# Effect of Hall current on Unsteady Flow of a Dusty Conducting Fluid through porous medium between Parallel Porous Plates with Temperature Dependent Viscosity and Thermal Radiation

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## Abstract

The purpose of present problem is to studies of effects of variable viscosity, hall current on unsteady laminar flow of dusty conducting fluid between parallel porous plates through porous medium with temperature dependent viscosity with thermal radiation. The fluid is considered as unsteady laminar flow through porous medium and acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates with hall current. It is assumed that the parallel plates are porous and subjected to a uniform suction from above and injection from below. It is also considered the viscosity is temperature dependent. The governing nonlinear partial differential equations are solved finite difference approximation. The results for temperature field and velocity for fluids and dust particles have been obtain numerically and displayed graphically.

**Key words:** MHD, Radiation parameter, hall current, porous medium, transient state, two-phase flow, dust particles, finite differences.

## Introduction

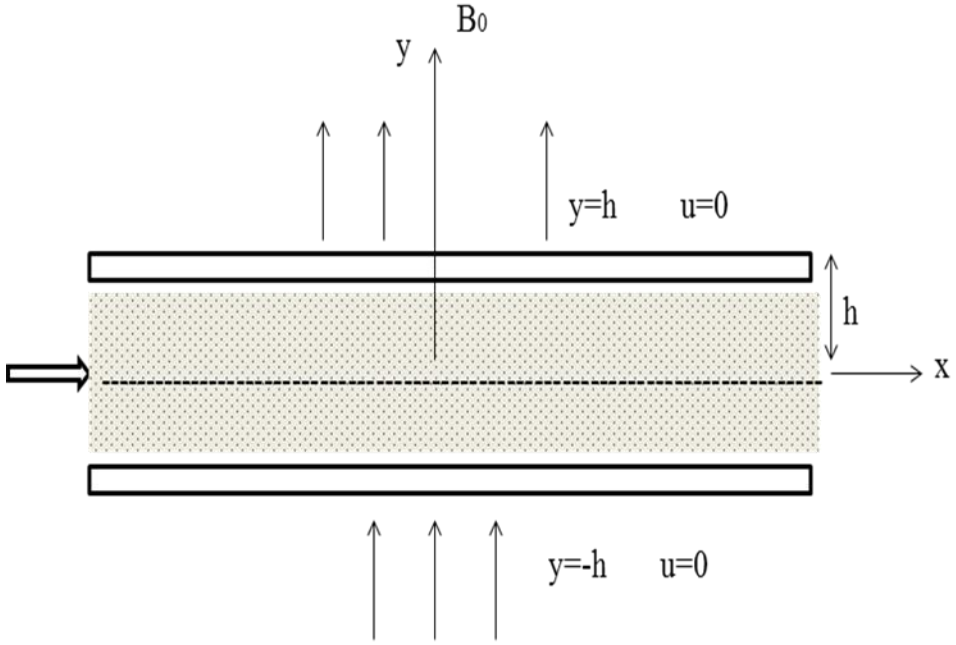
The flow of a dusty and electrically conducting fluid through a channel in the presence of a transverse magnetic field through porous medium has important application in such areas as magneto hydrodynamic generators, pumps, accelerators, cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, and fluid droplets sprays. On the other hand, flow through porous medium have numerous engineering and geophysical applications for examples, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs, in view of these application, the object of the present paper is to study the effect of parallel plates through a porous medium with temperature dependent viscosity performance and efficiency of these devices are influenced by the present of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators.

