Pattern Generation for Two Dimensional Cutting Stock Problem

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***Abstract* — Selection of feasible cutting patterns in order to minimize the raw material wastage which is known as cutting stock problem has become a key factor of the success in today's competitive manufacturing industries. In this paper, solving a two-dimensional cutting stock problem is discussed. Our study is restricted to raw material (main sheet) in a rectangular shape, and cutting items are also considered as rectangular shape with known dimensions. The *Branch and Bound* approach in solving integer programming problems is used to solve the problem.**

***Keywords*— Two-Dimensional cutting stock problem, Cutting patterns, Branch and Bound algorithm**

I. INTRODUCTION

Minimizing wastage is a key factor in improving productivity of a manufacturing plant. Wastage can occur in many ways and cutting stock problem can be described under the raw material wastage. An optimum cutting stock problem can be defined as cutting a main sheet into smaller pieces while minimizing total wastage of the raw material or maximizing overall profit obtained by cutting smaller pieces from the main sheet. Many researchers have worked on the cutting stock problem and developed different algorithms to solve the problem. Among them, Hifi et al (2000) and Coromoto et al (2007) have made an approach to cut a large rectangular stock of known dimensions to *n* types of smaller rectangles of known dimensions. Hifi has made assumptions that all pieces have fixed orientation (i.e a piece of length *l* and width *w* is different from a piece of length *w* and width *l*, for all $l\ (\neq w)$, and all applied cuts are of guillotine type (a cut from one edge of the rectangle to the opposite edge which is parallel to the two remaining edges). Hifi developed a mathematical model to maximize overall profit by cutting smaller rectangular pieces from the large rectangular stock[1].

Also, Coromoto has used a *Parallel Algorithm* and *Sequential Algorithm* to solve the mathematical model which maximizes the total profit incurred by cutting *n* number of rectangular items from a large rectangular main sheet. Coromoto has made an observation that all cutting patterns can be obtained by means of horizontal and vertical builds of meta-rectangles and used *Viswanathan and Bagchi Algorithm* to produce best horizontal and vertical builds[2].

In addition to above two studies, many researchers have introduced different approaches to maximize the utilization area of the main sheet or to minimize the waste area of the main sheet, and have assumed both main sheet and smaller pieces are in rectangular shape with known dimensions[3,4,5]. There are different arrangements to cut required pieces from the existing raw material to maximize the used area. Each arrangement is defined as a cutting pattern. In this study, modified *Branch and Bound Algorithm* is presented and a computer program using Matlab software package is developed to generate feasible cutting patterns for two-dimensional cutting stock problem.

II. MATERIALS AND METHODS

Prior to finding minimum raw material wastage of two-dimensional cutting stock problem, rectangular shaped main sheet with known dimensions and required items are selected. According to the selection, a mathematical model to minimize the wastage is formulated as follows:

Following notations are introduced to describe the model:

m = Number of items,

n = Number of patterns,

p_{ij} = Number of occurrences of the i^{th} item in the j^{th} pattern,

x_j = Number of main sheets being cut according to the j^{th} pattern,

c_j = Cutting loss for each j^{th} pattern,

d_i = Demand for the i^{th} item.

Mathematical Model:

$$\text{Minimize} \quad z = \sum_{j=1}^{n} c_j x_j \quad (\text{Total Cutting Loss})$$

$$\text{Subject to} \sum_{j=1}^{n} p_{ij} x_j \geq d_i \quad \text{for all } i = 1,2,\dots,m$$

$$(\text{Demand Constraints})$$

$$x_j,\ p_{ij} \geq 0 \text{ and integer for all } i,\ j,$$

The number of occurrences of the i^{th} piece in the j^{th} pattern (p_{ij}) needs to be determined to find the optimum solution (minimum-waste arrangement) for the given mathematical model. Therefore, modified *Branch and Bound Algorithm* is used to generate feasible cutting patterns.

Here, $\sum_{i=1}^{m} p_{ij} A_i \leq L \times W$ for all $j = 1,2,\dots,n$, where A_i, L and W are the area of the i^{th} item, length and width of the main sheet respectively.

A. Modified Branch and Bound Algorithm

Step 1: Arrange required lengths, l_i, $i = 1, 2, \dots, m$ in decreasing order, ie $l_1 > l_2 > \dots > l_m$,

where m = number of items.

