ are respectively the population means and population variances of X and Y.Let $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i$, $\bar{Y} = m^{-1}\sum_{j=1}^{m} Y_j$,

$S_1^2=(n-1)^{-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$, $S_2^2=(m-1)^{-1}\sum_{j=1}^{m}(Y_j-\bar{Y})^2$ be estimators of the population means and population variances of X and Y, respectively. We are interested in constructing the confidence interval for the the ratio and the difference of coefficients of variation, $\eta=\theta_1/\theta_2$, $\delta=\theta_1-\theta_2$ where $\theta_1=\sigma_1/\mu_1$ and $\theta_2=\sigma_2/\mu_2$. In some cases, when we known the ratio of variances, Shechman and Sherman [5], the parameter of interest $\eta=\theta_1/\theta_2$ reduces to $\eta=\frac{\sigma_1/\mu_1}{\sigma_2/\mu_2}=\frac{\mu_2}{\mu_1}.\frac{\sigma_1}{\sigma_2}=\sqrt{\frac{n}{m}}\frac{\mu_2}{\mu_1}=c\frac{\mu_2}{\mu_1}, c^2=\frac{n}{m}$. Also, the parameter of $\delta=\sigma_1/\mu_1-\sigma_2/\mu_2$ reduces to $\delta=\sigma_1/\mu_1-c^{-1}\sigma_1/\mu_2$.

A. *The Generalized Confidence Interval (GCI)*

In this subsection, we present the GCI for the ratio and the difference between the CVs of normal distributions based on Weerahandi [5]. Weerahandi [5] defined a generalized pivot (GP) as a statistic that has a distribution free of unknown parameters and an observed value of a generalized pivotal quantity does not depend on nuisance parameters. In a general setting, a generalized pivot can be defined as follows: Let X be a random quantity having a density function $f(X,\xi)$ where $\xi=(\theta,\tau)$ are unknown parameters; θ is the parameter of interest and τ is a nuisance parameter. Let x be the observed value of X. In the procedure to construct the confidence interval for θ, we start with a generalized pivotal quantity $R(X,x,\xi)$ which is a function of random variable X, its observed value x and the parameter ξ. Also $R(X,x,\xi)$ is required to satisfy the following conditions:

A1. For fix x, a probability distribution of $R(X,x,\xi)$ is free of unknown parameters.

A2. The observed pivot, which is defined as $R(x,x,\xi)$, does not depend on nuisance parameters.

Now, we construct the confidence interval for $\eta=\theta_1/\theta_2$ when we set $\theta_1=\sigma_1/\mu_1$, $\theta_2=\sigma_2/\mu_2$ and the ratio of variances $\frac{\sigma_1}{\sigma_2}=\sqrt{\frac{n}{m}}$. It is straightforward to see that

$$\delta=\frac{\sqrt{\frac{(n-1)S_1^2}{U_1}}}{\bar{X}-T_1S_1/\sqrt{n}}-\frac{c^{-1}\sqrt{\frac{(n-1)S_1^2}{U_1}}}{\bar{Y}-T_2S_2/\sqrt{m}} \quad \text{and} \quad \eta=c\frac{\bar{Y}-T_2S_2/\sqrt{m}}{\bar{X}-T_1S_1/\sqrt{n}} \tag{1}$$

where $T_1\sim t_{n-1}$, $T_2\sim t_{m-1}$, $U_1=\frac{(n-1)s_1^2}{\sigma_1^2}\sim\chi_{n-1}^2$ and $U_2=\frac{(m-1)s_2^2}{\sigma_2^2}\sim\chi_{m-1}^2$.

Using Equation (1), we can define the generalized pivotal quantity as

$$Q(X,Y;x,y,\xi)=\frac{\sqrt{\frac{(n-1)s_1^2}{U_1}}}{\bar{x}-T_1s_1/\sqrt{n}}-\frac{c^{-1}\sqrt{\frac{(n-1)s_1^2}{U_1}}}{\bar{y}-T_2s_2/\sqrt{m}} \text{ and } R(X,Y;x,y,\xi)=c\frac{\bar{y}-T_2s_2/\sqrt{m}}{\bar{x}-T_1s_1/\sqrt{n}} \tag{2}$$

where $s_1, s_2, \bar{x}, \bar{y}$ are the observed values of $S_1, S_2, \bar{X}, \bar{Y}$, respectively.