The hydrodynamic flow of dusty fluids has been studied by Saffman in (1962) on the stability of laminar flow of dusty gas. Soundalgekar (1973) studied on free convection effects on the oscillatory flow past on infinite vertical porous plates. Kim (2000) discussed on unsteady MHD convective heat transfer past a semi-infinite vertical porous plate with variable suction. Attia (2002) studied on influence of temperature dependent viscosity on MHD coquette flow of dusty fluid with heat transfer. Ramanathan and Suresh (2004) discussed on effect of magnetic field dependent viscosity and anisotropy of porous medium free convection. Attia (2005) have studied on unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity. Seeton (2006) studied on viscosity temperature correlation for liquid. EI-Amin *et. al* (2007) have discussed on effect of thermal radiation on natural convection in a porous medium. Attia (2008) studied on unsteady hydromagnetic Coquette flow of dusty fluid with temperature dependent viscosity and thermal conductivity. Ezzat *et. al* (2010) discussed on Space approach to the hydro-magnetic flow of dusty fluid through a porous medium. Chauhan and Agarwal (2011) studied on MHD through a porous medium adjacent to a stretching sheet numerical and an approximation solution. Recently in year Unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity.

In the present work, the effects of variable viscosity, hall current on the unsteady laminar flow of an electrically conducting, viscous, incompressible dusty fluid, variable viscosity and heat transfer between parallel non-conducting porous plates with thermal radiation is studied. The fluid is flowing through porous medium between two electrically insulating infinite plates maintained at two constant but different temperatures. An external uniform magnetic field is applied perpendicular to the plates with hall current. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected. The fluid is acted upon by a constant pressure gradient and its viscosity is assumed to vary exponentially with temperature. The flow and temperature distributions of both the fluid and dust particles are governed by the coupled set of the momentum and energy equations. The joule and viscous dissipation terms in the energy equation are taken into consideration. The governing coupled nonlinear partial differential equations are solved numerically using the finite difference approximations. The effects of thermal radiation, hall current and the temperature dependent viscosity on the time development of both the velocity and temperature distributions are discussed.

**Description of the Problem**

The dusty fluid is assumed to be flowing through porous medium between two infinite horizontal plates located at the $y = \pm h$ planes, as show in Figure. The dusty particles are assumed to be uniformly distributed throughout the porous fluid. The two plates are assumed to be electrically non-conducting and kept at two constant temperatures: $T_1$ for the lower plate and $T_2$ for the upper plate with $T_2 > T_1$. A constant pressure gradient is applied in the $x$- direction and the parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below. Thus they component of the velocity is constant and denoted by $v_0$. A uniform magnetic field $B_0$ is applied in the positive $y$- direction. By assuming a very small magnetic Reynolds number the induced magnetic field is neglected [15]. The fluid motion start from rest at $t = 0$ and the no-slip condition at the plates implies that the fluid and dust particles velocities have neither a $z$-axis nor an $x$- component at $y = \pm h$. The initial temperatures of the fluid and dust particles are assumed to be equal to $T_1$ and the fluid viscosity is assumed to vary exponentially with temperature. Since the plates are infinite in the $x$- and $z$-direction, the physical variables are invariant in these directions. The flow of the fluid through porous medium is governed by the Navier-Stokes equations [15].




The Geometry of the problem

$$\rho\frac{\partial u}{\partial t}+\rho v\frac{\partial u}{\partial y}=-\frac{dP}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B_\circ^2 u}{\left(1+m^2\right)}-\frac{\mu}{K_0'}u-KN\left(u-u_p\right) \quad ...(1)$$

Where $\rho$ is the density of clean fluid, $\mu$ is the viscosity of clean fluid, $u$ is the velocity of fluid, $u_p$ is the velocity of dust particles, $\sigma$ is the electric conductivity, $p$ is the pressure acting on the fluid, $N$ is the number of dust particles per unit volume, *m* is the hall current parameter, $K_0^{'}$ is porosity parameter and *K* is a constant. The first five terms in the right hand side are, respectively, the pressure gradient, viscous force, Lorentz force terms and Porous medium. The last term represents the force term due to the relative motion between fluid and dust particles. It is assumed that the Reynolds the force term due to the relative velocity is small. In such a case the force between dust and fluid is proportional to the relative velocity [1]. The motion of the dust particles is governed by Newton's second law [1] via

