Enhanced Vehicle Routing Problem with Time Windows a Real Case of Solid Waste Collection in Tafo Pankrono, Kumasi, Ghana

D. Otoo[#1], S. K. Amponsah[#2] and J. D. Ankamah[#3]
Department of Mathematics
[#2]*Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana*
[#1, #3]*Department of Mathematics, University of Energy and Natural Resources, Sunyani, Ghana*

ABSTRACT

Time in the twenty-first century plays an integral part in every sphere of our daily activities; this key factor in life is not an exception to solid waste management operators, whose operational cost account for about 80% of their total cost. In this paper, adopted the classical Vehicle Routing Problem with Time Window and enhanced it with two additional factors, such as the vehicle breaking distance/time which is absent in VRPTW and individual customer service time which is assumed to be constant in VRPTW. The VRPTW can be described as the problem of designing least cost routes from one depot to a set of geographically scattered points. The routes must be designed in such a way that each point is visited only once by exactly one vehicle within a given time interval, all routes start and end at the depot, and the total demands of all customers on one particular route must not exceed the capacity of the vehicle. Our enhanced VRPTW was applied to a real world problem in Tafo Pankrono and compared with the existing collection time by the waste management company in the area (Zoom Lion Company Limited). Our enhanced model saw a drastic reduction of 39% collection time as compared with the existing collection time.

Keywords: *Vehicle routing, Time windows, Customers, Routing, Vehicle breaking time*

1. INTRODUCTION

Because of real world application, the VRPTW continues to draw attention from researches and has been a well-known problem in network optimisation. Vehicle routing problem (VRP) is a class of well known NP-hard combinatorial optimisation problem concerned with the design of optimal routes, used by a fleet of identical vehicles stationed at a central depot to serve a set of customers with known demands. When the capacity constraint is considered, the problem is considered as a Capacitated VRP (CVRP) with the objective of minimizing total cost (distance) of routes. The Capacitated Vehicle Routing Problem with Time Windows (CVRPTW), is a generalization of the CVRP. In the CVRPTW, the vehicles must comply with constraints of time windows associated with each customer in addition to the capacity constraints.

Collection and transportation of solid waste in developing countries has come with high operational cost to successive government and waste management agencies alike. This problem is even more crucial for third class communities where most of the houses are built without proper road layout, poor accessibility to customers. The problem has even worsen due to the fact that waste collection operators do not have any mathematical model which could be used to optimally determine the amount of time needed to service a customer and eventually complete a tour for proper operations management and forecast.

2. STUDY AREA

The study area (Tafo Pankrono) has eleven (11) communities of which seven (7) of them are categorized as third class zones. The area is the smallest of the nine sub metropolitan areas in terms of land area but it is the second highest generator of solid waste after Subin. The area has a population of 157,226 with eleven communities within its domain. Four of these communities are categorised under class two whiles the remaining seven are under class three. The area shares boundary with Manhyia to the east, Suame to the west and Subin to the south and Kwabre to the north. The area generates about eighty-eight tones of solid waste a day. Our study considered five of the seven third class zones namely Old Tafo, Pankrono Dome, Pankrono West, Tafo Adompom and Ahenbronum constituting about 52% of the area population. This paper seeks to address the problem by introducing three main parameters in routing with time windows; one the stopping time, the breaking time and the deadheading time. The proposed method is implemented on third class communities in Tafo Pankrono which involves 2475 households and 3509 of 140 litre bins to optimally find the real time needed by a capacitated vehicle to service a set of customers.

3. RELATED WORKS

Vehicle routing problem with time windows has received extensive research works in the past two decades especially in the services sub sector including collection and transportation of solid waste but almost none of these research works take into accounts stooping time, the real time of service of a particular customer and deadheading time. Some of the research works related to our study is considered in this section. The VRPTW has been the subject of intensive research efforts for both heuristic and exact optimization approaches. Early results of solution techniques for the VRPTW can be found in Golden and Assad (1986), Desrochers et al. (1988), Golden and Assad (1988), and Cordeau et al. (2001) mostly focused on exact techniques. Further details on these exact methods can be found in Larsen (1999) and Cook and Rich (1999). Because of the high complexity level of the VRPTW and its wide applications to real-life situations, solution techniques capable of producing high-quality solutions in limited time, are of prime importance. Kim et al.(2006) include time windows and a driver break in the collection VRP-IF. The multi-objective genetic algorithm of Ombuki-Bermanet et al. (2007), the variable neighborhood tabu search of Benjamin (2011) and the adaptive large neighborhood search (ALNS) of Buhrkalet al.(2012) are tested on these instances. Compared to Kim et al.'s (2006) results, these algorithms improve average distance by approximately 15% and use fewer vehicles. Buhrkal et al.'s (2012) approach also leads to a distance improvement of 30-45% at a Danish waste collection company.

