A Topological Approach to Soft Covering Approximation Space

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Abstract — *Theories of rough sets and soft sets are powerful mathematical tools for modelling various types of vagueness. Hybrid model combining a rough set with a soft set which is called soft rough set proposed by Feng et al. [3] in 2010. In this paper, we study soft covering based rough sets from the topological view. We present under which conditions soft covering lower approximation operation become interior operator and the soft covering upper approximation become closure operator. Also some new methods for generating topologies are obtained. Finally, we study the relationship between concepts of topology and soft covering lower and soft covering upper approximations.*

Keywords — *Soft set, rough set, soft covering based rough set, topology*

I. Introduction

Mathematics is based on exact concepts and there is not vagueness for mathematical concepts. For this reason researchers need to define some new concepts for vagueness. The most successful approach is exactly Zadeh's fuzzy set [16] which is based on membership function. This theorical approach is used in several areas as engineering, medicine, economics and etc. Pawlak [11] initiated rough set theory in 1982 as a tool for uncertainty and imprecise data. The theory is based on partition or equivalence relation, which is rather strict. Covering based rough set [2, 13] is an important extension of rough sets. Compared with rough sets, it often gives a more reasonable description to a subset of the universe. In recent years, covering based rough set theory has attracted more attentions. The studies of Zhu et al. [17, 18, 19, 20] are fundamental and significant. In 1999 Molodtsov [10] gave soft set theory as a new tool for vagueness and showed in his paper that soft set theory can be applied to several areas. The hybrid models like fuzzy soft set [14], rough soft set [3], soft rough set [3] took attention from researchers. Feng et al. investigated the concept of soft rough set [3] which is a combination of soft set and rough set. It is known that the equivalence relation is used to form the granulation structure of the universe in the rough set model and also the soft set is used to form the granulation structure of the universe in the soft rough set model.

Topology is a branch of mathematics, whose concepts exist not only in almost all branches of mathematics, but also in many real life applications. Topology is also a mathematical tool to study rough sets [5, 6, 7, 8, 9]. It should be noted that the generation of topology by relation and the representation of topological concepts via relation will narrow the gap between topology and its applications.

The remaining part of this paper is arranged as follows:

In section 3, we give a new concept called as soft covering based rough sets and its basic properties. Also we investigate the conditions under which the soft covering lower and upper approximation operations are also interior and closure operators, respectively. In section 4, we discuss methods of setting up topology in soft covering approximation space. The relationship between concepts of topology and soft covering lower and upper approximations are studied in section 5. and the special condition of soft covering approximation space is investigated in section 6.

II. Preliminaries

In this section, we introduce the fundamental ideas behind rough sets, soft sets and topological spaces.

First, we recall some concepts and properties of the Pawlak's rough sets.

Definition 2.1 [11]:

Let U be a finite set and R be an equivalence relation on U. Then the pair (U, R) is called a Pawlak approximation space. R generates a partition $U/R=\{Y_1,Y_2,...,Y_m\}$ on U where $Y_1,Y_2,...,Y_m$ are the equivalence classes generated by the equivalence relation R. In the rough set theory, these are also called elementary sets of R. For any $X\subseteq U$, we can describe X by the elementary sets of R and the two sets:

$$R_-(X) = \cup\{Y_i\}\in U/R : Y_i\subseteq X\},$$
$$R^-(X) = \cup\{Y_i\in U/R : Y_i\cap X\neq\emptyset\}$$

which are called the lower and the upper approximation of X, respectively. In addition,

$$POS_R(X) = R_-(X),$$
$$NEG_R(X) = U-R^-(X),$$
$$BND_R(X) = R^-(X)-R_-(X)$$

are called the positive, negative and boundary regions of X, respectively.

Now, we are ready to give the definition of rough sets:

Definition 2.2 [12]:
Let (U,R) be a Pawlak approximation space. A subset $X \subseteq U$ is called definable (crisp) if $R_-(X)=R^-(X)$; in the opposite case, i.e., if $BND_R(X) \neq \emptyset$, X is said to be rough(or inexact). Any pair of the form $R(X)=(R_-(X),R^-(X))$ is called a rough set of X.