It is easy to see that $Q(X,Y;x,y,\xi)$ and $R(X,Y;x,y,\xi)$ satisfy for A1-A2. Therefore, the $100(1-\alpha)\%$ generalized confidence interval for η and δ are respectivery

$$CI_{gci1}=(Q_{\alpha/2},Q_{1-\alpha/2})$$

and

$$CI_{gci2} = \left(R_{\alpha/2}, R_{1-\alpha/2}\right) \qquad (3)$$

where $Q_{1-\alpha/2}$ and $R_{1-\alpha/2}$ are the $\left(1-\alpha/2\right)^{th}$ percentile of $Q(X,Y;x,y,\xi)$ and $R(X,Y;x,y,\xi)$ respectively.

B. The Method of Variance Estimates Recovery (MOVER)

Zou et al. [9] and Donner and Zhou [1] proposed a simple confidence interval for the sum of parameters $\theta_1+\theta_2$ is in the form of

$$CI_0 = \left(\left(\hat\theta_1+\hat\theta_2\right) - Z_{1-\alpha/2}\sqrt{\operatorname{var}\left(\hat\theta_1\right)+\operatorname{var}\left(\hat\theta_2\right)},\ \left(\hat\theta_1+\hat\theta_2\right) + Z_{1-\alpha/2}\sqrt{\operatorname{var}\left(\hat\theta_1\right)+\operatorname{var}\left(\hat\theta_2\right)} \right)$$

where $\hat\theta_1, \hat\theta_2$ are the estimators of parameters θ_1, θ_2 respectively. Suppose $CI_0 = \left(L, U\right)$ is the confidence interval for $\theta_1+\theta_2$, Zou et al. [9] and Donner and Zou [1] found that

$$L = \hat\theta_1+\hat\theta_2-\sqrt{\left(\hat\theta_1-l_1\right)^2+\left(\hat\theta_2-l_2\right)^2},\ U = \hat\theta_1+\hat\theta_2+\sqrt{\left(u_1-\hat\theta_1\right)^2+\left(u_2-\hat\theta_2\right)^2}, \qquad (4)$$

where $\theta_1 \in (l_1, u_1)$ and $\theta_2 \in (l_2, u_2)$ and the confidence interval for $\theta_1-\theta_2$ is $\theta_1+(-\theta_2)$. Note that a confidence interval for $-\theta_2$ is $\left(-u_2, l_2\right)$, replacing $\theta_2 \in \left(-u_2, l_2\right)$ in (4), we therefore have a confidence interval for $\theta_1-\theta_2$ which is

$$L_1 = \hat\theta_1-\hat\theta_2-\sqrt{\left(\hat\theta_1-l_1\right)^2+\left(u_2-\hat\theta_2\right)^2},\ U_1 = \hat\theta_1-\hat\theta_2+\sqrt{\left(u_1-\hat\theta_1\right)^2+\left(\hat\theta_2-l_2\right)^2}, \qquad (5)$$

Now, we set $\theta_1=\kappa_1=\sigma_1/\mu_1, \theta_2=\kappa_2=\sigma_2/\mu_2, \hat\theta_1=\hat\kappa_1=S_1/\bar{X}, \hat\theta_2=\hat\kappa_2=S_2/\bar{Y}$ and putting these in (5), we then have confidence intervals for $\theta_1-\theta_2$ as follows,

$$CI_G = \left(L_1, U_1\right)$$

Donner and Zou [1] also proposed the confidence interval for a ratio of parameters, θ_3/θ_4, by using the method of variance estimates recovery in 2010. They constructed the confidence interval for θ_3/θ_4 by using the confidence limits $\left(l_i, u_i\right), i=3,4$ where $\left(l_i, u_i\right)$ are the $100(1-\alpha)\%$ two-sided confidence intervals for θ_i. Then, the confidence interval for θ_3/θ_4 is given by $\left(\eta_L, \eta_U\right)$ where