Crevier et al. (2007) observe that when vehicles are stationed at several depots, inter-depot routes occur infrequently possibly because they are rarely economical. Hence, they create two sets of MDVRPI instances with 48 to 288 customers and a fixed homogeneous fleet stationed at one depot, with the rest of the depots acting only as intermediate facilities. These instances are used by Tarantilis et al. (2008) and Hemmelmayr et al.(2013) who propose, respectively, a hybrid guided local search and a variable neighborhood search (VNS) with a dynamic programming procedure for the insertion of the intermediate facilities in the tours. The former apply the MDVRPI framework to a distribution problem, while the latter to a solid waste collection problem. Both articles report small improvements over the results of Crevier et al.(2007) with computation times of only several minutes, even for the largest problems. In addition, Hemmelmayr et al. (2013) apply their approach to a PVRP-IF faced by a real waste collection company and obtain a 25% reduction in the routing cost.

4. PROBLEM AND MATHEMATICAL FORMULATIONS

The main objective of this paper can be stated as follows: formulate a formula that accounts for the stopping time of the service vehicle, include a formula that accounts for the real time needed to collect waste from a customer based on the volume of waste from that customer and the dead heading time. The collection of solid waste vehicle routing problem is mainly solved on residential or commercial (Industrial) waste. Both residential and commercial collection problems can be classified as variants of vehicle routing problem with time windows (VRPTW) but with additional constraints. A VRP comprises a set of vehicles, customer stops and a depot. Each vehicle starts from the depot, visits a number of customers and ends at the depot. A VRPTW is an extension of VRP by an additional time constraints associated with each customer. Solid waste vehicle routing problem with time windows can be summarized as follows: Minimize number of vehicles; Minimize total travel time and Balance workload among collecting vehicles.

With its constraints as vehicle capacity (volume, weight), Route capacity (maximum number of residential customers a vehicle can handle per trip), Routing time limit per vehicle, Time windows of the stopping times at customer point and the transfer depot and Clews' lunch break.

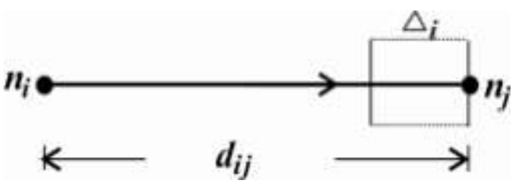
To explain the problem of VRPTW, we present a mathematical programming model for a simplified version by minimizing the travel and service time. We adopted the basic VRPTW model by Cordeau et al., (2002) and modified it by incorporating the transfer depot, clews' lunch time, stopping time and deadheading time. A simplified solid waste VRPTW is defined on the network $G = (V, A)$ where $A = \{(V_i, V_j); i \neq j \text{ and } i, j \in V)$ is an arc set and the vertex set $V = \{v_0, v_1, \ldots, v_{n+m}, v_{n+m+1})$ where v_0 and v_{n+m} denote the transfer depot at which vehicle of capacity Q start and end their tour and v_{n+m+1} is the node for the lunch break. Each vertex in V has an associated demand $q_i \geq 0$, a service time $t_s \geq 0$ and a service time window $\eta_i[t_i, T_i]$; where η_i is the number of bins to be emptied at customer i and the transfer depot time window $[T_0, T_t]$ representing earliest possible departure time from the transfer depot and the latest possible arrival time at the transfer depot. In particular, the transfer depot has $t_0 = 0$ and $q_0 = 0.$ The set of service points $C = \{v_1, v_2, \ldots, v_n\}$ specifies a set of n customers. The arrival time of a vehicle at customer $i, i \in C$ is denoted by t_{ai} and its departure time t_{pi}. An arc (V_i, V_j) has an associated distance $d_{ij} \geq 0$ and a travel time $t_{ij}(t_{pi})$ a function of the departure time from customer *i*. The set of available vehicles is denoted by *K*. The objective function is the minimization of total time. There are two decision variables; x_{ij}^k is a binary decision that indicates whether vehicle *k* travels between customers *i* and *j*. The real decision variable t_{sti} indicates service start time for customer *i* served by vehicle *k*. The VRPTW model is formulated as follows.

