Confidence Intervals for the Median of Lognormal Distribution with Restricted Parameter Mean

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Abstract-- In this paper, confidence intervals for the single median and the ratio of medians of lognormal distribution with restricted means are proposed. We derived analytic expressions to find the coverage probability and the expected length of the proposed confidence interval.

Keywords-- Coverage probability, mean, expected length, Monte Carlo simulation, ratio of medians

I. INTRODUCTION

Many environmental variables exhibit skewed distributions that can be approximated by the lognormal distribution. For lognormal distribution, the mean is different value from the median and when one use statistics to describe the data to summary statistic, the median is appropriated for skewed distributions, Parkin and Robinson [6]. For example, Hirano *et al*. [1] reported that in a study of epiphytic bacterial population on leaf surfaces, the median was chosen as the relevant summary statistics. Parkin and Robinson [7] also proposed the confidence intervals for the median of lognormal distribution. Our aim in this paper is to derive analytic expressions to find the coverage probability and the expected length of the confidence intervals for the median and the ratio of medians of lognormal distribution when the parameter means are restricted or bounded. See for example, Mandelkern [2], Roe and Woodroofe [8], Wang [9-11] and Niwitpong [5] and the references cited in these articles.

II. CONFIDENCE INTERVAL FOR THE MEDIAN OF LOGNORMAL DISTRIBUTION

In this section, let $X_i = (X_{i1}, X_{i2} \cdots, X_{in_i})$, $i=1,2$ be a random variable having a lognormal distribution, and let μ_i and σ_i^2 denote the mean and variance of $\ln(X_i)$ respectively, so that $Y_i = \ln(X_i) \sim N(\mu_i, \sigma_i^2)$. The probability density function the lognormal distribution is

$$f(x_i, \mu_i, \sigma_i^2) = \begin{cases} \dfrac{1}{x_i \sigma_i \sqrt{2\pi}} \exp\left(-\dfrac{(\ln(x_i) - \mu_i)^2}{2\sigma_i^2} \right); & \text{for } x_i > 0 \\ 0 & ; \text{ otherwise.} \end{cases}$$

The median of lognormal distribution is $M(X_i) = \exp(\mu_i) = \theta_i$ and the ratio of medians of lognormal distribution is $\dfrac{M(X_1)}{M(X_2)} = \dfrac{\exp(\mu_1)}{\exp(\mu_2)} = \dfrac{\theta_1}{\theta_2} = \delta$, when

$$\bar{Y}_1 = n^{-1}\sum_{j=1}^{n} Y_{1j},\ \bar{Y}_2 = m^{-1}\sum_{k=1}^{m} Y_{2k},\ S_1^2 = (n-1)^{-1}\sum_{j=1}^{n} (Y_{1j} - \bar{Y}_1)^2,\ S_2^2 = (m-1)^{-1}\sum_{k=1}^{m} (Y_{2k} - \bar{Y}_2)^2$$

are respectively the means and the variances of $Y_i = \ln(X_i)$. We are interested in constructing confidence intervals for θ_i and δ when parameter means lie between $[a_i, b_i]$, $0 < a_i < b_i$.

The confidence interval of the single median θ_i of lognormal distribution is constructed using $\ln(\theta_i) = \mu_i$. The confidence interval for each of $\ln(\theta_i) = \mu_i$ is carried out. Also the coverage probability and expected length of each confidence interval are derived. We then find the true coverage probability and expected length of the median of log normal distribution by transforming back using exponential function.

Assume that σ_1^2 is unknown, it is well known that the confidence interval for μ_1 is

$$CI_{\mu 1} = \left[\bar{Y}_1 - z_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}, \bar{Y}_1 + z_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}} \right]$$ when $S_1^2 = (n-1)^{-1}\sum_{j=1}^{n} \left(Y_{1j} - \bar{Y}_1 \right)^2$ and $z_{1-\alpha/2}$

is an upper percentiles of the standard normal distribution and for small sample sizes, it can be proved that the confidence interval for μ_1 is

$$CI_{\mu 1} = \left[\bar{Y}_1 - t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}, \bar{Y}_1 + t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}} \right]$$ where $t_{1-\alpha/2}$ is an upper percentiles of t-distribution with n-1

degrees of freedom. The confidence interval for $M\left(X_i\right) = \exp\left(\mu_i\right) = \theta_i$ is exp($CI_{\mu 1}$) . The coverage probability and the expected length of $CI_{\mu 1}$ is proved in Theorem 1 below.

