An Improved Ant Colony Optimisation Algorithm to a Real World Application in Solid Waste Collection; a Case of Tafo Pankrono, Kumasi, Ghana

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ABSTRACT

Solid waste collection is an important issue in vehicle routing problem especially in the developing countries where most of the road network in the residential areas is very poor. A well planned path routing will go a long way to help waste management agencies to cut down cost of operations. This paper proposes a new vehicle routing path planning problem that assigns a tricycle to collect waste from individual household and dispose off into a skip bin within a defined cluster zone. The paper presents an Ant Colony optimisation algorithm that incorporates some new factors of selecting nodes such as weight, angle, saving and visibility to solve the vehicle routing path problem. The study is motivated by a real case in Ghana with the aim to optimize the collection of solid waste. The implementation of our improved algorithm gave about 18% reduction in distance travel compared with the traditional Ant Colony algorithm for vehicle routing path planning method.

Keywords: *Solid waste, Waste management, Vehicle routing, Algorithm, Path routing, visibility*

1. INTRODUCTION

Collection of solid waste in developing countries has been a challenge to successive government, waste management agencies and the residents alike. This problem is even more crucial for third class communities where most of houses are built without proper road layout making it almost impossible to access customers. Due to the problems outlined, skip containers are used to collect waste from such communities, but this method of waste collection put a lot of customers at a disadvantage due to an unscientific way of locating the facility compelling some of the customers to walk long distances to access the facility, and thereby making some to dump waste in gutters, streams or even burn then which has its own health issues to the environment.

This paper introduces an improved ant colony algorithm to optimize the routing of customers by using a tricycle. Our proposed method is implemented on third class community in Tafo Pankrono, one of the nine sub-metropolitan areas in Kumasi Metropolitan Assembly, Ghana, and West Africa.

2. STUDY AREA

The study area (Tafo Pankrono) has eleven (11) communities of which seven (7) of them are categorized as third class zones. The area is the smallest of the nine sub metropolitan areas in terms of land area but it is the second highest generator of solid waste after Subin. The area has a population of 157,226 with eleven communities within its domain. Four of these communities are categorised under class two whiles the remaining seven are under class three. The area shares boundary with Manhyia to the east, Suame to the west and Subin to the south and Kwabre to the north. The area generates about eighty-eight tones (88 tones) of solid waste a day (Waste management department Kumasi Metropolitan Assembly, 2013). Our study considered five of the seven third class zones namely Old Tafo, Pankrono Dome, Pankrono West, Tafo Adompom and Ahenbronum constituting about 52% of the area population.