$$m_p \frac{\partial u_p}{\partial t} = KN\left(u - u_p\right) \qquad ...(2)$$

Where $m_p$ is the average mass of dust particles. The initial and boundary conditions on the velocity fields are respectively given by

$$t = 0; \qquad u = u_p = 0 \qquad ...(3)$$

For $t > 0$, the no-slip condition at the plates implies that

$$y = -h: \qquad u = u_p = 0 \qquad ...(4)$$

$$y = +h: \qquad u = u_p = 0 \qquad ...(5)$$

Heat transfer takes place from the upper hot plate towards the lower cold plate by conduction through porous medium the fluid. Also there is a heat generation due to both the joule and viscous dissipations. The dust particles gain heat energy from the fluid by conduction through their spherical surface. Two energy equations are required which describe the temperature distribution for both the fluid and dust particles and are respectively given by [16]

$$\rho c \frac{\partial T}{\partial t} + \rho c \upsilon_\circ \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 - \frac{\partial q_r}{\partial y} + \sigma B_\circ^2 u^2 + \frac{\rho_p C_s}{\gamma T}\left(T_p - T\right) \qquad ...(6)$$

$$\frac{\partial T_p}{\partial t} = -\frac{1}{\gamma T}\left(T_p - T\right) \qquad ...(7)$$

Where *T* is the temperature if the fluid, $T_p$ is the temperature of the particles, *c* is the specific heat capacity of the of fluid at constant pressure, $C_s$ is the specific heat capacity of the particles, $k$ is the thermal conductivity of the fluid, $q_r$ is the local radiative heat flux, $\gamma T$ is the temperature relaxation time $\left(= 3\Pr \gamma_p C_s / 2c\right)$, $\gamma_p$ is the velocity relaxation time $\left(= 2\rho_s D^2 / 3\mu\right)$, $\rho_s$ is the material density of the dust particles $\left(= 3\rho_p \mu / 2D^2 KN\right)$, right-hand side of Eq.(6) represent the viscous dissipation, thermal radiation, the joule dissipation and the heat conduction between the fluid and dust particles, respectively. The initial and boundary conditions on the temperature fields are given as

$$t \le 0: \qquad T = T_p = 0, \qquad ...(8)$$

$$t > 0, \qquad y = -h: \qquad T = T_p = T_{1}, \qquad ...(9)$$

$$t > 0, \qquad y = h: \qquad T = T_p = T_{2}, \qquad ...(10)$$

The viscosity of the fluid is assumed to depend on temperature and is defined as $\mu = \mu_\circ f\left(T\right)$. By assuming the viscosity to vary exponentially with temperature, the function $f\ (T)$ Takes the form [13, 14] $f\left(T\right) = c^{-b(T-T_1)}$, where the parameter $b$ has the dimension of $T^{-1}$ and such that at $T = T_1,\ f(T_1) = 1$ and then $\mu = \mu_\circ$. This means that $\mu_\circ$ is the velocity coefficient at *T* = $T_1$. The parameter $b$ may take positive values for liquids such as water, benzene or crude oil. In some gases like air, helium or methane $a_1$ may be negative, *i.e.* the coefficient viscosity increases with temperature [13, 14].

The radiative heat flux term by using the Rosseland approximation is given by

$$q_r = -\frac{4\sigma}{3k_1}\frac{\partial T^4}{\partial y} \qquad \ldots(11)$$

Where $\sigma$ and $k_1$ are respectively the Stefan-Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference within the flow are sufficiently small such that $T^4$ may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about $T_\infty$ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T \;\; - 3T_\infty^4 \qquad \ldots(12)$$

By using equations (11) and (12), into equation (6) is reduced to

$$\rho c\frac{\partial T}{\partial t} + \rho c\upsilon_\circ \frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{16\sigma T_\infty^3}{3k_1}\frac{\partial^2 T}{\partial y^2}$$

$$+\sigma B_\circ^2 u^2 + \frac{\rho_p C_s}{\gamma T}\left(T_p - T\right) \qquad \ldots(13)$$

$$\frac{\partial T_p}{\partial t} = -\frac{1}{\gamma T}\left(T_p - T\right) \qquad \ldots(14)$$