5. MATHEMATICAL FORMULATION

The speed of a service vehicle depends on the nature of road, the present weight of the vehicle and the distance between two adjacent customers. In this paper we shall consider the time spend by a driver when he/she apply breaks to stop at a customer for service, while assuming a constant speed of the vehicle.

Consider the speed of the vehicle as α ms^{-1}

Final velocity at customer ***i*** is zero;



The distance from customer *j* when breaks are applied is given as Δ_i

Applying Newton's law of motion

$$0^2 = \alpha^2 + 2a\Delta_i$$

$$a = -\frac{\alpha^2}{2\Delta_i} \qquad (5.1)$$

Also; $0 = \alpha + \left(-\frac{\alpha^2}{2\Delta_i}\right)t$

$$t = \frac{2\Delta_i \alpha}{\alpha^2} = \frac{2\Delta_i}{\alpha} \qquad (5.2)$$

Time taken to travel from customer *i* to customer *j*

$$(t_{ij}) = \frac{\alpha}{d_{ij} - \Delta_i} + \frac{2\Delta_i}{\alpha} \qquad (5.3)$$

a_0 = time taken to alight from the vehicle before the start of service and get onto the vehicle after service

Time taken to service customer *i* is

$$t_{si} = \eta_i t'_{si} + a_0 \qquad (5.4)$$

Time for deadheading T_d (traversing an edge *i* to *j* without collection) is $T_d = \frac{d_{ij}}{\alpha}$

Objective:

$$\text{Min} \sum_{k\in K} \sum_{(i,j)\in A} t_{ij}^k x_{ij}^k + \sum_{k\in K} \sum_{j\in C} (t_{s,n+ms}^k - t_{s0}^k) x_{0j}^k \qquad (5.5)$$

Subject to

$$\sum_{i\in C} q_i \sum_{j\in V} x_{ij}^k \le Q, \quad \forall k \in K \qquad (5.6)$$

$$\sum_{k\in K} \sum_{j\in V} x_{ij}^k = 1, \quad \forall i \in C \qquad (5.7)$$

$$\sum_{i\in V} x_{il}^k - \sum_{j\in V} x_{lj}^k = 0, \ \forall l \in C, \forall k \in K \qquad (5.8)$$

$$x_{i0}^k = 0, x_{n+m,i}^k = 0, \forall i \in V, \forall k \in K \qquad (5.9)$$

$$\sum_{j \in V} x_{0j}^k = 1, \ \forall k \in V \qquad (5.10)$$

$$\sum_{j \in V} x_{j,m+n}^k = 1, \ \forall k \in V \qquad (5.11)$$

$$\sum_{i=1}^{n+m} x_{i,n+m+1i}^k = 1, \ \forall k \in K \qquad (5.12)$$

$$\sum_{j=1}^{n+m} x_{n+m+1,j}^k = 1, \ \forall k \in K \qquad (5.13)$$

$$t_i \sum_{j \in V} x_{ij}^k \le t_{sti}^k, \ \forall i \in V, \forall k \in K \qquad (5.14)$$

$$\eta_i (T_i - t_i) \sum_{j \in V} x_{ij}^k, \ \forall k \in K \qquad (5.15)$$

$$x_{ij}^k (t_{sti}^k + t_{si} + t_{ij} (t_{sti}^k + t_{si}) \le t_{stj}^k, \ \forall (i, j) \in A, \forall k \in K \qquad (5.16)$$

$$\sum_{i=0}^{n+m} t_{ij} x_{ij}^k + \sum_{i=0}^{n+1} t_i^k x_{ij}^k + \sum_{i=1}^{n} T_{di} x_{ij}^k \le (T_t - T_0) \qquad (5.17)$$

$$x_{ij}^k \in \{0,1\}, \forall (i, j) \in A, \forall k \in K \qquad (5.18)$$

$$t_{sti}^k \in R, \forall i \in V, \forall k \in K \qquad (5.19)$$

Constraint (5.6) impose the rule that the vehicles capacity cannot be exceeded, (5.7) ensures that all customers are served, if a vehicle arrives at a customer it must also depart from that customer (5.8), route must start and end at the depot (5.9), each vehicle leaves from and returns to the transfer depot once (5.10) and (5.11) respectively. Constraints (5.12) and (5.13) are introduced to add the lunch break for each route; service times must satisfy time window start (5.14) and ending (5.15) times; and service start time must allow for travel time between customers (5.16). Constraint (5.17) ensures that the transfer depot time window is not violated. Decision variables type and domain are indicated in (5.18) and (5.19)

6. RESULTS OF THE ENHANCED VRPTW MODEL ON A REAL WORLD PROBLEM

Our enhanced vehicle Routing Problem with time windows was implemented on the study area which has four zones. The table(s) below gives the total time required to collect waste from each sub-cluster.