Arrange required widths, w_i, $i = 1, 2, \dots, m$ according to the corresponding length l_i, $i = 1, 2, \dots, m$.

Step 2: For $i = 1, 2, \dots, m$ and $j = 1$ do Steps 3 to 5.

Step 3: Set $a_{11} = \left[\left[L/l_1 \right]\right]$;

$$a_{ij} = \left[\left[\left(L - \sum_{z=1}^{i-1} a_{zj} l_z \right) \Big/ l_i \right]\right] \quad (1),$$

where L is the length of the main sheet.

Here, a_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern along the length of the main sheet and $[[y]]$ is the greatest integer less than or equal to y.

Step 4: If $a_{ij} > 0$, then set $b_{ij} = \left[\left[W/w_i \right]\right]$ (2)

else set $b_{ij} = 0$,

where W is the width of the main sheet.

Here, b_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern along the width of the main sheet.

Step 5: Set $p_{ij} = a_{ij} b_{ij}$,

where p_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern in the main sheet.

Step 6: Cutting Loss

(i) Cut loss along the length of the main sheet:

$$c_u = \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \times W$$

For $i = 1, 2, ..., m$

If $\left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \geq w_i$ and $W \geq l_i$, then

(Considering 90^o rotation for the given cutting items.)

$$\text{set } A_{ij} = \left\lfloor \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \Big/ w_i \right\rfloor ;$$

$$B_{ij} = \begin{cases} \left\lfloor W / l_i \right\rfloor, & \text{if } A_{ij} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$p_{ij} = p_{ij} + A_{ij}\, B_{ij}.$$

else set $A_{ij} = 0;$

$B_{ij} = 0;$

$P_{ij} = P_{ij}.$

If $A_{ij} > 0$, then

$$\text{set } C_u = \left[\left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) - A_{ij}\, w_i \right] \times B_{ij}\, l_i ;$$

$$C_v = \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \times \left(W - B_{ij} l_i \right).$$

$$\text{else} \quad C_u = \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \times W,$$

where, A_{ij} and B_{ij} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the c_u rectangle respectively and C_u and C_v are the total cut loss area along the length and width of the main sheet respectively.

(ii) Cut loss along the width of the main sheet:

$$c_v = \left(a_{ij}\, l_i \right) \times k_{ij}.$$

Here, $k_{ij} = W - \left(b_{ij}\, w_i \right);$

If $\left(b_{ij}\, w_i \right) = 0$, then

set $k_{ij} = 0$,

where k_{ij} is the remaining width of each item in each pattern.

For $z \neq i$

If $\left(a_{ij}\, l_i \right) \geq l_z$ and $k_{ij} \geq w_z$, then

$$\text{set } A_{zj} = \left\lfloor \left(\left(a_{ij}\, l_i \right) / l_z \right) \right\rfloor ;$$

$$B_{zj} = \begin{cases} \left\lfloor \left(k_{ij} / w_z \right) \right\rfloor, & \text{if } A_{zj} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$p_{zj} = p_{zj} + A_{zj}\, B_{zj}$$

else set $A_{ij} = 0;$

$B_{ij} = 0;$

$P_{ij} = P_{ij}.$

If $A_{zj} > 0$, then

set $C_u = \left(a_{ij}\, l_i - A_{zj}\, l_z \right) \times B_{zj}\, w_z ;$

$C_v = a_{ij}\, l_i \times \left(k_{ij} - B_{zj} l_z \right).$

else $C_v = \left(a_{ij}\, l_i \right) \times k_{ij},$

where, A_{zj} and B_{zj} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the c_v rectangle respectively.

Step 7: Set $r = m - 1$.

While $r > 0$, do Step 8.

Step 8: While $a_{rj} > 0$

set $j = j + 1$ and do Step 9.

Step 9: If $a_{rj} \geq b_{rj}$, then generate a new pattern according to the following conditions:

For $z = 1, 2, \ldots, r-1$
set $a_{zj} = a_{z\,j-1}$;
$b_{zj} = b_{z\,j-1}$.

For $z = r$
set $a_{z\,j} = a_{z\,j-1} - 1$;
if $a_{z\,j} > 0$, then set $b_{z\,j} = \left[\left[W \big/ w_z \right]\right]$;

else set $b_{z\,j} = 0$.

For $z = r+1, \ldots, m$
calculate $a_{z\,j}$ and $b_{z\,j}$ using Equations (1) and (2).