The problem is simplified by writing the equations in dimensionless form. The characteristic length is taken to be h, and the characteristic time is $\rho h^2 / \mu_\circ$, while the characteristic velocity is $\mu_\circ / h\rho$. Thus we define the following non-dimensional quantities:

$$\left(x', y', z'\right) = \frac{1}{h}\left(x, y, z\right),$$

$$t' = \frac{t\mu_\circ}{\rho h^2}, \qquad p' = \frac{P\rho h^2}{\mu_0^2}, \qquad \alpha = \frac{dp'}{dx'}, \quad \frac{1}{K_0} = \frac{\mu h^2}{K_0' \mu_0},$$

$$\left(u', \upsilon', w'\right) = \frac{\rho h}{\mu_\circ}\left(u, \upsilon, w\right), \qquad \left(u_p', \upsilon_p', w_p'\right) = \frac{\rho h}{\mu_\circ}\left(u_p, \upsilon_p, w_p\right)$$

$$T' = \frac{T - T_1}{T_2 - T_1}, \qquad T_p' = \frac{T_p - T_1}{T_2 - T_1},$$

$$f\left(T'\right) = e^{-b(T_2 - T_1)T'} = e^{-aT'},$$ Where $a$ is the viscosity parameter

$$H_a^2 = \frac{\sigma B_0^2 h^2}{\mu_\circ}$$ Where $H_a$ is the Hartmann number

$$R = \frac{KNh^2}{\mu_\circ}$$ is the particle concentration parameter

$$G = \frac{m_p \mu_\circ}{\rho h^2 K}$$ is the particle mass parameter

$$\xi = \frac{\rho h\upsilon_\circ}{\mu_\circ}$$ is the suction parameter

$R_a = \frac{4\sigma T_\infty^3}{k_1 k}$ is the thermal radiation parameter

$\Pr = \frac{\mu_\circ c}{k}$ is the Prandtl number

$Ec = \frac{\mu_\circ^2}{hcp^2 (T_2 - T_1)}$ is the Eckert number

$L_0 = \frac{\rho h^2}{\mu_\circ \gamma T}$ is the temperature relaxation time parameters.

In terms of the above dimensionless variables and parameters, equation equations (1)-(5), (13)-(14) with boundary condition (8)-(10) take the following form (where we have dropped the hats for convenience):

$$\frac{\partial u}{\partial t} + \xi \frac{\partial u}{\partial y} = a + f(T)\frac{\partial^2 u}{\partial y^2} + \frac{\partial f(T)}{\partial y}\frac{\partial u}{\partial y} - \left( \frac{H_a^2}{(1+m^2)} + \frac{1}{K_0} \right) u - R(u - u_p) \quad \text{...(15)}$$

$$G\frac{\partial u_p}{\partial t} = (u - u_p) \quad \text{...(16)}$$

$$t \le 0; \qquad u = u_p = 0 \quad \text{...(17)}$$

$$t > 0, \quad y = -1, \quad u = u_p = 0 \quad \text{...(18)}$$

$$t > 0, \quad y = 1, \quad u = u_p = 0 \quad \text{...(19)}$$

$$\frac{\partial T}{\partial t} + \xi \frac{\partial T}{\partial y} = \frac{1}{\Pr}\left(1 + \frac{4R_a}{3}\right)\frac{\partial^2 T}{\partial y^2} + Ecf(T)\left(\frac{\partial u}{\partial y}\right)^2 + EcH_a^2 u^2 + \frac{2R}{3\Pr}(T_p - T) \quad \text{...(20)}$$

