AN APPLICATION OF FUZZY SOFT RELATION IN DECISION MAKING PROBLEMS

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***Abstract*— Here we present the concept of soft relations and fuzzy soft relations and then applied in a decision making problem. In approximate reasoning, fuzzy soft relations have shown to be of a primordial importance. Human judgements are often based on comparison between couples of faced data. D. Molodtsov gave definition of fuzzy soft relation. Afterwards A. Chaudhuri gave extended definition of fuzzy soft relation. Here we discuss the concept of soft relation and fuzzy soft relation and the definition of A. Chaudhuri in the simplified form as an algorithm and apply then in decision making problem taking a separate example.**

***Keywords*— Soft Relation, Fuzzy Soft Relation, Decision Making**

I. INTRODUCTION

Most of our of real life problems in engineering, social and medical sciences, economics etc. involve imprecise data and their solution involves the use of mathematical principles based on uncertainty and imprecision. To handle such uncertainties a number of theories have been proposed for dealing with such systems in an effective way. Some of these are probability, fuzzy sets, intuitionistic fuzzy sets, interval mathematics and rough sets etc. All these theories, however, are associated with an inherent limitation, which is the inadequacy of the parameterization tool associated with these theories. In 1999, Molodtsov [7] introduced soft sets and established the fundamental results of the new theory. It is a general mathematical tool for dealing with objects which have been defined using a very loose and hence very general set of characteristics. A soft set is a collection of approximate descriptions of an object. Each approximate description has two parts: a predicate and an approximate value set. In classical mathematics, we construct a mathematical model of an object and define the notion of the exact solution of this model. Usually the mathematical model is too complicated and we cannot find the exact solution. So, in the second step, we introduce the notion of approximate solution and calculate that solution. In the Soft Set Theory, we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution. The absence of any restrictions on the approximate description in Soft Set Theory makes this theory very convenient and easily applicable in practice. We can use any parameterization we prefer with the help of words and sentences, real numbers, functions, mappings and so on. It means that the problem does not arise in Soft Set Theory. In [7], besides demarcating the basic contours of Soft Set Theory, Molodtsov also showed how Soft Set Theory is free from parameterization inadequacy syndrome of Fuzzy Set Theory, Rough Set Theory, Probability Theory and Game Theory. Soft Set Theory is a very general framework. Many of the established paradigms appear as special cases of Soft Set Theory.

Applications of Soft Set Theory in other disciplines and real life problems are now catching momentum. Molodtsov [7] successfully applied the soft theory into several directions, such as smoothness of functions, Perron integration, theory of probability, theory of measurement, and so on. Maji et al. [4] gave first practical application of soft sets in decision making problems. It is based on the notion of knowledge, reduction in rough set theory. Maji et al [1] defined and studied several basic notions of soft set theory in 2003. In 2005, Pei and Miao [3] and Chen et al. [2] improved the work of Maji et al. [4, 6].

A decision is the selection from two or more courses of action. Decision Making can be regarded as an outcome of mental processes which are basically cognitive in nature leading to the selection of a course of action among several alternatives. Every Decision Making process produces a final choice [5]. The output can be an action or an opinion of choice. Decision Making is vital choice. The problems which may be either long-range or short-range in nature; or the problems may be at relatively high or low level managerial responsibility. The Decision Theory provides a rich set of concepts and techniques to aid Decision Maker in dealing with complex decision problems.

In the present paper the concept of soft relations and fuzzy soft relations are discussed and then applied in a decision making problem. In approximate reasoning, fuzzy soft relations have shown to be of a primordial importance. Human judgements are often based on comparison between couples of faced data. Molodtsov [7] gave definition of fuzzy soft relation. Afterwards Chaudhuri gave extended definition of fuzzy soft relation. Here we discuss the concept of soft relation and fuzzy soft relation and the definition of Chaudhuri in the simplified form as an algorithm and apply then in decision making problem taking a separate example.

II. PRELIMINARIES

In this section we present the notion of soft relation and fuzzy soft relation introduced by Molodtsov in [7], extended by Chaudhuri in and some useful definitions.

Definition 1 [7]

Let X and Y are two non empty crisp sets of some Universal set U and E is a set of parameters, then, a soft relation denoted as (R, E) is defined as a mapping from E to $P(X \otimes Y)$.

Example 1:

Let

U= {Professors teaching in a college}

X={Male Professors in U}={m1, ………..,m9}

Y={Female Professors in U}={f1, …………..,f9}

E_1= {is father of, is uncle of, is husband of, is grandfather of, is son of, is nephew of}

E_2= {is mother of, is aunt of, is wife of, is grandmother of, is daughter of, is niece of}

Then (R, P) is a soft relation over $P(X \otimes Y)$ corresponding to E_1 may be given as $(R, E_1) = \{R$ *(is father of) = {(m_1, f_1), (m_2, f_3), (m_4, f_6), (m_6, f_7)}, R (is uncle of) ={(m_2, f_1), (m_3, f_5), (m_5, f_6))}, R (is husband of) = {(m_3, f_1), (m_4, f_7), (m_9, f_6)}, R (is grandfather of)= {(m_1, f_4), (m_5, f_4), (m_6, f_6)}, R (is son of)= {(m_1, f_2), (m_1, f_5), (m_4, f_3)}, R (is nephew of)= {(m_2, f_8), (m_1, f_9), (m_7, f_8)}}.*