Go to Step 5.

else generate a new pattern according to the following conditions:

For $z = 1, 2, \ldots, r-1$
set $a_{zj} = a_{z\,j-1}$;
$b_{zj} = b_{z\,j-1}$.

For $z = r$
set $a_{zj} = a_{z\,j-1}$;
$b_{zj} = b_{z\,j-1} - 1$.

For $z = r+1, \ldots, m$
calculate $a_{z\,j}$ and $b_{z\,j}$ using Equations (1) and (2).

Go to Step 5.

Step 10: Set $r = r - 1$.

Step 11: STOP.

B. Illustrative Example

Following example will illustrate how to generate feasible cutting patterns by minimizing total cutting waste:

A floor tile manufacturing plant uses rectangular shaped marble sheets of length 3000 mm and width 1400 mm as raw material to cut tiles according to the given specifications. The company has received an order for bathroom tiles according to the dimensions given in Table I:

TABLE I
Required item dimensions and demand

Item No	1	2	3	4	5	6
Required Dimensions (mm)	2200×600	1000×800	1400×800	1400×600	1000×600	1600×800
Demand	1	2	1	1	2	1

Below illustrates the method described in the research paper to cut the main sheet according to the dimensions so that the total raw material wastage is minimized.

III. RESULT

Modified *Branch and Bound Algorithm* is applied to the above example to generate feasible cutting patterns as given below:

Step 1: For i = 1, 2, 3, 4, 5, 6
lengths l_i = 2200, 1600, 1400, 1400, 1000, 1000 ;
widths w_i = 600, 800, 800, 600, 800, 600.

Length (L) and width (W) of the raw material are 3000 mm and 1400 mm respectively.

Dimensions of each item:

Item no (i)	Length l_i (mm)	Width w_i (mm)
1	2200	600
2	1600	800
3	1400	800
4	1400	600
5	1000	800
6	1000	600

Step 2: For i = 1, 2, …, 6 and j = 1 do Steps 3 to 5.

Step 3: Set $a_{11} = \left[\left[L/l_1 \right]\right] = 1;$

$$a_{21} = \left[\left[\left[L - (l_1 a_{11})\right] / l_2 \right]\right] = 0;$$

$$a_{31} = \left[\left[\left[L - (l_1 a_{11}) - (l_2 a_{21})\right] / l_3 \right]\right] = 0;$$

$$a_{41} = \left[\left[\left[L - (l_1 a_{11}) - (l_2 a_{21}) - (l_3 a_{31})\right] / l_4 \right]\right] = 0;$$

$$a_{51} = \left[\left[\left[L - (l_1 a_{11}) - (l_2 a_{21}) - (l_3 a_{31}) - (l_4 a_{41})\right] / l_5 \right]\right] = 0;$$

$$a_{61} = \left[\left[\left[L - (l_1 a_{11}) - (l_2 a_{21}) - (l_3 a_{31}) - (l_4 a_{41}) - (l_5 a_{51})\right] / l_6 \right]\right] = 0.$$

Step 4: $a_{11} > 0$, then set $b_{11} = \left[\left[W/w_{11} \right]\right] = 2;$

$a_{21} = 0$, then $b_{21} = 0;$

$a_{31} = 0$, then $b_{31} = 0;$

$a_{41} = 0$, then $b_{41} = 0;$

$a_{51} = 0$, then $b_{51} = 0;$

$a_{61} = 0$, then $b_{61} = 0.$

Step 5:

$$\text{Set Pattern 1} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Step 6: Cut loss

(i) Cutting loss along the length of the main sheet:

$$c_u = [L - (l_1 a_{11}) - (l_2 a_{21}) - (l_3 a_{31}) - (l_4 a_{41}) - (l_5 a_{51}) - (l_6 a_{61})] \times W$$
$$c_u = 800 \times 1400 = 1{,}120{,}000 \text{ mm}^2.$$

For i = 1, 2

set $A_{ij} = 0;$

$B_{ij} = 0$. (Conditions are not satisfied given in Step 6 part (i))

For i = 3, dimensions of Item 3 are of length (l_3) 1400 mm and width (w_3) 800 mm and conditions are satisfied given in Step 6 part (i).

