Effect of Resistance to flow and wall shear stress on axially symmetric artery for Non-Newtonian fluid Model of blood flow through a stenosis

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***Abstract*— An effort has been made to explore the non-Newtonian behaviour on blood flow through a stenosed artery with Power-law fluid model. Numerical illustration presented at the end of the paper provides the consequences for resistance to flow and wall shear stress through their graphical representations. It is seen that resistance to flow ($\bar{\lambda}$) increases with stenosis size for different values of flow index behavior (n). We have also shown the variations in wall shear stress with the axial distance (z/l_0) for different index behavior and stenosis size.**

***Keywords*— Resistance to flow, wall shear stress, axial distance, index behavior**

I. INTRODUCTION

Arteriosclerotic vascular disease (ASVD) is a situation in which an artery wall thickens as a result of the gathering of fatty materials such as cholesterol. It is a syndrome affecting arterial blood vessels, a chronic inflammatory response in the walls of arteries, caused largely by the accumulation of macrophage white blood cells and promoted by low-density lipoproteins (plasma proteins that carry cholesterol and triglycerides) without sufficient removal of fats and cholesterol from the macrophages by functional high density lipoproteins (HDL).It is commonly referred to as a hardening or furring of the arteries. It is caused by the formation of multiple plaques within the arteries. Different mathematical models have been studied by some researchers to explore the various aspects of blood flow in stenosed artery **(smith et al. 2002 and shukla et al. (1980a, b). Srivastava (2002)** investigated the effects of stenosis shape and red cell concentration (hematocrit) on blood flow characteristics due to the presence of stenosis. **Ponalagusamy (2007)** considered a mathematical model for blood flow through stenosed arteries with axially variable peripheral layer thickness and variable slip at the wall. **Mishra et al. (2008)** studied that as the height of stenosis increases in uniform or non-uniform portion of the artery, the resistance parameter also increases.**Sapna Ratan Shah (2011)** shown that the resistance to flow, apparent viscosity and wall shear stress increases with the size of the stenosis. **Singh et al. (2010)** formulated a model of blood flow through an artery for improved generalized geometry of multiple stenoses located at equispaced points. **Verma and Parihar (2010)** presented a mathematical model to study the effect of stenosis and Hematocrit on flow rate, wall shear stress and resistance parameter through tapered artery under stenotic conditions by considering laminar flow, rigid walls and Newtonian fluid. In this study, the effect of non-Newtonian behaviour on blood flow through a stenosed artery with Power-law fluid model has been investigated.

II. MATHEMATICAL FORMULATION:

Let us consider an axially symmetric, laminar, steady, one dimensional and fully developed flow of blood. The Geometry of the stenosis is given by

$$\frac{R}{R_0} = 1 - \frac{\delta}{2R_0}\left[1 + COS\,\frac{2\pi}{l_0}\left(z - d - \frac{l_0}{2}\right)\right]; d \le z \le d + l_0$$

$$=1;\ \text{Otherwise} \qquad (1)$$

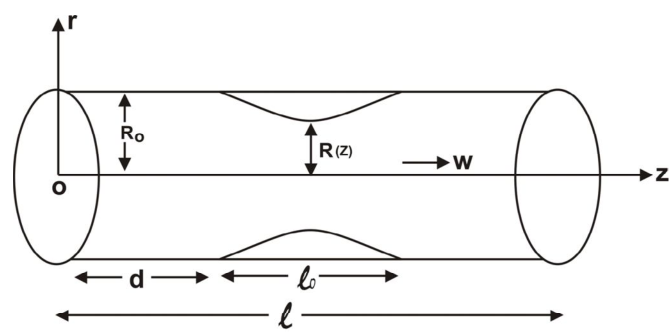

Fig 1. – Geometry of the Stenosis according to the problem

The constitutive equation for power law fluid is

$$\tau = \mu e^n, n<1 \quad \tau = \mu e^n, n<1 \qquad (2)$$

$$\tau = \frac{1}{2}\mathrm{Pr} \qquad (3)$$

$$e = -\frac{du}{dr} \qquad (4)$$

From equation (2), (3) and (4), we have

$$\frac{du}{dr} = -\left(\frac{1}{2}\frac{P}{\mu}r\right)^{\frac{1}{n}} \qquad (5)$$

Integrating (5) to obtain

$$u = \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right) \qquad (6)$$

$$Q = \int_0^R 2\pi r u dr = \left(\frac{1}{2}\frac{P}{\mu}\right)^{\frac{1}{n}} \frac{n\pi}{3n+1} R^{\frac{3n+1}{n}} \qquad (7)$$

Pressure gradient from equation can be obtained from eq. (7)

$$\frac{dp}{dz} = 2\mu\left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{1}{R^{3n+1}} \qquad (8)$$