The confidence interval of the ratio of medians of lognormal distribution is $\frac{M\left(X_1\right)}{M(X_2)} = \frac{\exp\left(\mu_1\right)}{\exp(\mu_2)} = \frac{\theta_1}{\theta_2} = \delta.$

Therefore, ln(δ) = $\mu_1 - \mu_2$, the confidence interval for $\mu_1 - \mu_2$ is well known in 3 cases.

a) Case I, when σ_1^2 and σ_2^2 are known

The pivotal quantity for this case is

$$Z = \frac{\bar{Y}_1 - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}}$$

when Z is a standard normal distribution. The confidence interval for $\mu_1 - \mu_2$ is

$$CI_1 = \left[(\bar{Y}_1 - \bar{Y}_2) - Z_{1-\alpha/2}\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}, (\bar{Y}_1 - \bar{Y}_2) + Z_{1-\alpha/2}\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}} \right]$$

b) Case II, when σ_1^2 and σ_2^2 are unknown but $\sigma_1^2 = \sigma_2^2$

The pivotal quantity for this case is

$$T_1 = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}}$$

when T_1 is the t-distribution with $n+m-2$ degrees of freedom,

and S_p^2 is the pooled estimate of the sample variance;

$$\frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}.$$

A 100(1-α) % confidence interval for $\ln(\delta) = \mu_1 - \mu_2$ is

$$CI_2 = \left[(\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2, n1+n2-2} S_p\sqrt{\frac{1}{n} + \frac{1}{m}}, (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2, n1+n2-2} S_p\sqrt{\frac{1}{n} + \frac{1}{m}} \right]$$

when $t_{1-\alpha/2}$ is a $(1-\alpha/2)100th$ percentile of the *t*-distribution with n1+n2-2 degrees of freedom.

c) Case III, when σ_1^2 and σ_2^2 are unknown but $\sigma_1^2 \neq \sigma_2^2$

The pivotal quantity for this case is

$$T_2 = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}}$$

when T_2 is an approximated t-distribution with

$$\nu = \frac{(A+B)}{\frac{A^2}{n-1} + \frac{B^2}{m-1}}, A = \frac{S_1^2}{n}, B = \frac{S_2^2}{m}$$

degrees of freedom.

A 100(1-α) % confidence interval for $\ln(\delta) = \mu_1 - \mu_2$ is

$$CI_3 = \left[(\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2,\nu}\sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}, (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2,\nu}\sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}} \right]$$

A final process is to use exponential function to transform CI_1, CI_2, CI_3 back to δ, we then have $\exp\ CI_1$, $\exp\ CI_2$ and $\exp\ CI_3$ respectively.

III. COVERAGE PROBABILITY AND EXPECTED LENGTH OF EACH CONFIDENCE INTERVAL

In this section, we present only the coverage probability and the expected length of confidence intervals $CI_{\mu 1}$, CI_2 and CI_3, since a confidence interval CI_1 is never use in practice.

Theorem 1 (Niwitpong [4]) The coverage probability and expected length of $CI_{\mu 1}$ are respectively,

$$E\ \Phi(A) - \Phi(-A) \text{ and } E(CI_{\mu 1}) = \frac{2c}{\sqrt{n}} \frac{\sqrt{2}\Gamma(n/2)}{\sqrt{n-1}\Gamma((n-1)/2)}\sigma_1, \text{ where } A = cS_1/\sigma_1, \Phi(\bullet) \text{ is a}$$

standard normal function and *c* is $t_{1-\alpha/2}$, an upper $(1-\alpha/2)100\%$ percentiles of the *t*-distribution with n-1 degrees of freedom,

Proof See proof of this theorem in Niwitpong [4]**.**

Theorem 2 (Niwitpong [4]) The coverage probability and the expected length of CI_2 when the variances are equal, $\sigma_1^2 = \sigma_2^2$, are respectively

$$E[\Phi(W_1) - \Phi(-W_1)] \text{ and } 2^{3/2} d\sigma_1 \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{1}{n_1 + n_2 - 2}} \frac{\Gamma(\frac{n+m-1}{2})}{\Gamma(\frac{n+m-2}{2})}$$

where $W_1 = d_1\sigma_1^{-1}S_p, d_1 = t_{1-\alpha/2,n+m-2}$, $\Gamma[.]$ is the gamma function and $\Phi[.]$ is the cumulative distribution function of *N* (0, 1).

Proof. Similarly proof to Niwitpong [4].