$$\text{set } A_{31} = \left[\left[800/w_3 \right]\right] = 1;$$

$$B_{31} = \left[\left[1400/l_3 \right]\right] = 1.$$

$A_{31} > 0$, then

$$\text{set } C_u = [800 - (A_{31} w_3)] \times B_{31} l_3 = 0 \times 1400 = 0 \text{ mm}^2;$$

$$C_v = 800 \times [1400 - (B_{31} l_3)] = 800 \times 0 = 0 \text{ mm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + \begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}2\\0\\1\\0\\0\\0\end{bmatrix}.$$

(ii) Cutting loss along the width of the main sheet:

$$c_v = a_{11} l_1 \times [1400 - (b_{11} w_1)]$$
$$= 2200 \times 200 = 440{,}000 \text{ mm}^2$$

For $z = 2, \ldots, 6$

set $A_{ij} = 0$;

$B_{ij} = 0$. (Conditions are not satisfied given in Step 6 part (ii))

$$C_v = 2200 \times 200 = 440{,}000 \text{ mm}^2.$$

$$\text{Pattern 1} = \begin{bmatrix}2\\0\\1\\0\\0\\0\end{bmatrix} \text{ and}$$

total cutting loss = 440, 000 mm^2.

Step 7: Set $r = 6 - 1 = 5 > 0$

Step 8: $a_{51} = 0$, then go to Step 10.

Step 10: Set $r = 5 - 1 = 4 > 0$

Step 8: $a_{41} = 0$, then go to Step 10.

Step 10: Set $r = 4 - 1 = 3 > 0$

Step 8: $a_{31} = 0$, then go to Step 10.

Step 10: Set $r = 3 - 1 = 2 > 0$

Step 8: $a_{21} = 0$, then go to Step 10.

Step 10: Set $r = 2 - 1 = 1 > 0$

Step 8: $a_{11} > 0$, then

set $j = j + 1 = 2$ and go to Step 9.

Step 9: $a_{11} < b_{11}$, then generate a new pattern j (= 2) according to the following conditions:

set $a_{12} = a_{11} = 1$; $b_{12} = b_{11} - 1 = 1$.

$$a_{22} = \left[\left[[L - (l_1 a_{12})] \big/ l_2 \right]\right] = 0;\ b_{22} = 0.$$

$$a_{32} = \left[\left[[L - (l_1 a_{12}) - (l_2 a_{22})] \big/ l_3 \right]\right] = 0;\ b_{32} = 0.$$

$$a_{42} = \left[\left[[L - (l_1 a_{12}) - (l_2 a_{22}) - (l_3 a_{32})] \big/ l_4 \right]\right] = 0;\ b_{42} = 0.$$

$$a_{52} = \left[\left[[L - (l_1 a_{12}) - (l_2 a_{22}) - (l_3 a_{32}) - (l_4 a_{42})] \big/ l_5 \right]\right] = 0;\ b_{52} = 0.$$

$$a_{62} = \left[\left[[L - (l_1 a_{12}) - (l_2 a_{22}) - (l_3 a_{32}) - (l_4 a_{42}) - (l_5 a_{52})] \big/ l_6 \right]\right] = 0;\ b_{62} = 0.$$

Step 5:

$$\text{Set Pattern 2} = \begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix}.$$

Step 6: Cut loss

(i) Cutting loss along the length of the main sheet:

$$c_u = [L - (l_1 a_{12}) - (l_2 a_{22}) - (l_3 a_{32}) - (l_4 a_{42}) - (l_5 a_{52}) - (l_6 a_{62})] \times W$$
$$c_u = 800 \times 1400 = 1{,}120{,}000 \text{ mm}^2.$$

For $i = 1, 2$

set $A_{ij} = 0$;

$B_{ij} = 0$.

For $i = 3$, dimensions of Item 3 are of length (l_3) 1400 mm and width (w_3) 800 mm and conditions are satisfied given in Step 6 part (i).

$$\text{set } A_{32} = \left[\left[{}^{800}\!/_{w_3} \right]\right] = 1;$$

$$B_{32} = \left[\left[{}^{1400}\!/_{l_3} \right]\right] = 1.$$

$A_{32} > 0$, then

$$\text{Set } C_u = [800 - (A_{32} w_3)] \times B_{32} l_3$$
$$= 0 \times 1400 = 0 \text{ mm}^2;$$

$$C_v = 800 \times [1400 - (B_{32} l_3)]$$
$$= 800 \times 0 = 0 \text{ mm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