Let U be an initial universe set and E be the set of all possible parameters with respect to U. Usually, parameters are attributes, characteristics or properties of the objects in U. The notion of a soft set is defined as follows:

Definition 2.3 [10]:
A pair G=(F,A) is called a soft set over U, where $A \subseteq E$ and $F:A \to P(U)$ is a set-valued mapping.

Theorem 2.1 [1]:
Every rough set may be considered as a soft set.

The following result indicates that soft sets and binary relations are closely related.

Theorem 2.2 [3]:
Let G=(F,A) be a soft set over U. Then G induces a binary relation $R_G \subseteq A \times U$, which is defined by

$$(x,y) \in R_G \Leftrightarrow y \in F(x)$$

where $x \in A$, $y \in U$. Conversely, assume that R is a binary relation from A to U. Define a set valued mapping $F_R:A \to P(U)$ by

$$F_R(x) = \{y \in U:(x,y) \in R\},$$

where $x \in A$. Then $G_R=(F_R,A)$ is a soft set over U.

Definition 2.4 [3]:
Let G=(F,A) be a soft set over U. Then the pair $S=(U,R_G)$ is called a soft approximation space.

III. Soft Covering Based Rough Sets

We know that a soft set is determined by the set-valued mapping from a set of parameters to the powerset of the universe. In this section, we will use a special soft set and by using this soft set, we will establish a soft covering approximation space.

Definition 3.1 [3]:
A soft set G=(F,A) over U is called a full soft set if $\cup_{a \in A} F(a)=U$.

Definition 3.2 [3]:
A full soft set G=(F,E) over U is called a covering soft set if $F(e) \neq \emptyset$, $\forall e \in E$.

We discussed some properties of soft covering upper and lower approximations in our previous work [15]. Following definitions are given in this paper.

Definition 3.3:
Let G=(F,E) be a covering soft set over U. The ordered pair $S=(U,C_G)$ is called a soft covering approximation space.

Definition 3.4:
Let $S=(U,C_G)$ be a soft covering approximation space, for any $x \in U$, the soft minimal description of x is defined as following:

$$Md_S(x)=\{F(e):e \in E \wedge x \in F(e) \wedge (\forall a \in E \wedge x \in F(a) \subseteq F(e) \Rightarrow F(a)=F(e))\}.$$

Definition 3.5:
Let $S=(U,C_G)$ be a soft covering approximation space. For a set $X \subseteq U$, the soft covering lower and upper approximations are respectively defined as

$$S_-(X) = \cup\{F(e):e \in E \wedge F(e) \subseteq X\}$$
$$S^-(X) = S_-(X) \cup \{Md_S(x):x \in X - S_-(X)\}.$$

In addition,

$$POS_S(X) = S_-(X)$$
$$NEG_S(X) = U - S^-(X)$$
$$BND_S(X) = S^-(X) - S_-(X)$$

are called the soft covering positive, negative and boundary regions of X, respectively.

Definition 3.6:
Let $S=(U,C_G)$ be a soft covering approximation space. A subset $X \subseteq U$ is called definable if $S_-(X)=S^-(X)$; in the opposite case, i.e., if $S_-(X) \neq S^-(X)$, X is said to be soft covering based rough set. The pair $(S_-(X),S^-(X))$ is called soft covering based rough set of X and it is showed that $X=(S_-(X),S^-(X))$.

Example 3.1:
Let $S=(U,C_G)$ be a soft covering approximation space, where U={a,b,c,d,e,f,g,h}, $E=\{e_1,e_2,e_3,e_4,e_5\}$, $F(e_1)=\{a,b\}$, $F(e_2)=\{b,c,d\}$, $F(e_3)=\{e,f\}$, $F(e_4)=\{g\}$ and $F(e_5)=\{g,h\}$. For $X_1=\{a,b,c\} \subseteq U$, since $S_-(X_1) \neq S^-(X_1)$, X_1 is a soft covering based rough set. For $X_2=\{e,f,g\} \subseteq U$, since $S_-(X_2)=S^-(X_2)$, X_2 is a definable set.

We give following two theorems in our previous work [15].