Theorem 3 (Niwitpong [4]) The coverage probability and the expected length of CI_3 are respectively

$$E[\Phi(W)-\Phi(-W)] \text{ and } \begin{cases} 2d\sigma_1\sigma_2(nm)^{-1/2}\delta\sqrt{r_1}F\left[\frac{-1}{2},\frac{m-1}{2},\frac{m+n-2}{2},\frac{r_1-r_2}{r_1}\right], \text{if } r_2<2r_1 \\ 2d\sigma_1\sigma_2(n\text{m})^{-1/2}\delta\sqrt{r_2}F\left[\frac{-1}{2},\frac{n-1}{2},\frac{n+m-2}{2},\frac{r_2-r_1}{r_2}\right], \text{if } 2r_1\le r_2 \end{cases}$$

where

$$W_2=\frac{d\sqrt{\frac{S_1^2}{n}+\frac{S_2^2}{m}}}{\sqrt{\sigma_1^2n^{-1}+\sigma_2^2m^{-1}}}, d=t_{1-\alpha/2,v}, \delta=\frac{\sqrt{2}\Gamma\left(\frac{n+m-1}{2}\right)}{\Gamma\left(\frac{n+m-2}{2}\right)}$$

$$r_1=\frac{m}{\sigma_2^2(n-1)}, r_2=\frac{n}{\sigma_1^2(\text{m}-1)}, \nu=\frac{(A+B)}{\frac{A^2}{n-1}+\frac{B^2}{m-1}}, A=\frac{S_1^2}{n}, B=\frac{S_2^2}{m} \text{ and}$$

$E(.)$ is an expectation operator, $F(a; b; c; k)$ is the hypergeometric function,

defined by $F(a; b; c; k) = 1+\frac{ab}{c}\frac{k}{1!}+\frac{a(a+1)b(b+1)}{c(c+1)}\frac{k^2}{2!}+\dots$ where $|k| < 1$, $\Gamma[.]$ is the gamma function and $\Phi[.]$ is the cumulative distribution function of $N(0, 1)$.

Proof. Similarly proof to Niwitpong [4].

IV. CONFIDENCE INTERVAL FOR THE MEDIAN OF LONGNORMAL DISTRIBUTION WITH RESTRICTED MEANS

Wang [9-11] proposed to construct the confidence interval for θ when $0<a<\theta<b$ and this confidence interval is given by

$$CI_r = \left[\max\left(a,\hat{\theta}-z_{1-\alpha/2}\sqrt{Var(\hat{\theta})}\right), \min\left(b,\hat{\theta}-z_{1-\alpha/2}\sqrt{Var(\hat{\theta})}\right)\right].$$

Similarly to Wang [9-11], we now propose to construct confidence intervals for $CI_{\mu1}$, CI_2 and CI_3.

There are the following,

$$CI_{\mu1R}=\left[\max\left(\text{a}_1,\bar{Y}_1-t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}\right),\min\left(b_1,\bar{Y}_1+t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}\right)\right], 0<a_1<\mu_1<b_1,$$

$$CI_{2R}=\left[\max\left(\text{a}_1-\text{b}_2,(\bar{\text{Y}}_1-\bar{Y}_2)-t_{1-\alpha/2}S_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right),\min\left(\text{b}_1-\text{a}_2,(\bar{\text{Y}}_1-\bar{Y}_2)+t_{1-\alpha/2}S_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right)\right],$$

$$CI_{3R}=\left[\max\left(\text{a}_1-\text{b}_2,(\bar{\text{Y}}_1-\bar{Y}_2)-t_{1-\alpha/2,\nu}\sqrt{\frac{S_1^2}{n}+\frac{S_2^2}{m}}\right),\min\left(\text{b}_1-\text{a}_2,(\bar{\text{Y}}_1-\bar{Y}_2)+t_{1-\alpha/2,\nu}\sqrt{\frac{S_1^2}{n}+\frac{S_2^2}{m}}\right)\right].$$

IV. COVERAGE PROBABILITY AND EXPECTED LENGTH OF EACH CONFIDENCE INTERVAL WITH RESTRICTED MEANS

In this section, we are interested to construct confidence intervals $CI_{\mu1R}$, CI_{2R} and CI_{3R} when the parameters are bounded.

There are four possible results for a confidence interval $CI_{\mu 1R}$ when $0 < a_1 < \mu_1 < b_1$:

Case I, $a_1 > \bar{Y}_1 - t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}$ and $b_1 > \bar{Y}_1 + t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}$ then $CI_{\mu 1R}$ is reduced to

$$CI_{\mu 1a} = \left[a_1, \bar{Y}_1 + t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}} \right], 0 < a_1 < \mu_1 < b_1$$

Case II, $a_1 > \bar{Y}_1 - t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}$ and $b_1 < \bar{Y}_1 + t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}$ then $CI_{\mu 1R}$ is reduced to

$$CI_{\mu 1b} = a_1, \mathrm{b}_1 \ , 0 < a_1 < \mu_1 < b_1 .$$

Case III, $a_1 < \bar{Y}_1 - t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}$ and $b_1 > \bar{Y}_1 + t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}$ then $CI_{\mu 1R}$ is reduced to

$$CI_{\mu 1c} = \left[\bar{Y}_1 - t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}, \bar{Y}_1 + t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}} \right], 0 < a_1 < \mu_1 < b_1 .$$

Case IV, $a_1 < \bar{Y}_1 - t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}$ and $b_1 < \bar{Y}_1 + t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}$ then $CI_{\mu 1R}$ is reduced to

$$CI_{\mu 1d} = \left[\bar{Y}_1 - t_{1-\alpha/2}\sqrt{\frac{S_1^2}{n}}, \mathrm{b}_1 \right], 0 < a_1 < \mu_1 < b_1 .$$

The coverage probabilities and expected lengths of these 4 confidence intervals are already proved see e.g. Niwitpong [3].