(ii) Cutting loss along the width of the main sheet:

$$c_v = a_{12} l_1 \times [1400 - (b_{12} w_1)]$$
$$= 2200 \times 800 = 1{,}760{,}000 \text{ mm}^2.$$

For $z = 2$, dimensions of Item 2 are of length (l_2) 1600 mm and width (w_2) 800 mm and conditions are satisfied given in Step 6 part (ii).

$$\text{set } A_{22} = \left[\left[{}^{2200}\!/_{l_2} \right]\right] = 1;$$

$$B_{22} = \left[\left[{}^{800}\!/_{w_2} \right]\right] = 1.$$

$A_{22} > 0$, then

$$C_u = [2200 - (A_{22} w_2)] \times B_{22} l_2$$
$$= 600 \times 800 = 480{,}000 \text{ mm}^2,$$

$$C_v = 2200 \times [800 - (B_{22} l_2)]$$
$$= 2200 \times 0 = 0 \text{ mm}^2.$$

$$\text{Pattern 2} = \text{Pattern 2} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\text{Pattern 2} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and}$$

total cutting loss = 480,000 mm^2.

The algorithm proceeds in the same manner to generate all the cutting patterns shown in Table II for the $3000\text{ mm} \times 1400\text{ mm}$ standard dimension.

The Table II exhibits the generated cutting patterns for optimum waste:

TABLE II

Generated cutting patterns.

Required Dimensions	Cutting patterns																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$2200 \times 600\text{ mm}^2$	2	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
$1600 \times 800\text{ mm}^2$	0	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
$1400 \times 800\text{ mm}^2$	1	1	1	1	0	1	0	0	0	2	1	2	1	1	1	0	0	0	1	0
$1400 \times 600\text{ mm}^2$	0	0	0	2	3	2	2	1	1	0	3	2	2	2	2	4	3	3	2	2
$1000 \times 800\text{ mm}^2$	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	1	1
$1000 \times 600\text{ mm}^2$	0	0	2	0	0	0	1	0	2	0	0	0	1	0	2	0	0	2	0	1
Cut Loss ($x10^4$) mm^2	44	48	46	12	40	12	59	128	78	64	56	28	75	60	10	84	88	38	60	107

Required Dimensions	Cutting patterns				
	21	22	23	24	25
$2200 \times 600\text{ mm}^2$	0	1	0	0	0
$1600 \times 800\text{ mm}^2$	0	0	0	0	0
$1400 \times 800\text{ mm}^2$	1	0	0	0	0
$1400 \times 600\text{ mm}^2$	2	0	1	0	0
$1000 \times 800\text{ mm}^2$	0	3	2	1	0
$1000 \times 600\text{ mm}^2$	2	0	2	4	6
Cut Loss ($x10^4$) mm^2	10	48	46	80	30

There are 25 feasible cutting patterns available to cut raw material with the dimensions $3000\text{ mm} \times 1400\text{ mm}$ into required rectangular shaped items. The mathematical model is developed to design generated cutting patterns so that waste (cut loss) will be minimized and the optimum solution to the model is given in Table III:

TABLE III

Optimum Solution

Required Dimensions	Optimal Patterns		Demand
	2	23	
$2200 \times 600\text{ mm}^2$	1	0	1
$1600 \times 800\text{ mm}^2$	1	0	1
$1400 \times 800\text{ mm}^2$	1	0	1
$1400 \times 600\text{ mm}^2$	0	1	1
$1000 \times 800\text{ mm}^2$	0	2	2
$1000 \times 600\text{ mm}^2$	0	2	2
# of sheets from			

each pattern	1	1	

Z_{min} = 940, 000 cm^2 (Total cut loss = 940, 000 mm^2).

IV. CONCLUSION

In this study, a cutting stock problem is formulated as a mathematical model based on the concept of cutting patterns. As given in Table II, twenty five cutting patterns are generated and only two cutting patterns are selected as given in Table III to cut the main sheet according to the requirements. In this case study, the plant assumes that all the extra pieces from each item as wastage. Also, dimensions of cutting items are large and the total cut loss can be decreased if there are smaller rectangular shaped cutting items.

Twenty three feasible cutting patterns can be generated by applying 90^o rotation to the main sheet. In this case study, it is better to use cutting patterns without permitting rotation to the main sheet because Item 1 (2200 mm × 600 mm) and Item 2 (1600 mm × 800 mm) cannot be cut, if 90^0 rotation is applied to the main sheet.

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