$$\eta_L = \frac{\hat\theta_3\hat\theta_4-\sqrt{\left(\hat\theta_3\hat\theta_4\right)^2-l_3u_4\left(2\hat\theta_3-l_3\right)\left(2\hat\theta_4-u_4\right)}}{u_4\left(2\hat\theta_4-u_4\right)} \qquad (6)$$

and

$$\eta_U = \frac{\hat\theta_3\hat\theta_4+\sqrt{\left(\hat\theta_3\hat\theta_4\right)^2-u_3l_4\left(2\hat\theta_3-u_3\right)\left(2\hat\theta_4-l_4\right)}}{l_4\left(2\hat\theta_4-l_4\right)}. \qquad (7)$$

Following Donner and Zou [1], the confidence interval for $\delta = \left(\sigma_1/\mu_1\right) - \left(c^{-1}\sigma_1/\mu_2\right)$ is starting from the following:

A confidence interval for $\theta_1 = \sigma_1/\mu_1$ which is, see e.g. Mahmoudvand and Hassani [3],

$$l_1, u_1 = \left(\frac{\hat{\kappa}_1}{2 - c_n + z_{1-\alpha/2}\sqrt{1-c_n^2}}, \frac{\hat{\kappa}_1}{2 - c_n - z_{1-\alpha/2}\sqrt{1-c_n^2}} \right), c_n = \sqrt{\frac{2}{n-1}} \frac{\Gamma(n/2)}{\Gamma((n-1)/2)},$$

$\hat{\theta}_1 = \hat{\kappa}_1, \hat{\kappa}_1 = S_1 / \bar{X}$.

Following with a confidence interval for $c^{-1}\sigma_1 / \mu_2$ which is

$l_2, u_2 = \eta_L, \eta_U$, $\theta_3 = c^{-1}\sigma_1, \theta_4 = \mu_2$ and confidence intervals for $\theta_3 = c^{-1}\sigma_1, \theta_4 = \mu_2$ are respectively

$$l_3, u_3 = \left(c^{-1}\sqrt{\frac{(n-1)S_1^2}{\chi^2_{1-\alpha/2,(n-1)}}} \le c^{-1}\sigma_1 \le c^{-1}\sqrt{\frac{(n-1)S_1^2}{\chi^2_{\alpha/2,(n-1)}}} \right) \text{ and}$$

$l_4, u_4 = \bar{Y} - z_{1-\alpha/2} S_2 / \sqrt{m} \le \mu_2 \le \bar{Y} + z_{1-\alpha/2} S_2 / \sqrt{m}$, $\hat{\theta}_2 = c^{-1}S_1 / \bar{Y}, \hat{\theta}_3 = c^{-1}S_1, \hat{\theta}_4 = \bar{Y}$, putting these l_3, u_3 , l_4, u_4 in (6) and (7), we then have $l_2, u_2 = \eta_L, \eta_U$, and finally putting l_1, u_1 , l_2, u_2 in (5) , we then have a confidence interval for $\delta = \sigma_1 / \mu_1 - c^{-1}\sigma_1 / \mu_2$. This confidence interval is called CI_{rov1}.

The confidence interval for $\eta = \frac{\sigma_1 / \mu_1}{\sigma_2 / \mu_2} = \frac{\mu_2}{\mu_1} . \frac{\sigma_1}{\sigma_2} = \sqrt{\frac{n}{m}} \frac{\mu_2}{\mu_1} = c \frac{\mu_2}{\mu_1}, c^2 = \frac{n}{m}$ is therefore constructed from (6) and (7) by replacing

$$l_3 = c\left(\bar{Y} - z_{1-\alpha/2} \frac{S_2}{\sqrt{m}} \right), u_3 = c\left(\bar{Y} + z_{1-\alpha/2} \frac{S_2}{\sqrt{m}} \right) \text{ and } l_4 = \bar{X} - z_{1-\alpha/2} \frac{S_1}{\sqrt{n}}, u_4 = \bar{X} - z_{1-\alpha/2} \frac{S_1}{\sqrt{n}},$$

$\hat{\theta}_3 = c\bar{Y}, \hat{\theta}_4 = \bar{X}$. This confidence interval is called CI_{rov2}.