Similarly to a confidence interval $CI_{\mu 1R}$, we can do four possible results for the confidence intervals CI_{2R} and CI_{3R}. For the confidence interval CI_{2R}, we have four possible results which are

Case Ia, if $\mathrm{a}_1 - \mathrm{b}_2 > (\bar{\mathrm{Y}}_1 - \bar{\mathrm{Y}}_2) - t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$ and $\mathrm{b}_1 - \mathrm{a}_2 > (\bar{\mathrm{Y}}_1 - \bar{\mathrm{Y}}_2) + t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$

Then a confidence interval CI_{2R} is reduced to

$$CI_{2Ra} = \left[\mathrm{a}_1 - \mathrm{b}_2, (\bar{\mathrm{Y}}_1 - \bar{\mathrm{Y}}_2) + t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}} \right],$$

Case IIa, if $\mathrm{a}_1 - \mathrm{b}_2 > (\bar{\mathrm{Y}}_1 - \bar{\mathrm{Y}}_2) - t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$ and $\mathrm{b}_1 - \mathrm{a}_2 < (\bar{\mathrm{Y}}_1 - \bar{\mathrm{Y}}_2) + t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$

Then a confidence interval CI_{2R} is reduced to

$$CI_{2Ra} = \mathrm{a}_1 - \mathrm{b}_2, \mathrm{b}_1 - \mathrm{a}_2 \ ,$$

Case IIIa, if $a_1 - b_2 < (\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$ and $b_1 - a_2 > (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$

Then a confidence interval CI_{2R} is reduced to

$$CI_{2Ra} = \left[(\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}, (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}} \right],$$

Case IVa, if $a_1 - b_2 > (\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$ and $b_1 - a_2 < (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$

Then a confidence interval CI_{2R} is reduced to

$$CI_{2Ra} = \left[(\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2} S_p \sqrt{\frac{1}{n} + \frac{1}{m}}, b_1 - a_2 \right].$$

The coverage probabilities and expected lengths of these 4 confidence intervals are simple and based on Theorem 3. So we skip the proof of these Theorems

Similarly to the confidence interval CI_{2R}, we will have four possible results for the confidence interval CI_{3R}:

Case Ib, if $a_1 - b_2 > (\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}$ and $b_1 - a_2 > (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}$

Then a confidence interval CI_{3R} is reduced to

$$CI_{3Ra} = \left[a_1 - b_2, t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}} \right],$$

Case IIb, if $a_1 - b_2 > (\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}$ and $b_1 - a_2 < (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}$

Then a confidence interval CI_{3R} is reduced to

$$CI_{3Ra} = a_1 - b_2, b_1 - a_2 \quad ,$$

Case IIb, if $a_1 - b_2 < (\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}$ and $b_1 - a_2 > (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}$

Then a confidence interval CI_{3R} is reduced to

$$CI_{3Ra} = \left[(\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}, (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}} \right],$$

Case IIb, if $a_1 - b_2 < (\bar{Y}_1 - \bar{Y}_2) - t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}$ and $b_1 - a_2 < (\bar{Y}_1 - \bar{Y}_2) + t_{1-\alpha/2,v} \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}$

Then a confidence interval CI_{3R} is reduced to

$$CI_{3Ra}=\left[(\bar{Y}_1-\bar{Y}_2)-t_{1-\alpha/2,\nu}\sqrt{\frac{S_1^2}{n}+\frac{S_2^2}{m}},b_1-a_2\right].$$

The coverage probabilities and expected lengths of these 4 confidence intervals are simple and based on Theorem 3. So we skip the proof of these Theorems

V.CONCLUSIONS

In this paper, we derived the coverage probability and the expected length of $CI_{\mu 1}$, $CI_2, CI_3, CI_{\mu 1R}, CI_{2R}$ and CI_{3R}. The coverage probabilities of these confidence intervals approach $1-\alpha$, when α is a level of significance and for large sample sizes. The expected lengths for each interval, shown in Theorems 1,2 and 3, can be compared. So we do not need to use the simulation to show the results.

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