$p = p_0$ at $z = 0$ and $p = p_l$ at $z = l$, we have

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = 1 - \frac{l_0}{l} + \frac{1}{l}\int_d^{d+l_0} \frac{1}{(R/R_0)^{3n+1}} dz$$

$$p_l - p_0 = \frac{2\mu}{R_0^{3n+1}}\left[\frac{(3n+1)}{n\pi}Q\right]^n \int_0^l \frac{1}{(R/R_0)^{3n+1}} dz \qquad (9)$$

$$\lambda = \frac{p_l - p_0}{Q} = \frac{2\mu}{QR_0^{3n+1}}\left[\frac{(3n+1)}{n\pi}Q\right]^n \int_0^l \frac{1}{(R/R_0)^{3n+1}} dz \qquad (10)$$

$$\lambda = \frac{p_l - p_0}{Q}$$

$$= \frac{2\mu}{QR_0^{3n+1}}\left(\frac{(3n+1)Q}{n\pi}\right)^n \left[\int_0^d dz + \int_d^{d+l_0} \frac{1}{(R/R_0)^{3n+1}} + \int_{d+l_0}^{l} dz\right] \qquad (11)$$

In the normal condition,

$$\lambda_N = \left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{2\mu}{QR_0^{3n+1}} l \qquad (12)$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = 1 - \frac{l_0}{l} + \frac{1}{l}\int_d^{d+l_0} \frac{1}{(R/R_0)^{3n+1}} dz \qquad (13)$$

$$a = 1 - \frac{\delta}{2R_0},\; b = \frac{\delta}{2R_0},\; n\ \theta = \pi - \frac{2\pi}{l_0}\left(z - d - \frac{l_0}{2}\right)$$

$$\frac{R}{R_0} = 1 - b(1 - \cos\theta) \qquad (14)$$

As $z = d$ implies that $\theta = 2\pi$ and $z = d + l_0$ implies that $\theta = 0$

The substitution of eqn. (14) in (13) gives

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = 1 - \frac{l_0}{l} + \frac{l_0}{2\pi l}\int_0^{2\pi} \frac{1}{(a + b\cos\theta)^{3n+1}} dz \qquad (15)$$

Now the ratio of shear stress on the wall can be obtained as

$$\bar{\tau}_R = \left(\frac{1}{(R/R_0)}\right)^{-3n} \qquad (16)$$

III. Numerical Results:

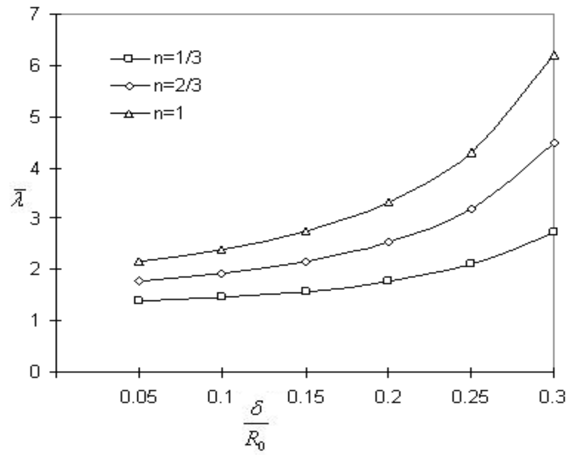

Fig 2(a)

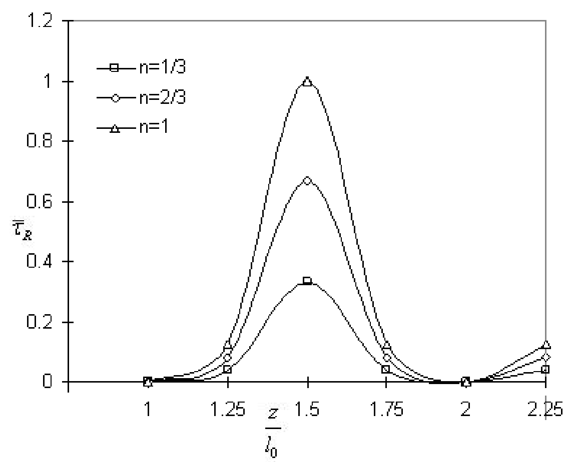

Fig. 2(b)

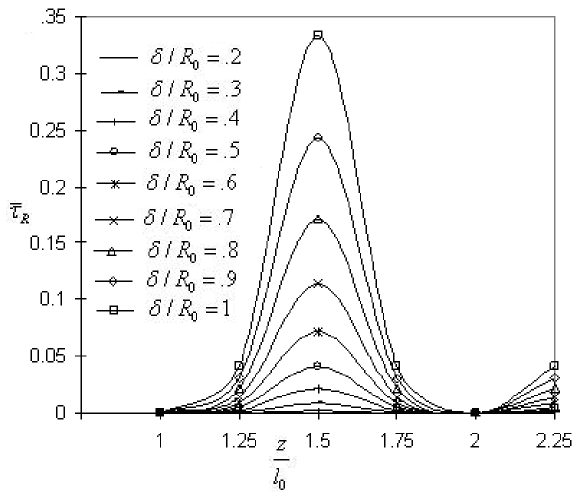

Fig. 2(c)

Fig.**2 (a)** shows that the resistance to flow increases as the stenosis height increases for different index behaviour. Fig. **2(b)** and 2**(c)** demonstrate the variation of wall shear stress with the axial distance for different values of index behaviour and stenosis heights.

IV. CONCLUSION:

In this study, we have developed a mathematical model to study the effects of stenosis height on the resistance to flow and wall shear stress for various index behaviour through an artery under stenotic conditions by considering axially symmetric, laminar, steady, one dimensional, non-Newtonian and fully developed flow of blood. The effects of variations in axial distance on the wall shear stress for different stenosis height has also been investigated. The numerical results for these expressions also have been carried out.

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