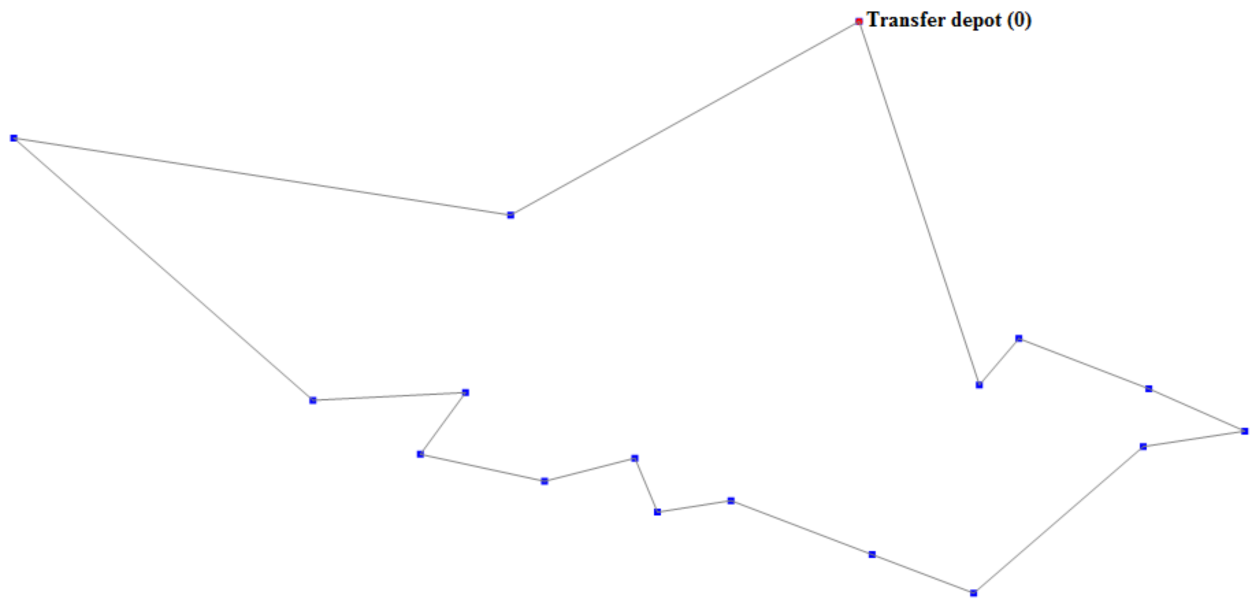


Figure 6.1: Best tour of customers in a sub-cluster

A typical vehicle routing in a sub-cluster with a sequence of customers visited gave a total collection time of 1407.16sec, a tour length of 844.371m, visited 16 customers and emptied 35 bins from the customers.

$$0 \to 60a \to 84a \to 85a \to 86a \to 94a \to 52a \to 52ai \to 53a \to 45a \to 54a \to 44a \to 43a \to 55a \to 42a \to 65a \to 57a$$

Table 1: Total collection time in each sub-cluster of zone 1

Sub-cluster One			
Optimal sequential tour of customers	Customers visited	Total bins emptied	Enhanced model routing time (sec)
$0 \to 26a \to \ldots \to 63a \to 0$	9	35	1636.81
$0 \to 31 \to \ldots \to 75 \to 0$	21	35	1712.18
$0 \to 74 \to \ldots \to 27a \to 0$	17	35	1687.67
$0 \to 73 \to \ldots \to 30 \to 0$	20	35	1707.28
$0 \to 95 \to \ldots \to 94 \to 0$	16	25	1382.67
Total	**83**	**165**	**8126.61**

Sub-cluster two			
Optimal sequential tour of customers	Customers visited	Total bins emptied	Enhanced model routing time (sec)
$0 \to 62c \to \ldots \to 83c \to 0$	23	35	1724.88
$0 \to 2 \to \ldots \to 47 \to 0$	17	35	1686.42
$0 \to 68c \to \ldots \to 59 \to 0$	20	35	1707.16
$0 \to 81c \to \ldots \to 12 \to 0$	25	35	1738.60