Theorem 3.1:

Let $G=(F,E)$ be a soft set over U, $S=(U,C_G)$ be a soft covering approximation space and $X,Y\subseteq U$. Then the soft covering lower and upper approximations have the following properties:

1. $S_-(U)=S^-(U)=U$
2. $S_-(\emptyset)=S^-(\emptyset)=\emptyset$
3. $S_-(X)\subseteq X\subseteq S^-(X)$
4. $X\subseteq Y\Rightarrow S_-(X)\subseteq S_-(Y)$
5. $S_-(S_-(X))=S_-(X)$
6. $S^-(S^-(X))=S^-(X)$
7. $\forall e\in E$, $S_-(F(e))=F(e)$
8. $\forall e\in E$, $S^-(F(e))=F(e)$

Theorem 3.2:

Let $G=(F,E)$ be a soft set over U, $S=(U,C_G)$ be a soft covering approximation space and $X,Y\subseteq U$. Then the soft covering lower and upper approximations do not have the following properties:

1. $S_-(X\cap Y)=S_-(X)\cap S_-(Y)$
2. $S^-(X\cup Y)=S^-(X)\cup S^-(Y)$
3. $X\subseteq Y\Rightarrow S^-(X)\subseteq S^-(Y)$
4. $S_-(X)=-(S^-(-X))$
5. $S^-(X)=-(S_-(-X))$
6. $S_-(-S_-(X))=-S_-(X)$
7. $S^-(-S^-(X))=-S^-(X)$

The symbol "-" denotes the complement of the set. The following examples show that the equalities mentioned above do not hold.

Example 3.2:

Let $S=(U,C_G)$ be a soft covering approximation space, where $U=\{a,b,c,d,e,f,g\}$, $E=\{e_1,e_2,e_3,e_4\}$, $F(e_1)=\{a,b,c\}$, $F(e_2)=\{b,c,d\}$, $F(e_3)=\{d,e\}$ and $F(e_4)=\{f,g\}$. Suppose that $X=\{a,b,c,d\}\subseteq U$ and $Y=\{d,e\}$. The properties 1, 4, 5, 6, 7 of Theorem 3.2 do not hold.

Example 3.3:

Let $S=(U,C_G)$ be a soft covering approximation space and (F,E) be a soft set given in the Example 3.2. Suppose that $X=\{a,b\}\subseteq U$ and $Y=\{c,d\}\subseteq U$. The property 2 of Theorem 3.2 does not hold.

Example 3.4:

Let $S=(U,C_G)$ be a soft covering approximation space and (F,E) be a soft set given in the Example 3.2. Suppose that $X=\{d\}\subseteq U$ and $Y=\{b,c,d\}\subseteq U$. The property 3 of Theorem 3.2 does not hold.

Now, we consider under which conditions soft covering lower and upper approximations satisfy properties 1, 2, 3 of Theorem 3.2.

The continuation of the paper, the parameter set E is supposed to be finite.

Proposition 3.1:

$S_-(X)=X$ if and only if X is a union of some elements of C_G. Similarly, $S^-(X)=X$ if and only if X is a union of some elements of C_G.

Theorem 3.3:

Let $S=(U,C_G)$ be a soft covering approximation space and $X,Y\subseteq U$. $S_-(X\cap Y)=S_-(X)\cap S_-(Y)$ if and only if $\forall e_1,e_2\in E$, $F(e_1)\cap F(e_2)$ is a finite union of elements of C_G.

Proof: $\Rightarrow$: Since $F(e_1)\cap F(e_2)=S_-(F(e_1))\cap S_-(F(e_2))=S_-(F(e_1)\cap F(e_2))$ and $S_-(F(e_1)\cap F(e_2))$ is a finite union of elements of C_G, $F(e_1)\cap F(e_2)$ is a finite union of elements of C_G.

$\Leftarrow$: By 4 of Theorem 3.1, it is easy to see that $S_-(X\cap Y)\subseteq S_-(X)\cap S_-(Y)$. Now we shall show that $S_-(X)\cap S_-(Y)\subseteq S_-(X\cap Y)$. Let $S_-(X)=F(e_1)\cup F(e_2)\cup\ldots\cup F(e_m)$ and $S_-(Y)=F(e_1')\cup F(e_2')\cup\ldots\cup F(E_n')$ where $e_i, e_j'\in E$, $1\le i\le m$, $1\le j\le n$. For any $1\le i\le m$ and $1\le j\le n$, $F(e_i)\cap F(e_j')\subseteq X\cap Y$ and $F(e_i)\cap F(e_j')$ is a finite union of elements of C_G, let us say $F(e_i)\cap F(e_j')=F(p_1)\cup\ldots\cup F(p_l)$ where $F(p_h)\in C_G$, $1\le h\le l$, so $F(p_h)\subseteq S_-(X\cap Y)$ for $1\le h\le l$. Thus $F(e_i)\cap F(e_j')\subseteq S_-(X\cap Y)$ for $1\le i\le m$ and $1\le j\le n$. From