Another soft relation R over $P(X \otimes Y)$ corresponding to E_2 may be given as *(R, E_2)= {R (is mother of)= {(f_2, m_1), (f_5, m_1), (f_3, m_4)}, R (is aunt of)= {(f_8, m_2), (f_9, m_1), (f_8, m_7)}, R (is wife of)= {(f_1, w_3), (f_7, w_4), (f_6, w_9)}, R (is grandmother of)= {(f_9, m_6), (f_8, m_5), (f_7, m_9)}, R (is daughter of)= {(f_1, m_1), (f_3, m_2), (f_6, m_4), (f_7, m_6)}, R (is niece of)= {(f_2, m_2), (f_3, m_3)}}*

Definition 2

Considering the sets E and $P(X \otimes Y) = V$ and any subset of the Cartesian product $E \times V$ is called a soft binary relation denoted by T. $\forall e \in E, v \in V, if \langle e, v \rangle \in T$, then e and v satisfy relation T i.e., eTv; otherwise $\langle e, v \rangle \notin T$, i.e., e and v do not satisfy relation *T*.

Definition 3

Let T be relation on a non empty set E. Then $\forall a, b, c \in E$, *(T, E)* is called

(i) reflexive iff aTa

(ii) symmetric iff $aTb \Rightarrow bTa$

(iii) transitive iff $aTb, bTc \Rightarrow aTc$

Definition 4

A soft relation is said to be soft equivalence relation if it satisfies reflexivity, symmetry and transitivity.

Definition 5

A soft relation is said to be soft tolerance if it is reflexive and symmetric.

Definition 6

Let X and Y are two non empty crisp sets of some Universal set U and E is a set of parameters, then, a fuzzy soft relation denoted as *(R, E)* is defined as a mapping from E to fuzzy power set $P(X \otimes Y)$

Example 2: Let *P={Paris, Berlin, Amsterdam}* and *Q={Rome, Madrid, Lisbon}* be two set of cities and *E={far, very far, near, crowed, well managed}*. Let R be the fuzzy soft relation over the sets P and Q given by *(R.E)= {R(far)= {(Paris, Rome)/0.60, (Paris, Madrid)/0.45, (Paris, Lisbon)/ 0.40, (Berlin, Rome)/0.55, (Berlin, Madrid)/0.65, (Berlin, Lisbon)/0.70, (Amsterdam, Rome)/ 0.75, (Amsterdam, Madrid)/ 0.50, (Amsterdam, Lisbon)/0.80}}.*

Definition 7

Let T be relation on a non empty set E. The *(T, E)* is called

(i) Reflexive iff $T(a, a) = 1$

(ii) Symmetric iff $T(a, b) = T(b, a)$

(iii) Transitive iff $T(a, c) \geq MAXc(T(a, b) \wedge T(b, c))$, where $\wedge$ is a t norm.

Definition 8

A fuzzy soft relation is said to be fuzzy soft equivalence relation if it satisfies reflexivity, symmetry and t- transitivity.

Definition 9

A fuzzy soft relation is said to be fuzzy soft similarity relation if it is reflexive and symmetric.

Definition 10

Let T_1 and T_2 are two soft fuzzy relations on set E. The union of T_1 and T_2 is also fuzzy soft relation denoted by

$$(T_1 \cup T_2)(a, b) = \max\{T_1(a, b), T_2(a, b)\}, \forall a, b \in E$$

Definition 11

Let T_1 and T_2 are two soft fuzzy relations on set E. The intersection of T_1 and T_2 is also fuzzy soft relation denoted by

$$(T_1 \cap T_2)(a, b) = \min\{T_1(a, b), T_2(a, b)\}, \forall a, b \in E$$

Definition 12

Let T_1 and T_2 are two soft fuzzy relations on set E. The containment of T_1 in T_2 is defined as

$$T_1 \subseteq T_2 \Rightarrow T_1(a, b) \leq T_2(a, b), \forall a, b \in E$$

Definition 13

The family of fuzzy soft equivalence classes $[e_i]_T$, written as $E/T = \{[e_i]_T \mid e_i \in T\}$ is called a fuzzy soft quotient set of *E* induced by *T*.

Molodtsov [7] introduced soft set theory and showed its applications in several different directions like game theory, operation research, smoothness of functions, Perron integration, probability, theory of measurement etc. Maji et al. [4] presented applications of soft set and fuzzy soft set decision making problems. In this section, we discuss application of fuzzy soft relation.