$0 \to 11 \to \ldots \to 10 \to 0$	14	21	1250.13
Total	**99**	**161**	**8107.19**

Sub-cluster three			
Optimal sequential tour of customers	Customers visited	Total bins emptied	Enhanced model routing time (sec)
$0 \to 82a \to \ldots \to 40a \to 0$	20	35	1766.18
$0 \to 60a \to \ldots \to 57a \to 0$	16	35	1741.94
$0 \to 94a \to \ldots \to 5b \to 0$	18	35	1756.00
$0 \to 1b \to \ldots \to 98a \to 0$	18	35	1754.16
$0 \to 4b \to \ldots \to 100a \to 0$	13	25	1422.66
Total	**85**	**165**	**8440.94**

Sub-cluster four			
Optimal sequential tour of customers	Customers visited	Total bins emptied	Enhanced model routing time (sec)
$0 \to 21d \to \ldots \to 54c \to 0$	21	35	1711.80
$0 \to 39d \to \ldots \to 55c \to 0$	21	35	1711.25
$0 \to 37c \to \ldots \to 38b \to 0$	30	35	1770.29
$0 \to 25b \to \ldots \to 39c \to 0$	23	35	1726.84
$0 \to 20c \to \ldots \to 22c \to 0$	18	26	1423.39
Total	**113**	**166**	**8343.57**

Sub-cluster five			
Optimal sequential tour of customers	Customers visited	Total bins emptied	Enhanced model routing time (sec)
$0 \to 76b \to \ldots \to 94b \to 0$	22	35	1719.03
$0 \to 68e \to \ldots \to 2c \to 0$	28	35	1757.57
$0 \to 99b \to \ldots \to 57e \to 0$	31	35	1776.81
$0 \to 64e \to \ldots \to 99d \to 0$	29	35	1764.08
$0 \to 11e \to \ldots \to 12e \to 0$	26	26	1477.88
Total	**136**	**166**	**8107.19**

Sub-cluster six			
Optimal sequential tour of customers	Customers visited	Total bins emptied	Enhanced model routing time (sec)
$0 \to 19d \to \ldots \to 18d \to 0$	22	35	1718.00
$0 \to 52d \to \ldots \to 45e \to 0$	31	35	1776.18
$0 \to 94f \to \ldots \to 1g \to 0$	31	35	1776.80
$0 \to 4g \to \ldots \to 99f \to 0$	34	35	1796.46
$0 \to 5g \to \ldots \to 87f \to 0$	12	12	967.68
Total	**130**	**152**	**8035.12**

Sub-cluster seven			
Optimal sequential tour of customers	Customers visited	Total bins emptied	Enhanced model routing time (sec)
$0 \rightarrow 62c \rightarrow \ldots \rightarrow 83c \rightarrow 0$	25	35	1740.24
$0 \rightarrow 2 \rightarrow \ldots \rightarrow 47 \rightarrow 0$	31	35	1775.57
$0 \rightarrow 68c \rightarrow \ldots \rightarrow 59 \rightarrow 0$	28	35	1757.54
$0 \rightarrow 11 \rightarrow \ldots \rightarrow 10 \rightarrow 0$	19	20	1250.26
Total	**103**	**125**	**6823.61**

Table 2: Summary of total collection time in zone one

Zone one	Customers visited	Total bins emptied	Optimal routing distance (m)	Enhanced model routing time (sec)	Existing operational routing time (sec.)
Sub-cluster one	83	165	3467.342	8126.61	11340.00
Sub-cluster two	99	161	5191.136	8107.19	10836.00
Sub-cluster three	85	165	3593.204	8440.94	11448.00
Sub-cluster four	113	166	6357.110	8343.57	11592.00
Sub-cluster five	136	166	5191.136	8107.19	11700.00
Sub-cluster six	130	152	6693.328	8035.12	11088.00
Sub-cluster seven	103	125	4827.158	6823.61	9216.00
Total	**749**	**1100**	**35320.414**	**55984.24**	**77220.00**

Table 3: Summary of total collection time in zone two

Zone Two	Customers visited	Total bins emptied	Optimal routing Distance (m)	Enhanced model routing time (sec)	Existing operational routing time (sec.)
Sub-cluster one	141	165	5731.855	8499.00	11592.00
Sub-cluster two	134	166	6384.504	8486.42	11808.00
Sub-cluster three	99	166	3329.486	8260.99	11736.00
Sub-cluster four	84	166	3703.957	8164.93	11880.00
Sub-cluster five	103	165	5757.303	8252.96	11916.00
Total	**561**	**828**	**24907.105**	**41664.30**	**58932.00**