$$S_-(X)\cap S_-(Y)=\cup_{i=1}^{m}\cup_{j=1}^{n}[F(e_i)\cap F(e_j')]$$

hence $S_-(X)\cap S_-(Y)\subseteq S_-(X\cap Y)$.

Theorem 3.4:

Let $S=(U,C_G)$ be a soft covering approximation space and $X,Y\subseteq U$. $X\subseteq Y\Rightarrow S^-(X)\subseteq S^-(Y)$ if and only if $\forall e_1,e_2\in E$, $F(e_1)\cap F(e_2)$ is a finite union of elements of C_G.

Proof: $\Rightarrow$: $S^-(F(e_1)\cap F(e_2))\subseteq S^-(F(e_1))=F(e_1)$ and $S^-(F(e_1)\cap F(e_2))\subseteq S^-(F(e_2))=F(e_2)$, so $S^-(F(e_1)\cap F(e_2))\subseteq F(e_1)\cap F(e_2)$. By property 3 of Theorem 3.1, $F(e_1)\cap F(e_2)\subseteq S^-(F(e_1)\cap F(e_2))$, so $F(e_1)\cap F(e_2)=S^-(F(e_1)\cap F(e_2))$. Hence, $F(e_1)\cap F(e_2)$ is a finite union of elements of C_G.

$\Leftarrow$: By the definition of soft covering upper approximation, $S^-(X)$ can be expressed as $S^-(X)=S_-(X)\cup F(e_1)\cup\ldots\cup F(e_m)$ where $y_i\in F(e_i)\not\subseteq X$ and $F(e_i)\in Md_S(y_i)$ for some $y_i\in X-S_-(X)$, $1\le i\le m$. It is obvious that $y_i\in Y$. If $y_i\in Y-S_-(Y)$, it is easy to see that $F(e_i)\subseteq S^-(Y)$, $1\le i\le m$. If $y_i\notin Y-S_-(Y)$, then $y_i\in S_-(Y)$. Thus, there exists a $F(e_j)\in C_G$ such that $y_i\in F(e_j)\subseteq S_-(Y)$. By the assumption of this Theorem, $F(e_i)\cap F(e_j)$ is a finite union of elements in C_G. Let us say $F(e_i)\cap F(e_j)=F(e_1)\cup\ldots\cup F(e_l)$ where $F(e_h)\in C_G$, $1\le h\le l$, so there exists $1\le j\le l$ such that $y_i\in F(e_j)$. Since $F(e_i)\in Md_S(y_i)$, $F(e_i)=F(e_j\})$, thus $F(e_i)\subseteq F(e_j)$. Therefore, $F(e_i)\subseteq S_-(Y)\subseteq S^-(Y)$, $1\le i\le m$. From property 3 and property 4 of Theorem 3.1, $S_-(X)\subseteq S_-(Y)\subseteq S^-(Y)$, so $S^-(X)\subseteq S^-(Y)$.

Theorem 3.5:
Let $S=(U,C_G)$ be a soft covering approximation space and $X,Y\subseteq U$. $X\subseteq Y\Rightarrow S^-(X)\subseteq S^-(Y)$ if and only if $S^-(X\cup Y)=S^-(X)\cup S^-(Y)$.

Proof: $\Rightarrow$: By the assumption of this Theorem, $S^-(X)\subseteq S^-(X\cup Y)$ and $S^-(Y)\subseteq S^-(X\cup Y)$, so $S^-(X)\cup S^-(Y)\subseteq S^-(X\cup Y)$. Now we shall show that $S^-(X\cup Y)\subseteq S^-(X)\cup S^-(Y)$. By property 3 of Theorem 3.1, $X\cup Y\subseteq S^-(X)\cup S^-(Y)$. By the assumption of this Theorem, $S^-(X\cup Y)\subseteq S^-(S^-(X)\cup S^-(Y))$. By Proposition 3.1, $S^-(S^-(X)\cup S^-(Y))=S^-(X)\cup S^-(Y)$, so $S^-(X\cup Y)\subseteq S^-(X)\cup S^-(Y)$.