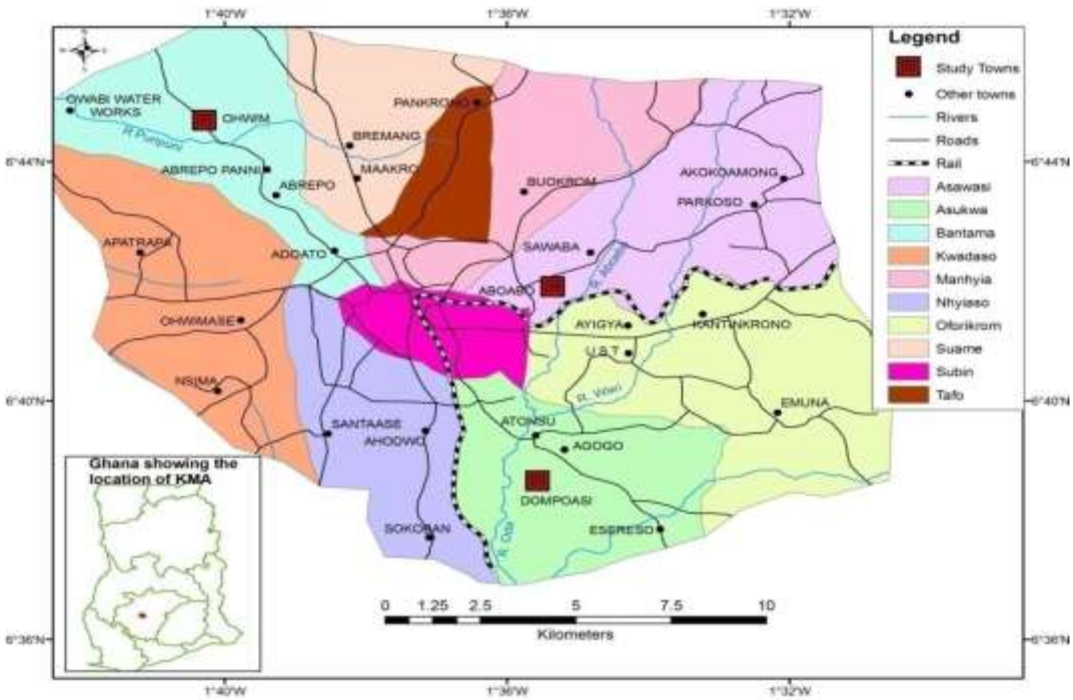

Figure 1: Map of Kumasi Metropolis

3. RELATED WORKS

Extensive research works has gone into vehicle routing problems especially in the area of solid waste collection but very few of these research works investigated the collection of waste in the lower income communities (third class). Some of the research works related to our study is considered in this section. Chang, Lu and Wei (1997) used a revised multi-objective mixed-integer programming model (MIP) to analyze the optimal path in a waste collection within an environment of geographic information system (GIS). They demonstrated the integration of the MIP and GIS for the management of solid waste in Kaohsiung, Taiwan.

Mourao and Almeida (2000) solved a capacitated arc routing problem (CARP) with side

constraints for a solid waste collection VRP using two lower-bounding methods to incorporate the side constraints and a three-phase heuristic to generate a near optimal solution. Mourao and Amado (2005) presented a heuristic method for a mixed CARP, inspired by the refuse collection problem in Lisbon. The proposed heuristic holds on both directed and mixed cases. Mixed cases indicate that waste may be collected on customers on both sides of the road at the same time (i.e. narrow street), whereas waste for the directed cases is on one side of the road. Mourao, Nunes and Prins (2009) proposed two two-phase heuristics and one best insertion method for solving a sectoring arc routing problem (SARC) in a municipal waste collection problem. In SARC, the street network is partitioned into a number of sectors, and then a set of vehicle trips is built in each sector that aims to minimize the total duration of the trips.

Ogwueleka (2009) proposed a heuristic procedure which consists of a route first, cluster second method for solving a solid waste collection problem in Onitsha, Nigeria.

For example Bautista, Fernandez and Pereira (2008) transformed the arc routing into a node routing problem due to the road constraint such as forbidden turns for solving an urban waste collection problem in the municipality of Sant Boi de Llobregat, Barcelona with 73917 inhabitants using an ant colonies heuristic which is based on nearest neighbour and nearest insertion methods. Cordeau and Laporte studied simulated annealing algorithm for the VRP, they proposed a simulated annealing method which is suitable for solving the vehicle routing problem and shows the advantage of accuracy and the speed of search convergence. Zulvia et al. use genetic algorithm (GA) encoding to solve VRP problem.

4. PRPBLEM DEFINITION

The area of the community where collection of solid waste route are planned is represented by means of a graph $G=(V,E,A)$ where V is associated with street junctions, including dead-ends and the transfer depot E, the nodes and A is associated with existing street sections between junctions. Arcs represent single-way streets while edges represent two way streets. There is a single transfer depot denoted by O, where $O \in V$. Each link $i \in E \cup A$ with length d_{ij}. The set of links $L = E^L \cup A^L \in E \cup A$ is defined as the set of links with a minimum of one collection point, where each required $i \in E^L \cup A^L$ has a demand q_i which represents the quantity of solid waste that must be collected. There are m vehicles each with capacity Q to provide service to the required customers in a cluster (link). Each cluster (link) must be serviced by one single vehicle. Each of the serviced vehicles starts and end at the transfer depot, the total waste to be collected by a vehicle in a cluster must not exceed the capacity of the vehicle. Each customer is serviced once by only one vehicle in the time constraint. If the load stored by the ant exceeds the vehicle capacity, the ant must return to the transfer depot, we then obtain a complete route for a vehicle.