C. The Confidence Interval for θ Based on Fieller's Method

We now applied Fieller [3]'s method to construct the confidence interval for $\eta = c\mu_2 / \mu_1$ as follow,

$$T = \frac{(c\bar{Y} - \theta\bar{X}) - (\mu_1 - c\mu_2)}{\sqrt{\left(\frac{\theta^2}{n} + \frac{c}{m} \right) S^2}}, S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2 + \sum_{j=1}^{m} (Y_i - \bar{Y})^2}{n+m}, \tag{8}$$

a statistic $\sqrt{(n+m-2)/(n+m)}T$ follows t-distribution with $n+m-2$ degrees of freedom.
A 100(1-α) % confidence interval for θ, by solving (8), which is

$$CI_{F^{**}} = \left[\theta : \sqrt{\frac{(n+m-2)}{n+m}} |T| < t_{1-\alpha/2} \right] = \left[\theta : |c\bar{y} - \theta\bar{x}| \le \sqrt{\frac{(n+m)(c/m + \theta^2/n)}{n+m-2}} S^2 \right]$$

where $t_{1-\alpha/2}$ is the $(1-\alpha/2)$ *th* percentile of *t*-distribution with *n*+*m*-2 degrees of freedom. Note that T_1 has an exact *t*-distribution with *n* + *m*- 2 degrees of freedom. The confidence interval $CI_{F^{**}}$ can be transformed to

$$CI_F = \left[\frac{c\bar{Y}}{\bar{X}} - d\frac{\sqrt{w}}{\bar{X}}, \frac{c\bar{Y}}{\bar{X}} + d\frac{\sqrt{w}}{\bar{X}} \right], d = t_{1-\alpha/2,n+m-2}, w = \sqrt{\frac{(n+m)(\mathrm{c/\,m} + \hat{\theta}^2/\mathrm{n})S^2}{n+m-2}}, \hat{\theta} = \frac{\bar{Y}}{\bar{X}}.$$

III. SIMULATION STUDIES

In this section, we evaluated the performance of all confidence intervals : $CI_{gci1}, CI_{rov1}, CI_F, CI_{gci2}, CI_{rov2}$ via Monte Carlo simulation. The simulation studies are carried out to evaluate coverage probabilities and expected lengths of each confidence interval. The number of simulation runs is equal to 10,000 and the design values of the nominal confidence level is 0.95. For the generalized pivotal approach, in each of 10,000 simulations, 500 pivotal quantities are used. All data are generated from normal distribution with means $\mu_1 = \mu_2 = 1$ and standard deviations $\sigma_1 = 1, \sigma_2 = 1$, c=1,2,4,8,16 = and sample sizes n,m =10, 20,40 and 100. The results were performed using a program written in the R statistic software (The R Development Core Team, [4]). The simulation results are shown in Tables 1-2. As seen in Table 1, the confidence interval CI_{gci1} has coverage probabilities approximately nominal confidence level (0.95) in all cases but the coverage probabilities of the confidence interval CI_{rov1} are slightly decreasing from the nominal value as c is increasing. This means the coverage probability of the confidence interval CI_{rov1} is not stable, i.e. it is depending on the values of c. We therefore choose the confidence interval CI_{gci1} for the ratio of CVs with a known ratio of variances, $\eta = \theta_1/\theta_2$. From Table 2, the confidence interval using the Filler's method CI_F, has produces a wider confidence interval as usual since coverage probabilities of this confidence interval are over the nominal level at 0.95. We therefore do not consider the confidence interval CI_F. The confidence interval CI_{gci2} have much coverage probabilities than that the confidence interval from MOVER method, CI_{rov2} for all sample sizes and values of c. We therefore choose the confidence interval CI_{gci2} for the difference between CVs when we know a ratio of variances.