$\Leftarrow$: If $X\subseteq Y$, $S^-(Y)=S^-(X\cup Y)=S^-(X)\cup S^-(Y)$, so $S^-(X)\subseteq S^-(Y)$.

Corollary 3.1:
Let $S=(U,C_G)$ be a soft covering approximation space and $X,Y\subseteq U$. $S^-(X\cup Y)=S^-(X)\cup S^-(Y)$ if and only if $\forall e_1,e_2\in E$, $F(e_1)\cap F(e_2)$ is a finite union of elements in C_G.

Proof: The proof is obvious by Theorem 3.4 and Theorem 3.5.

IV. SOME METHODS TO SET UP TOPOLOGIES DEFINED IN SOFT COVERING APPROXIMATION SPACE

Theorem 4.1:
Let U be a nonempty universe set and $S=(U,C_G)$ be a soft covering approximation space. For each $e_1,e_2\in E$, $F(e_1)\cap F(e_2)$ is a finite union of elements of C_G.

$$\tau=\{X\subseteq U:S_-(X)=X\}$$

be a collection of subsets of U. Then τ is called a topology over U.

Proof:
O_1) If $X=\emptyset$, then by Theorem 3.1, $S_-(\emptyset)=\emptyset$. Hence $\emptyset\in\tau$. If X=U, then by Theorem 3.1, $S_-(U)=U$. Hence $U\in\tau$.

O_2) Let for each $i\in I$, $A_i\in\tau$, i.e., $S_-(A_i)=A_i$. Then there exists a $j\in I$, $A_j\in\tau$ such that $A\subseteq\bigcup_{i\in I} A_i$. From Theorem 3.1, $S_-(A_j)\subseteq S_-(\bigcup_{i\in I} A_i)$. Since $A_j\in\tau$, $S_-(A_j)=A_j$. Hence $A_j\subseteq S_-(\bigcup_{i\in I} A_i)$. Since this property is satisfied for each $j\in I$, we get

$$\bigcup_{i\in I} A_i \subseteq S_-(\bigcup_{i\in I} A_i) \quad (1)$$

Also by Theorem 3.1, we know that

$$S_-(\bigcup_{i\in I} A_i)\subseteq \bigcup_{i\in I} A_i \quad (2)$$

From (1) and (2), we get $S_-(\bigcup_{i\in I} A_i)=\bigcup_{i\in I} A_i$. And so we conclude that $\bigcup_{i\in I} A_i\in\tau$.

O_3) Let $A,B\in\tau$. Hence we get $S_-(A)=A$ and $S_-(B)=B$. By Theorem 3.3, $S_-(A\cap B)=S_-(A)\cap S_-(B)=A\cap B$. Hence $S_-(A\cap B)=A\cap B$. Therefore $A\cap B\in\tau$.

Theorem 4.2:
Let U be a nonempty universe set and $S=(U,C_G)$ be a soft covering approximation space. For each $e_1,e_2\in E$, $F(e_1)\cap F(e_2)$ is a finite union of elements of C_G.

$$K=\{X\subseteq U:S^-(X)=X\}$$

be a collection of subsets of U. Then K is called a topology over U.

Proof:
C_1) If $X=\emptyset$, then by Theorem 3.1, $S^-(\emptyset)=\emptyset$. Hence $\emptyset\in K$. If X=U, then by Theorem 3.1, $S^-(U)=U$. Hence $U\in\tau$.

C_2) Let for each $i\in I$, $A_i\in K$, i.e., $S^-(A_i)=A_i$. Then there exists $j\in I$, $A_j\in K$ such that $\bigcap_{i\in I} A_i\subseteq A_j$. From Theorem 3.4, $S^-(\bigcap_{i\in I} A_i)\subseteq S^-(A_j)$. Since $A_j\in K$, $S^-(A_j)=A_j$. Hence $S^-(\bigcap_{i\in I} A_i)\subseteq A_j$. Since this property is satisfied for each $j\in I$, we get

$$S^-(\bigcap_{i\in I} A_i)\subseteq \bigcap_{i\in I} A_i \quad (3)$$

Also by Theorem 3.1, we know that

$$\bigcap_{i\in I} A_i \subseteq S^-(\bigcap_{i\in I} A_i) \quad (4)$$

From (3) and (4), we obtain that $S^-(\bigcap_{i\in I} A_i)=\bigcap_{i\in I} A_i$. Therefore $\bigcap_{i\in I} A_i\in K$.