5. MATHEMATICAL FORMULATION

In this section an improved Ant Colony Heuristics is used to construct routing path for the vehicle routing problem. The improved heuristics has the following parameter notations:

$K = \sum_{i=1}^{n} b_i$: Total number of ants

N: Set of customers to be visited

$tabu_k$: Tabu list of the k-th ant

$tabu_k(s)$: s-th customer visited by the k-th ant in the tour

τ_{ij} (*t*): Intensity of trail on edge between customer *i* and customer *j* at time *t*

η_{ij} : Visibility of edge between customer *i* and customer *j*

$\omega_{ij}(t)$: Weight factor, which is the ratio of the current weight including weight of customer *j* to the capacity of the service vehicle

$u_{ij}(t)$: Saving heuristic and

$\psi_{ij}(j)$: Angle factor

The initial trial pheromone level on each edge is given by $\tau_{ij}(0) = \dfrac{1}{(n+1)L}$ (5.1)

where *n* is the number of nodes in a given route (link) and *L* is the tour length generated by the nearest neighbor heuristic which is an improvement over the usual initial trial level $\tau_{ij}(0) = \dfrac{1}{n}$. The nearest neighbor heuristic is obtained by the following steps;

i) Randomly start with one node that has not been visited at the beginning of a route;

ii) Select the not yet visited closest feasible customer as the next customer to be visited.

iii) Repeat step 2, until the vehicle capacity is violated, then go back to the transfer depot.

The visibility function η_{ij} from customer *i* to customer *j* is given by $\eta_{ij} = \left(\dfrac{1}{d_{ij} + d_{oj}}\right)$ (5.2)

replacing the traditional visibility function $\eta_{ij} = \dfrac{1}{d_{ij}}$ where d_{ij} s the distance between customer *i* and customer *j* and the value of d_{oj} is the distance between the affected node *j* and the transfer depot. The saving heuristic takes in account the distance from the current customer *i* from the transfer depot and that of the next probable customer *j* and weight factor of the next probable customer. The saving heuristic is given by

$$U_{ij} = d_{oi} + d_{oj} - \tfrac{1}{(q_i+1)} d_{ij} \qquad (5.3)$$

Weight factor has been factored into the probabilistic rule to guide the ant of the capacity constraints of the service vehicle. The weight factor is given by

$$w_{ij}(t) = \left(\frac{Q_i + q_j}{Q}\right) \qquad (5.4)$$

where q_j is the number of waste to be collected from customer *j*, Q_i is the total capacity of waste in the ant tabu list after servicing customer *i*.

$\psi_{ij}(j)$ is the angle factor, which is the angle between the vector $\left[\overrightarrow{\text{node}_i^1\text{node}_i}, \overrightarrow{\text{node}_i\text{node}_j}\right]$ where node_i^1 is the previous node before visiting the affected node_i, node_j represents the current visiting affected node after node_i. For escape of an ant when it faced a dead end, an angle in the range $0 \le \theta \le \frac{\pi}{2}$ is assigned a constant value of 0.001 and an angle in the range $\frac{\pi}{2} < \theta \le \pi$ its value is $\dfrac{\theta}{\pi}$

and $\theta = \cos^{-1}\left(\dfrac{\overrightarrow{\text{node}_i^1\text{node}_i} \cdot \overrightarrow{\text{node}_i\text{node}_j}}{\left|\overrightarrow{\text{node}_i^1\text{node}_i}\right|\left|\overrightarrow{\text{node}_i\text{node}_j}\right|}\right)$

The state transition rule that ants uses to select the next affected node are modified, which is calculated as follows. An ant *k*, positioned on a vertex *i* , chooses the next vertex *j* to visit by applying the probabilistic rule p_{ij}^k and is given by

$$P_{ij}^{k}(t)=\begin{cases}\dfrac{\left[\tau_{ij}(t)\right]^{\alpha}\times\left[\eta_{ij}\right]^{\beta}\times\left[U_{ij}(t)\right]^{\gamma}\times\left[w_{ij}(t)\right]^{\gamma}\times\left[\psi_{ij}(j)\right]}{\sum_{k\in \text{allowed}_k}\tau_{ik}(t)^{\alpha}\times\eta_{ik}^{\beta}\times U_{ik}(t)^{\gamma}\times w_{ik}(t)^{\gamma}\times\psi_{ik}(k)}, & \text{if } j\in \text{allowed}_k\\ 0 & \text{otherwise}\end{cases}\tag{5.6}$$

where α, β and γ are adjustable parameters that determine the relative influence of the pheromone and the visibility in the transition probabilities and allowed_k denote the neighbor affected node of affected node *i* that the *k*th ant has not yet visited.