Suppose $U=\{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ be a set of seven objects having different colours, size and surface texture feature and *E={ large, course, dark brown, small, moderately course, fine, blackish, extra fine}* be the set of parameters. Let (F_1, A_1) be the fuzzy soft set which describes the size of the objects given by $(F_1, A_1)=\{F_1(small)=\{u_1/1, u_2/0, u_3/1, u_4/0.2, u_5/1, u_6/0.2, u_7/1\}, F_1(large)=\{ u_1/0, u_2/0.9, u_3/0.1, u_4/0.7, u_5/0.2, u_6/0.8, u_7/0.2\}\}$. Let (F_2, A_2) be the fuzzy soft set which describes the colour of the objects by $(F_2,A_2)=\{F_2(dark\ brown)=\{ u_1/0.8, u_2/0.2, u_3/0.4, u_4/0.2, u_5/0.4, u_6/0.6, u_7/0.5\}, F_2(blackish)=\{ u_1/0.3, u_2/0.7, u_3/0.5, u_4/0.6, u_5/0.2, u_6/0.3, u_7/0.4\}\}$. Let (F_3, A_3) be the fuzzy soft set which describes the surface texture of the object given by $(F_3, A_3)= \{F_3(course)=\{ u_1/0.2, u_2/0.7, u_3/0.9, u_4/1, u_5/0.8, u_6/0, u_7/0.5\}, F_3(moderately\ course)=\{ u_1/0.4, u_2/0.6, u_3/1, u_4/1, u_5/0.9, u_6/0.4, u_7/0.6\}, F_3(fine)=\{ u_1/0.7, u_2/0.5, u_3/0, u_4/0.6, u_5/0.3, u_6/0.2, u_7/0.4\}, F_3(extra\ fine)=\{ u_1/0.8, u_2/0.2, u_3/0, u_4/0.3, u_5/0.1, u_6/0.1, u_7/0.2\}\}$

Suppose that one person is interested in buying an object on the basis of his choice of parameters dark brown, course, small. This implies that from the houses available in *U*, he should select the object that satisfies with all the parameters of his choice.

The above problem obtained by Maji et al [8] requires calculating the row sum, column sum and membership scare for each house. This requires more computational time compared to the situation obtained by using fuzzy soft relation.

The definition of relation on two fuzzy soft set are as below.

Definition 14

Let *(F,A)* and *(G,B)* be two fuzzy soft sets over a common universal set. Then a relation *R* of *(F,A)* on *(G,B)* may be defined as a mapping $R: A\times B \to P(U^2)$ such that for each $e_i \in A, e_j \in B$ and for all $u_p \in F(e_i), u_q \in G(e_j)$, the relation *R* is characterized by the following membership function, $\mu_R(u_l, u_k) = \mu_{F(e_i)}(u_l) \times \mu_{G(e_j)}(u_k)$, where, $u_l \in F(e_i), u_k \in G(e_j)$.

Chaudhuri gave an alternative definition of fuzzy soft relation as follows.

Definition 15

If *(F,A)* and *(G,B)* are two fuzzy soft sets then the fuzzy soft subset (R,C) of (F,A) x (G,B) is called a fuzzy soft relation. Here $C \subset A\times B$ and $\forall (x,y) \in A\times B, R(x,y)$ is a fuzzy subset of $P(x,y)$ where. $P(x,y) = F(x) \cap G(y)$

Definition 16

The fuzzy soft set *(R,C)* of (F_i, A_i) is called an n-ary fuzzy soft relation. Here, $C \subset A_1 \times ......\times A_n$ $\forall (x_1, ..., x_n) \in A_1 \times ....\times A_n$, $R(x_1, ..., x_n) \subset O$ where, $O(x_1, ..., x_n) = F_1(x_1) \cap .... \cap F_2(x_2)$.

Thus the above mapping is well defined. Higher the value of the membership grade in the relation *R* for a pair, stronger is the parametric character present between the pair.

To simplify the above definition, we present below in the form of an algorithm.

Algorithm for selection of interested object

Step 1: Input the fuzzy soft set *(F, A)*

Step 2: Input the fuzzy soft set *(G, B)*

Step3: Calculate the fuzzy soft relation *(R, C)* of $(F,A)\times(G,B)$.

Here $C \subset A\times B$ and $\forall (x,y) \in A\times B, R(x,y)$ is a fuzzy subset of *P(x, y)* where $P(x,y) = F(x) \cap G(y)$.

Step 4: Now the person will select that object which has the largest membership value.

Here for the above problem, a fuzzy soft relation *(R, C)* among the fuzzy soft sets (F_1, A_1), (F_2, A_2) and (F_3, A_3) of the objects of *U* which are dark brown, course, small. Here the fuzzy soft relation *(R, C)* is given by $(R, C) =\{R\ (dark\ brown, course, small) = \{u_1/0.2, u_2/0, u_3/0.4, u_4/0.2, u_5/0.6, u_6/0, u_7/0.5\}$.

Thus, the object which has the largest membership value in the relation will be selected. Here, u_7 has the largest membership value equal to 0.5; hence the person will select the object u_7.

III. CONCLUSION

The concept of soft relations and fuzzy soft relations introduced by Molodtsov and Chaudhuri are discussed and express the method with the help of an algorithm applied in decision making problem with separate examples.

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