C_3) Let $A,B\in K$. Hence $S^-(A)=A$ and $S^-(B)=B$. By Corollary 3.1, we obtain that $S^-(A\cup B)=S^-(A)\cup S^-(B)=A\cup B$. Hence $S^-(A\cup B)=A\cup B$. Therefore $A\cup B\in K$.

Remark 4.1:
Let U be a nonempty universe set and $S=(U,C_G)$ be a soft covering approximation space. We can set up a topology over U when we consider the soft covering of the universe as a subbase.

Example 4.1:
Let $S=(U,C_G)$ be a soft covering approximation space where $U=\{h_1,h_2,h_3,h_4,h_5\}$, $E=\{e_1,e_2,e_3\}$, $F(e_1)=\{h_1,h_2,h_3\}$, $F(e_2)=\{h_3,h_4\}$, $F(e_3)=\{h_4,h_5\}$. Then

$$S=C_G=\{\{h_1,h_2,h_3\},\{h_3,h_4\},\{h_4,h_5\}\}$$
$$\cap$$
$$\beta=\{\{h_1,h_2,h_3\},\{h_3,h_4\},\{h_4,h_5\},\{h_3\},\{h_4\}\}$$
$$\cap$$
$$\tau=\{\emptyset,U,\{h_1,h_2,h_3,h_4\},\{h_3,h_4,h_5\},\{h_1,h_2,h_3\},\{h_3,h_4\},\{h_4,h_5\},\{h_3\},\{h_4\}\}$$

V. RELATIONSHIP BETWEEN CONCEPTS OF TOPOLOGY AND SOFT COVERING LOWER AND UPPER APPROXIMATIONS

In soft covering based rough set theory the reference space is the soft covering approximation space. We will consider the soft covering of the universe as a subbase for topology and we will obtain the closure, the interior and the boundary of a set with respect to this topology, then we will compare these concepts with the soft covering upper approximation, the soft covering lower approximation and the soft covering boundary region of a set.

Proposition 5.1:

Let $S=(U,C_G)$ be a soft covering approximation space and $X\subseteq U$. The soft covering lower approximation is contained in the interior of a set defined by taking this soft covering as a subbase for topology.

Proof: Let C_G be a soft covering of the universe U, $X\subseteq U$ and $x\in S_{\_}(X)$. Then, $\exists F(e)\in S_{\_}(X)$ such that $x\in F(e)$. Since F(e) is an element of subbase for the topology defined on U then every $F(e)\in C_G$ is open hence $x\in\cup\{F(e)\subseteq U{:}F(e)\subseteq X \text{ open}\}$. Thus $x\in int(X)$ and $S_{\_}(X)\subseteq int(X)$.

Let C_G be a soft covering of the universe U, $X\subseteq U$ and $x\in S_{\_}(X)$. Then, $\exists F(e)\in S_{\_}(X)$ such that $x\in F(e)$. Since F(e) is an element of subbase for the topology defined on U then every $F(e)\in C_G$ is open hence $x\in\cup\{F(e)\subseteq U{:}F(e)\subseteq X \text{ open}\}$. Thus $x\in int(X)$ and $S_{\_}(X)\subseteq int(X)$.

Proposition 5.2:

Let $S=(U,C_G)$ be a soft covering approximation space and $X\subseteq U$. The soft covering upper approximation of X cannot be compared with the closure of X with respect to the topology induced by soft covering.

Corollary 5.1:

Let $S=(U,C_G)$ be a soft covering approximation space and $X\subseteq U$. The soft covering boundary region of X cannot be compared with the boundary of X with respect to the topology induced by soft covering.