Ants chooses the next affected node *j* in accordance with the equation

$$j=\begin{cases}\underset{u\in JJ_i^k}{\text{argmax}}\ \tau_{iu}(t)^{\alpha}\times\eta_{iu}(t)^{\beta}\times U_{iu}(t)^{\gamma}\times w_{iu}(t)^{\gamma}, & \text{if } q\le q_0\\ JJ, & \text{if } q>q_0\end{cases}\tag{5.7}$$

Where $JJ=P_{ij}^{k}(t)$ and q, q_0 are random variables such that $q, q_0\in(0,1)$. When $q>q_0$ the ant chooses the next node in accordance with *JJ*

When an ant has completed a tour, we use a 2-optimal heuristic to improve the solution in each tour. In each iteration, the algorithm examines each two distinct arcs $a_i\to a_{i+1}$ and $a_j\to a_{j+1}$ in the route *R*. These two arcs are replaced by the arcs $a_i\to a_j$ and $a_{i+1}\to a_{j+1}$ provided the distance decreases and there is actually an arc joining them, then the original arcs are replaced by the arcs $a_i\to a_j$ and $a_{i+1}\to a_{j+1}$

5.1 Local and Global Pheromone Updates

In order to avoid the probability of selecting a customer repeatedly, the amount of pheromone on an arc is reduced through evaporation. This is done by applying a local pheromone updating rule given by

$$\tau_{ij}^{\text{new}}=(1-\rho)\tau_{ij}^{\text{old}}+\rho\tau_{ij}(0)\ \text{where } \rho\in(0,1)\tag{5.8}$$

and ρ is the trial evaporation constant, τ_{ij}^{new} is the pheromone on the link (*i*, *j*), τ_{ij}^{old} is the pheromone on the link (*i*, *j*) before updating. When no feasible customer is available due to vehicles capacity constraints then the transfer depot is chosen and a new route is started. This process is executed until all customers have been visited. When all ants construct their tours, the best ant tours are chosen and the global pheromone updating rule is applied by the rule given by

$$\tau_{ij}^{\text{new}}=(1-\rho)\tau_{ij}^{\text{old}}+\rho\sum_{r=1}^{m}\Delta\tau_{ij}^{r}\tag{5.9}$$

where $\Delta\tau_{ij}^{r}$ is the increased in pheromone on link (*i*, *j*) of route *r* found by the ant. The pheromone increment updating rule is given by

$$\Delta\tau_{ij}^{r} = \begin{cases} \dfrac{P}{\omega \times \sum_{r} D^{r}} \times \dfrac{D^{r} - d_{ij}}{m^{r} \times D^{r}} & \text{if link } (i,\ j) \text{ is on the } r\text{th route} \\ 0 & \text{otherwise} \end{cases} \tag{5.10}$$

Where P is a constant, L the total length of all routes in the solution, D^{r} is the length of the rth route in the solution, d_{ij} is the length of link (i, j) and m^{r} the number of customers in the rth route and ω is the number of routes in the solution and $\omega > 0$.

5.2 DATA COLLECTION AND ANALYSIS

The study area had 2475 customers and (3509) of 140 litre bins comprising of four zones, with zone 1 having 749 customers and 1100 bins, zone two 561customers and 828 bins, zone three 542 customers and 792 bins and the final zone four had 623 customers and 789 bins.

Per the capacity of the skip containers of 166 bins, our probabilistic location model gave the number of sub-clusters needed in each zone as in the table below.

Table 3.1: Summary of customers and bins in each zone

Zone one			Zone Two			Zone Three			Zone Four		
Sub-cluster	**Cust.**	**bins**	**Sub-cluster**	**Customers**	**bins**	**Sub-cluster**	**Cust.**	**bins**	**Sub-cluster**	**Cust.**	**bins**
1	83	165	1	141	165	1	84	139	1	102	158
2	99	161	2	134	166	2	134	165	2	156	166
3	85	165	3	99	166	3	98	166	3	119	135
4	113	166	4	84	166	4	124	164	4	106	166
5	136	166	5	103	165	5	102	158	5	140	164
6	130	152									
7	103	125									

5.3 IMPROVED AND COLONY ALGORITHM VERSUS ANT COLONY ALGORITHM

In this section we compared with the results from our improved model with the classical Ant colony algorithm. The two models were implemented on all the five zones of the study area, but for the purposes of this paper result of only zone one is displaced. We had twenty-two (22) clusters in which the skip containers are to be located to collect solid waste from capacitated tricycle of volume 3.5m^{3} and can hold 35 of 140 litre bins.