Table 1. Simulation results of 95% confidence intervals for the ratio of two independent coefficients of variation in normal distribution

n,m	c	GCI		MOVER		Ratio
		COV	Ex. Length	COV	Ex. Length	(MOVER/GCI)
(10,10)	1	**0.9524**	1.9095	0.9982	0.9754	0.5108
	2	**0.9382**	0.2344	0.9863	0.6992	2.9829
	4	**0.9497**	0.4532	0.9525	0.5897	1.3011
	8	**0.9542**	0.6940	0.9256	0.5496	0.7919
	16	**0.9546**	0.4397	0.9045	0.5419	1.2324
(10,20)	1	**0.9507**	0.2573	0.9992	0.8981	3.4904
	2	**0.9440**	0.1966	0.9859	0.6687	3.4014
	4	**0.9482**	0.3370	0.9544	0.5756	1.7080
	8	**0.9536**	1.6735	0.9238	0.5467	0.3266
	16	**0.9594**	0.9662	0.9057	0.5433	0.5623
(20,10)	1	**0.9490**	0.5249	0.9930	0.6615	1.2600
	2	**0.9455**	0.4308	0.9850	0.4560	1.0583
	4	**0.9464**	0.4974	0.9517	0.3789	0.7617
	8	**0.9458**	0.3702	0.9210	0.3513	0.9488
	16	**0.9521**	0.6224	0.9061	0.3446	0.5536
(20,40)	1	**0.9471**	0.4302	0.9996	0.5403	1.2558
	2	**0.9447**	0.6600	0.9873	0.4058	0.6148
	4	**0.9509**	0.3359	0.9518	0.3585	1.0672
	8	**0.9542**	0.4422	0.9275	0.3451	0.7805

	16	**0.9480**	0.3574	0.8986	0.3434	0.9607
(40,20)	1	**0.9494**	0.2106	0.9933	0.4168	1.9787
	2	**0.9455**	0.2436	0.9829	0.2931	1.2031
	4	**0.9472**	0.1952	0.9523	0.2475	1.2683
	8	**0.9481**	0.3081	0.9248	0.2344	0.7609
	16	**0.9526**	0.3373	0.9084	0.2307	0.6839
(40,40)	1	**0.9489**	0.1933	0.9984	0.3752	1.9406
	2	**0.9428**	0.2265	0.9861	0.2760	1.2181
	4	**0.9442**	0.2949	0.9478	0.2417	0.8194
	8	**0.9474**	0.2825	0.9204	0.2328	0.8241
	16	**0.9494**	0.3027	0.9085	0.2301	0.7602

n,m	c	GCI		MOVER		Ratio (MOVER/GCI)
		COV	Ex. Length	COV	Ex. Length	
(20,20)	1	**0.9465**	0.3104	0.9982	0.5768	1.8580
	2	**0.9441**	0.2985	0.9877	0.4195	1.4051
	4	**0.9476**	0.4294	0.9528	0.3627	0.8448
	8	**0.9437**	0.3933	0.9204	0.3461	0.8801
	16	**0.9500**	0.3161	0.9042	0.3427	1.0843
(50,50)	1	**0.9489**	0.2298	0.9982	0.3303	1.4375
	2	**0.9432**	0.2412	0.9840	0.2424	1.0051
	4	**0.9438**	0.3625	0.9489	0.2138	0.5897
	8	**0.9476**	0.2106	0.9210	0.2056	0.9765
	16	**0.9522**	0.2290	0.9095	0.2037	0.8897
(100,100)	1	**0.9517**	0.1658	0.9985	0.2261	1.3638
	2	**0.9470**	0.1905	0.9838	0.1674	0.8786
	4	**0.9443**	0.1664	0.9498	0.1481	0.8902
	8	**0.9464**	0.2155	0.9210	0.1427	0.6625
	16	**0.9527**	0.1581	0.9101	0.1417	0.8959

Table 2. Simulation results of 95% confidence intervals for the ratio of two independent coefficients of variation in normal distribution