Example 5.1:

Let $S=(U,C_G)$ be a soft covering approximation space, where $U=\{h_1,h_2,h_3,h_4,h_5\}$, $E=\{e_1,e_2,e_3,\}$, $F(e_1)=\{h_1,h_2,h_3\}$, $F(e_2)=\{h_3,h_4\}$, $F(e_3)=\{h_4,h_5\}$. Suppose that $X=\{h_2,h_3,h_4\}$, then $S_{\_}(X)=\{h_3,h_4\}$, $S^{-}(X)=\{h_1,h_2,h_3,h_4\}$, $BND_S(X)=\{h_1,h_2\}$ and by using the Example 4.1, we get $int(X)=\{h_3,h_4\}$, $cl(X)=U$, $Bnd(X)=\{h_1,h_2,h_5\}$. Thus we obtain, $S_{\_}(X)\subseteq int(X)$, $S^{-}(X)\subseteq cl(X)$ and $BND_S(X)\subseteq Bnd(X)$.

Also, suppose that $Y=\{h_1,h_4,h_5\}$, then $S_{\_}(Y)=\{h_4,h_5\}$, $S^{-}(Y)=U$, $BND_S(Y)=\{h_1,h_2,h_3\}$ and by using the Example 4.1, we get $int(Y)=\{h_4,h_5\}$, $cl(Y)=\{h_1,h_2,h_4,h_5\}$, $Bnd(Y)=\{h_1,h_2\}$. Thus we obtain, $S_{\_}(Y)\subseteq int(Y)$, $cl(Y)\subseteq S^{-}(Y)$ and $Bnd(Y)\subseteq BND_S(Y)$.

VI. SPECIAL CONDITION OF SOFT COVERING APPROXIMATION SPACE

Definition 6.1 [4]:

A soft set G=(F,E) over U is called a partition soft set if $\{F(e){:}e\in E\}$ forms a partition of U.

Theorem 6.1 [4]:

Let G=(F,E) be a partition soft set over U and P=(U,G) be a soft covering approximation space. Define an equivalence relation R on U by

$$(x,y)\in R\Leftrightarrow \exists e\in E,\ \{x,y\}\subseteq F(e)$$

for all $x,y\in U$. Then, for all $X\subseteq U$,

$$R_{\_}(X)=P_{\_}(X) \text{ and } R^{-}(X)=P^{-}(X).$$

Theorem 6.2:

Let $S=(U,C_G)$ be a soft covering approximation space and $X\subseteq U$. If G=(F,E) is a partition soft set then the soft covering upper approximation and the soft covering lower approximation of X are equal to the closure and the interior of the set with respect to the topology induced by this covering, respectively.

Proof: Let G=(F,E) be a partition soft set then $R_{\_}(X)=S_{\_}(X)$ and $R^{-}(X)=S^{-}(X)$. And we know that $R_{\_}(X)=int(X)$ and $R^{-}(X)=cl(X)$. Hence we conclude that $S_{\_}(X)=int(X)$ and $S^{-}(X)=cl(X)$.

Example 6.1:

Let G=(F,E) be a partition soft set and $S=(U,C_G)$ be a soft covering approximation space, where $U=\{h_1,h_2,h_3,h_4,h_5,h_6,h_7\}$, $E=\{e_1,e_2,e_3\}$, $F(e_1)=\{h_1,h_2\}$, $F(e_2)=\{h_3,h_4\}$ and $F(e_3)=\{h_5,h_6,h_7\}$. Then we get

$$S=C_G=\{\{h_1,h_2\},\{h_3,h_4\},\{h_5,h_6,h_7\}\}$$
$$\cap$$
$$\beta=\{\{h_1,h_2\},\{h_3,h_4\},\{h_5,h_6,h_7\}\}$$
$$\cap$$
$$\tau=\{\{h_1,h_2\},\{h_3,h_4\},\{h_5,h_6,h_7\}\}$$

Suppose that $X=\{h_1,h_2,h_3\}$, then $S_{\_}(X)=\{h_1,h_2\}$, $S^{-}(X)=\{h_1,h_2,h_3,h_4\}$, $BND_S(X)=\{h_3,h_4\}$ and $int(X)=\{h_1,h_2\}$, $cl(X)=\{h_1,h_2,h_3,h_4\}$, $Bnd(X)=\{h_3,h_4\}$. Hence $S_{\_}(X)=int(X)$, $S^{-}(X)=cl(X)$ and $BND_S(X)=Bnd(X)$.

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