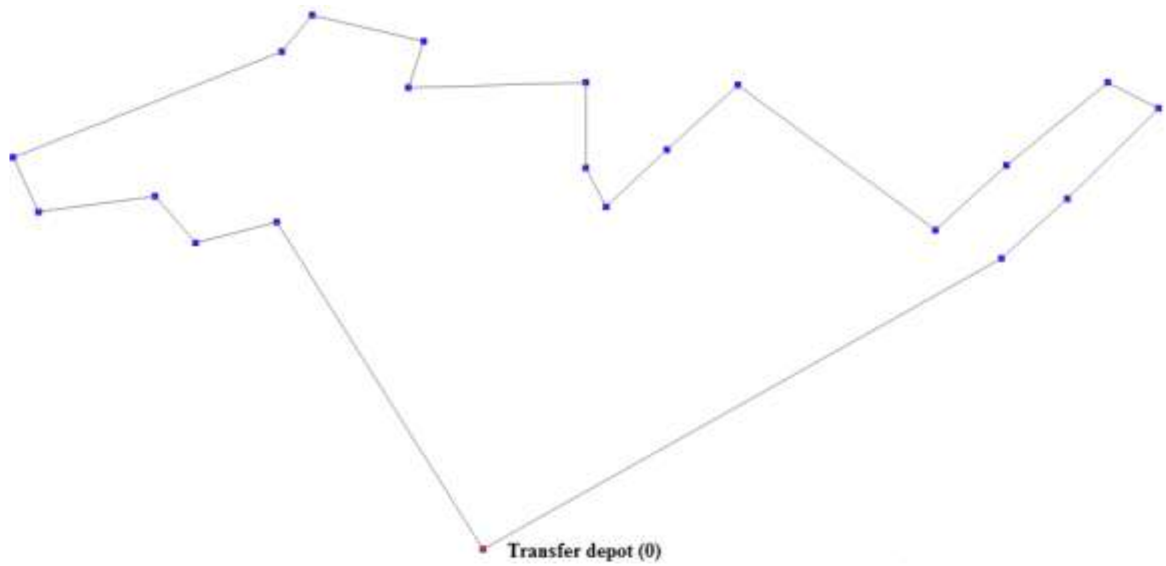

Figure 3.1: Third Ant best tour from sub-cluster two in zone one, by the improved model

The third Ant best tour obtained from sub-cluster two is as shown above with a total collection time of 1707.16sec, a tour length of 844.371m, visited 20 customers and emptied 35 bins from the customers. The chain of directed numbers below shows the path from by the tricycle as chosen by the best ant, from the cluster centre named (0) to the first customer (68c) in that sequence till the tricycle is full after collecting from customer (59) and then back to cluster centre (0) to empty into the skip container.

$$0 \to 68c \to 69c \to 70c \to 81 \to 82 \to 83 \to 79 \to 78 \to 77 \to 61 \to 70 \to 62 \to 69 \to 68 \to 63$$
$$\to 57 \to 55 \to 58 \to 54 \to 59 \to 0$$

3.2: Summary of Ant best tour, customers served and minimum distance covered in zone 1

Sub-cluster One				
Tour by improved algorithm	Customers visited	Total bins collected	Distance per improved Algorithm	Distance per classical Algorithm
$0 \to 26a \to \ldots \to 63a \to 0$	9	35	534.659	616.712
$0 \to 31 \to \ldots \to 75 \to 0$	21	35	896.562	958.129
$0 \to 74 \to \ldots \to 27a \to 0$	17	35	783.728	841.326
$0 \to 73 \to \ldots \to 30 \to 0$	20	35	774.556	833.767
$0 \to 95 \to \ldots \to 94 \to 0$	16	25	477.837	513.221
Total	**83**	**165**	**3467.342**	**3763.155**
Sub-cluster two				
Tour by improved algorithm	Customers	Total bins	Distance per improved Algorithm	Distance per classical Algorithm
$0 \to 62c \to \ldots \to 83c \to 0$	23	35	1488.572	1457.452