(n,m)	c	Filler		GCI		MOVER	
(10,10)	1	0.9968	1.4486	**0.9324**	1.3459	0.8877	1.0275
	2	0.9976	2.9047	**0.9327**	2.7029	0.8846	2.0654
	4	0.9969	5.8039	**0.9355**	5.3996	0.8893	4.1368
	8	0.9966	11.6020	**0.9361**	10.9313	0.8918	8.1377
	16	0.9965	23.1812	**0.9316**	21.4523	0.8847	17.1837
(10,20)	1	0.9974	1.3270	**0.9335**	1.1719	0.8973	0.8845
	2	0.9985	2.6499	**0.9305**	2.3890	0.8917	1.8384
	4	0.9980	5.2802	**0.9340**	4.7409	0.8997	3.6834
	8	0.9979	10.6300	**0.9340**	9.6618	0.8946	5.6075
	16	0.9978	21.2037	**0.9312**	19.0954	0.8969	3.8131
(20,10)	1	0.9983	1.2889	**0.9302**	0.9115	0.8937	0.7721
	2	0.9990	2.5790	**0.9338**	1.8189	0.9014	1.5436
	4	0.9982	5.1536	**0.9344**	3.6573	0.9019	3.1016
	8	0.9988	10.3360	**0.9351**	7.3068	0.9008	6.1925
	16	0.9973	20.6481	**0.9328**	14.6065	0.8931	12.3813
(20,40)	1	0.9996	1.0665	**0.9404**	0.6254	0.9258	0.5711

	2	0.9996	2.1283	**0.9404**	1.2428	0.9253	1.1350
	4	0.9997	4.2666	**0.9418**	2.5008	0.9259	2.2855
	8	0.9995	8.5451	**0.9391**	5.0129	0.9242	4.5824
	16	0.9997	17.0835	**0.9381**	10.0070	0.9236	9.1612
(40,20)	1	0.9998	1.0533	**0.9378**	0.5829	0.9219	0.5404
	2	0.9999	2.1101	**0.9426**	1.1648	0.9246	1.0798
	4	0.9998	4.2199	**0.9407**	2.3383	0.9269	2.1676
	8	0.9999	8.4298	**0.9384**	4.6579	0.9233	4.3197
	16	0.9998	16.8480	**0.9409**	9.3221	0.9253	8.6371
(40,40)	1	0.9999	0.9480	**0.9410**	0.4688	0.9326	0.4476
	2	1.0000	1.8923	**0.9408**	0.9336	0.9319	0.8910
	4	0.9999	3.7917	**0.9443**	1.8774	0.9356	1.7926
	8	0.9999	7.5991	**0.9420**	3.7573	0.9342	3.5883
	16	0.9998	15.1987	**0.9444**	7.5116	0.9365	7.1789

(n,m)	c	Filler		GCI		MOVER	
(20,20)	1	0.9990	1.1544	**0.9382**	0.7225	0.9180	0.6524
	2	0.9995	2.3132	**0.9374**	1.4475	0.9169	1.3064
	4	0.9997	4.6236	**0.9418**	2.8911	0.9193	2.6108
	8	0.9991	9.2202	**0.9379**	5.7490	0.9183	5.1909
	16	0.9990	18.4932	**0.9417**	11.5339	0.9236	10.4191
(50,50)	1	1.0000	0.8917	**0.9430**	0.4129	0.9359	0.3979
	2	1.0000	1.7847	**0.9441**	0.8260	0.9378	0.7957
	4	1.0000	3.5697	**0.9490**	1.6555	0.9421	1.5942
	8	1.0000	7.1428	**0.9449**	3.3072	0.9389	3.1877
	16	1.0000	14.2753	**0.9427**	6.6108	0.9330	6.3733
(100,100)	1	1.0000	0.7437	**0.9519**	0.2874	0.9506	0.2795
	2	1.0000	1.4883	**0.9510**	0.5685	0.9464	0.5586
	4	1.0000	2.9699	**0.9493**	1.1360	0.9478	1.1150
	8	1.0000	5.9517	**0.9440**	2.2778	0.9409	2.2348
	16	1.0000	11.8993	**0.9498**	4.5506	0.9463	4.4682

IV. CONCLUSION

In this paper, the confidence intervals for the difference and the ratio of coefficients of variation of normal distribution with a known ratio of variances are studied. The performances of these confidence intervals were assessed in terms of coverage probabilities and expected lengths through simulation studies. The simulation results indicated that the GCI method produce the coverage probability close to the nominal confidence level, 0.95 for all sample sizes and values of c. Therefore our recommendation for the difference and the ratio of CVs when we know a ratio of variances is to use the GCI to construct the confidence intervals.

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