Spherical symmetric space-time in Bimetric Relativity

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Abstract

Spherical symmetric Kantowski –Sachs space-time is studied in Rosen's bimetric relativity, considering the source of gravitation as cosmic strings coupled with perfect fluid distribution. It is shown that a macro cosmological model represented by cosmic string coupled with perfect distribution does not exist and only a vacuum model can be constructed.

Key Words- Spherical symmetric, Kantowski-Sachs model, cosmic string.

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Introduction

The spherical symmetry has its own importance in general relativity theory by virtue of its comparative simplicity. Many noteworthy space-time, such as the Schwarzschild solutions (exterior and interior) ,the Robertson-Walker model of expanding universe etc. are all spherically symmetric. Rosen N. [1, 2, 3, 4, and 5] introduced a new theory known as bimetric theory of relativity. It is based on two metric tensors g_{ij} and γ_{ij} . The first metric tensor described the curved space-time and thereby the gravitational field. The second metric tensor refers to the flat space-time and described the inertial force associated with acceleration of the frame of reference.
Israelit [6,7] studied several aspects of bimetric theory of gravitation . Recently Mohanty et-al. [8,9] constructed some physical viable models in this theory.
In this paper we have shown the spherical symmetric cosmological model with cosmic string does not exist. Mahurpawa and Ronghe[10] have shown that cosmic strings dose not exist in this modelin the context of bimetric relativity.

1Field equations and solutions

Accordingly, at each space-time point one has two line elements

$$ds^2 = g_{ij}\, dx^i dx^j \qquad (1.1)$$

and

$$d\sigma^2 = \gamma_{ij}\, dx^i dx^j \qquad (1.2)$$

This theory is based on a simple form of Lagrangian and has a simpler mathematical structure than that of the general relativity.

The field equations of the bimetric theory of gravitation formulated by Rosen N. are

$$K_i^{\ j} = N_i^{\ j} - \frac{1}{2} N g_i^{\ j} = -8\pi\kappa T_i^{\ j} \qquad (1.3)$$

where

$$N_i^{\ j} = \frac{1}{2}\gamma^{\alpha\beta}(g^{hi}g_{hj} \,|\, \alpha) \,|\, \beta \qquad (1.4)$$

$\kappa = (\frac{g}{\gamma})^{\frac{1}{2}}$ with g-the determinant of g_{ij} and γ -determinant of γ_{ij} .

The vertical bar (|)stands for γ -differentiation and $T_i^{\ j}$ is the energy-momentum tensor .

We considered here the spherically symmetric Kantowaski-Sachs space-time in the form

$$ds^2 = dt^2 - \lambda dr^2 - k^2(d\theta^2 - Sin^2\theta d\phi^2) \quad (1.5)$$

where λ and k are functions of "t" only
the background metric corresponding to the metric (1.5) is

$$d\sigma^2 = dt^2 - dr^2 - d\theta^2 - Sin^2\theta d\phi^2 \quad (1.6)$$

In this case we have taken the source of gravitation cosmic strings coupled with perfect fluid distribution.
The energy-momentum tensor for cosmic string coupled with perfect fluid distribution is given

$$T_i^j = T_{i\ string}^j + (\varepsilon + p)v_i v^j + pg_i^j \quad (1.7)$$

where $T_{i\ string}^j = \rho v_i v^j - \lambda x_i x^j$ (1.8)

Here ρ is energy density for a cloud of cosmic strings with particle attached to them, λ the string tensor density, v^i are four velocity vector of cosmic string distribution, x^i is an anisotropic direction or say direction of strings, and p are ε proper pressure and matter density.
The particle density associated with configuration is given by

$$\rho = \rho_p + \lambda \quad (1.9)$$

where ρ_p is the particle density in the string cloud

$-x_i x^j = v_i v^j = 1, v_i x^j - 0$ if $i \neq j$

We considered the anisotropic direction along x direction

$x_1 x^1 = -1, v_4 v^4 = 1$

So $T_1^1 = p + \lambda, T_2^2 = p = T_3^3, T_4^4 = \varepsilon + 2p + \rho$ (1.10)

Using equations (1.1) to (1.10) the field equations are

$$\left(\frac{\lambda_4}{\lambda}\right)_4 - 2\left(\frac{k_4}{k}\right)_4 = 16\pi\kappa(p + \lambda) \quad (1.11)$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = 16\pi\kappa(\rho) \quad (1.12)$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 + 2\left(\frac{k_4}{k}\right)_4 = 16\pi\kappa(\varepsilon + 2p + \rho) \quad (1.13)$$

Here suffix "4" following an unknown function denotes an ordinary differentiation with respect to time "t".
Equation (1.11) and (1.13) with help of (1.12) gives

$$p + \varepsilon + \lambda + \rho = 0 \quad (1.14)$$

Since $p \geq 0, \lambda \geq 0, \rho \geq 0, \rho \geq 0$

So $p = \lambda = \varepsilon = \rho = 0$ (1.15)

Using equation (1.14) in the field equations (1.11) to (1.13), we have

$$\left(\frac{\lambda_4}{\lambda}\right)_4 - 2\left(\frac{k_4}{k}\right)_4 = 0 \quad (1.16)$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = 0 \tag{1.17}$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 + 2\left(\frac{k_4}{k}\right)_4 = 0 \tag{1.18}$$

Equations (1.17) and (1.18), gives

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = \left(\frac{k_4}{k}\right)_4 = 0 \tag{1.19}$$

We have

$$\lambda = e^{c_{1t}} \tag{1.20}$$

And $$k = e^{c_{2t}} \tag{1.21}$$

Using equations (1.20) and (1.21) the line element (1.5) becomes

$$ds^2 = dt^2 - e^{2c_{1t}} dr^2 - e^{2c_{2t}} \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

By proper choice of coordinates this metric can be transform $ds^2 = d\tau^2 - e^{2\tau}\left(dr^2 + d\theta^2 + \sin^2\theta d\phi^2\right)$

Which is free from singularity at $\tau = 0$

2 Conclusions

We have studied spherically symmetric Kantowaski-Sachs cosmological model with cosmic string coupled with perfect fluid as energy-momentum tensor and observed that the cosmic string coupled with perfect fluid does not accommodate in this model only a vacuum model can be constructed. There is no singularity at $\tau = 0$

Reference

1 N. Rosen (1973) : Genl. Rela. Grav., 4,435.
2 N. Rosen (1974) : Ann.Phy.(N.Y.), 84,455.
3 N. Rosen (1977) : The Astrophysical journal, 221,357-360.
4 N. Rosen (1978) : The Astrophysical Journal ,221,284-285.
5 N. Rosen (1980) : Gen. Rela. Grav., 12,493-510.
6 M. Israelit (1979) : Gen. Rela. Grav. ,11,25.
7 M. Israelit (1976) : Gen. Rela. Grav. ,7,623-641.
8 G. Mohanty,P.K Sahoo (2002): Czech.J. Phys. 52,,1041-1047.
9 G.Mohanty ,P.K.Sahoo (2002) : Bulg.J.Phys., 28.
10 V. Mahurpawar and A.K. Ronghe(2011): ACTA CIENCIA INDICA, Vol.XXXVII M,N0 2