$0 \to 2 \to \ldots \to 47 \to 0$	17	35	917.236	1007.229
$0 \to 68c \to \ldots \to 59 \to 0$	20	35	844.371	928.215
$0 \to 81c \to \ldots \to 12 \to 0$	25	35	1204.497	1344.013
$0 \to 11 \to \ldots \to 10 \to 0$	14	21	736.460	776.889
Total	**99**	**161**	**5191.136**	**5513.798**
Sub-cluster three				
Tour by improved algorithm	Customers visited	Total bins collected	Distance per improved Algorithm	Distance per classical Algorithm
$0 \to 82a \to \ldots \to 40a \to 0$	20	35	1222.836	1308.129
$0 \to 60a \to \ldots \to 57a \to 0$	16	35	642.299	718.100
$0 \to 94a \to \ldots \to 5b \to 0$	18	35	723.667	750.235
$0 \to 1b \to \ldots \to 98a \to 0$	18	35	619.432	678.348
$0 \to 4b \to \ldots \to 100a \to 0$	13	25	384.970	409.359
Total	**85**	**165**	**3593.204**	**3863.171**
Sub-cluster four				
Tour by improved algorithm	Customers visited	Total bins collected	Distance per improved Algorithm	Distance per classical Algorithm
$0 \to 21d \to \ldots \to 54c \to 0$	21	35	1501.295	1498.087
$0 \to 39d \to \ldots \to 55c \to 0$	21	35	1729.982	1799.838
$0 \to 37c \to \ldots \to 38b \to 0$	30	35	1539.704	1632.259
$0 \to 25b \to \ldots \to 39c \to 0$	23	35	883.396	927.812
$0 \to 20c \to \ldots \to 22c \to 0$	18	26	702.733	768.328
Total	**113**	**166**	**6357.11**	**6626.324**
Sub-cluster five				
Tour by improved algorithm	Customers visited	Total bins collected	Distance per improved Algorithm	Distance per classical Algorithm
$0 \to 76b \to \ldots \to 94b \to 0$	22	35	1220.419	1309.449
$0 \to 68e \to \ldots \to 2c \to 0$	28	35	1294.256	1317.102
$0 \to 99b \to \ldots \to 57e \to 0$	31	35	1610.948	1643.589
$0 \to 64e \to \ldots \to 99d \to 0$	29	35	1251.813	1313.872
$0 \to 11e \to \ldots \to 12e \to 0$	26	26	729.788	781.879
Total	**136**	**166**	**6107.136**	**6365.891**
Sub-cluster six				
Tour by improved algorithm	Customers visited	Total bins collected	Distance per improved Algorithm	Distance per classical Algorithm
$0 \to 19d \to \ldots \to 18d \to 0$	22	35	1864.403	1934.231
$0 \to 52d \to \ldots \to 45e \to 0$	31	35	1840.000	1844.873
$0 \to 94f \to \ldots \to 1g \to 0$	31	35	1260.544	1321.501
$0 \to 4g \to \ldots \to 99f \to 0$	34	35	1353.738	1398.452
$0 \to 5g \to \ldots \to 87f \to 0$	12	12	374.643	441.560
Total	**130**	**152**	**6693.328**	**6940.617**

Sub-cluster seven				
Tour by improved algorithm	Customers visited	Total bins collected	Distance per improved Algorithm	Distance per classical Algorithm
$0 \to 62c \to \ldots \to 83c \to 0$	25	35	1179.574	1276.287
$0 \to 2 \to \ldots \to 47 \to 0$	31	35	1768.622	1699.760
$0 \to 68c \to \ldots \to 59 \to 0$	28	35	1179.954	1342.451
$0 \to 11 \to \ldots \to 10 \to 0$	19	20	699.008	756.889
Total	**103**	**125**	**4827.158**	**5075.337**

CONCLUSSION

Apart from a defined route provided by our improved algorithm for collection of solid waste in the study area, it also gave a better distance coverage as compared with the classical Ant Colony Algorithm. In sub-cluster one there was improved distance coverage of about 8.53% compared with the classical algorithm, in sub-cluster two we had 6.22% reduction, sub-cluster three 7.51%, sub-cluster four 4.23%, sub-cluster five 4.23%, sub-cluster six 3.69% and sub-cluster seven had a reduction of 5.14%.

In summary, our improved ant colony optimisation algorithm gave on average a reduction of 4.65% translating to a real saving distance of 279m in each sub-cluster and a a total saving distance of 1950m in zone one.

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