Table 4: Summary of total collection time in zone three

Zone Three	Customers visited	Total bins emptied	Optimal routing distance (m)	Enhanced model routing time (sec)	Existing operational routing time (sec.)
Sub-cluster one	134	139	5817.149	7443.19	9900.00
Sub-cluster two	98	166	5076.770	7054.62	10176.00
Sub-cluster three	124	164	5868.957	7156.92	9720.00
Sub-cluster four	102	158	3969.739	6840.78	9144.00
Sub-cluster five	84	165	3234.103	6935.91	9972.00
Total	**542**	**792**	**23966.718**	**35431.42**	**48912.00**

Table 5: Summary of total collection time in zone four

Zone Four	Customers visited	Total Bins emptied	Optimal routing distance (m)	Enhanced model routing time (sec)	Existing operational routing time (sec.)
Sub-cluster one	102	158	5233.28	6834.99	9252.00
Sub-cluster two	156	166	6400.496	7430.65	10260.00
Sub-cluster three	119	135	5605.844	5965.12	9180.00
Sub-cluster four	106	166	5700.891	7102.29	9936.00
Sub-cluster five	140	164	6696.442	7260.88	9756.00
Total	623	789	29636.953	**34593.93**	**48384.00**

7. RESULTS AND CONCLUSIONS

The results from our enhanced vehicle routing problem with time windows compared with the existing operational time saw a reduction of 37.93% in zone one, 41.44% in zone two, 38.04% in zone three and 39.86% time reduction in zone four. These reductions in collection time translate averagely to about 4.57 hours saving time in each zone.

References

[1] A. M. Benjamin‘‘Metaheuristics for the waste collection vehicle routing problem with time windows”, Ph.D. Thesis, Department of Mathematical Sciences, Brunel University (2011).

[2] K. Buhrkal, A. Larsen and S. Ropke ‘‘The waste collection vehicle routing problem with time windows in a city logistics context”, Procedia-Social and Behavioral Sciences,39,241–254 (2012)

[3] B. Crevier, J. F. Cordeau and G. Laporte ‘‘The multi-depot vehicle routing problem with inter- depot routes”, European Journal of Operational Research,176 (2) 756–773 (2007)

[4] M. Desrochers, J.K. Lenstra, M.W.P. Savelsbergh and F. Soumis “Vehicle Routing with Time Windows: Optimization and Approximation”, in Vehicle Routing: Methods and Studies (1988)

[5] B. Golden and A. Assad 65–84, Elsevier Science Publishers, (1988) Amsterdam.

[6] W. Cook and J.L. Rich “A Parallel Cutting-Plane Algorithm for the Vehicle Routing Problems with Time Windows”, Technical Report TR99-04 (1999) Department of Computational and Applied Mathematics, Rice University,

[7] Houston, J. F. Cordeau, G. Desaulniers, J. Desrosiers, M. M. Solomon and F. Soumis “The VRP with Time Windows”, in The Vehicle Routing Problem, SIAM Monographs on Discrete Mathematics and Applications, P. Toth and D. Vigo (eds), 157−194 (2001) SIAM, Philadelphia.

[8] B. L. Golden and A.A. Assad “Perspectives on Vehicle Routing: Exciting New Developments”, Operations Research 34, 803−809 (1986)

[9] J. Larsen ‘‘Parallelization of the Vehicle Routing Problem with Time Windows”, Ph.D. thesis, Institute of Mathematical Modelling, Technical University of Denmark, Lyngby, Denmark (1999)

[10] V. Hemmelmayr, K. F. Doerner, R. F. Hartl and S. Rath ‘‘A heuristic solution method for node routing based solid waste collection problems”, Journal of Heuristics,19 (2) 129–156 (2013)

[11] B. I. Kim, S. Kim and S. Sahoo ‘‘Waste collection vehicle routing problem with time windows”, Computers& Operations Research, 33 (12) 3624–3642 (2006)

[12] C. D. Tarantilis, E. E. Zachariadis and C. T. Kiranoudis ‘‘A hybrid guided local search for the vehicle- routing problem with intermediate replenishment facilities”. NFORMS Journal on Computing, 20 (1)154–168 (2008)