$$\frac{\partial T_p}{\partial t} = -L_\circ (T_p - T) \quad \text{...(21)}$$

$$t \le 0; T = T_p = 0 \quad \text{...(22)}$$

$$t > 0, \quad y = -1; \quad T = T_p = 0, \quad \text{...(23)}$$

$$t > 0, \quad y = 1; \quad T = T_p = 1. \quad \text{...(24)}$$

Equations (11), (12), (20) and (21) represent a system of coupled and nonlinear partial differential equations which must be solved numerically under the initial and boundary conditions (17)-(19) and (22)-(24) using finite difference approximations [17]. The nonlinear terms are first linearized and then an iterative scheme is used at every time-step to solve the linearized system of difference equations.

## Results and discussions

The exponential dependence of viscosity on temperature results in decomposing the viscous force term $=\frac{\partial}{\partial y}\left(\frac{\mu \partial u}{\partial y}\right)$ in Eq. (1) into two terms. the variations of those resulting terms with the viscosity parameter *a*, radiation parameter ($R_a$), hall current parameter ($m$), Magnetic field ($H_a$), permeability of porous medium ($K_0$), Concentration parameter ($R$), Suction parameter $(\xi)$ and Prandtl number (Pr) have an important effect on the flow and temperature fields. In the following discussion selected parameters are given the following fixed values $G$ = 0.8, α = 5, $Ec$ = 0.03 and $L_0$ = 0.7. Numerical calculations have been carried out for dimensionless velocity of dusty fluid ($u$), velocity of dust particle ($u_p$), temperature profiles $T$ and temperature of dust particle $T_p$ for different values of parameters which are displayed in Figures-(1) to (21).

Figures – (1) and (2) depict that with the increase in Hartmann ($Ha$) the velocity of fluid and dust particles decreases. This agrees with the natural phenomena because in the presence of magnetic field, Lorentz force sets in, which impedes the velocity of fluid and dust particles.

Figures – (3), (4) and (5) depict that with increase in porosity parameter ($K_0$) and hall current parameter ($m$), then increasing the value of the velocity of fluid and dust particles.

Figures – (6) and (7) indicate the variations of the velocity of fluid ($u$) and velocity of dust particles ($u_p$) at the centre of the channel $y$ = 0 with time for different values of the viscosity parameter *a*. The figures show that increasing viscosity parameter *a* increases the velocities and the time required to approach the steady state. This implies that higher velocities are obtained at lower viscosities. The effect of the viscosity parameter *a* on the steady state time is more pronounced for positive values of a than for negative values. Notice that the velocity of fluid ($u$) reaches the steady state more quickly than velocity of dust particles ($u_p$). This is because the fluid velocity is the source for the dust particles velocity.

Figures – (8) and (9) depict that with increase in suction parameter of particle $(\xi)$ increasing the value of velocity of fluid $u$ and dust particles ($u_p$) because; increase in suction parameter of particle $(\xi)$ reduces mass forces.

Figure – (10), (11), (12) and (13) show the variations of the temperatures profile of fluid ($T$) and fluid particles $T_p$ at the centre of the channel $y$ = 0 with time for different values of Suction parameter ( $\xi$ ) and Prandtl number (Pr). The figures show that increasing Suction parameter ( $\xi$ ) and Prandtl number (Pr), decreases the temperatures and the steady state times.

Figures – (14), (15), (16) and (17) indicate the variations of the both velocities $u$ and $u_p$ at the centre of the channel $y$ = 0 with time and both temperatures $T$ and $T_{\text{p}}$ for different values of the concentration parameter ($R$). It is clear that the suction velocity decreases both $u$ and $u_p$ and both temperatures $T$ and $T_{\text{p}}$ are decreases with increasing the value of concentration parameter ($R$).

Form figures – (18) and (19) that the velocity of fluid ($u$) and the temperature profile of fluid ($T$) is increasing with increasing the value of radiation parameter ($R_a$). The boundary layer and thermal boundary layer thicknesses increase with increase the radiation parameter ($R_a$).

## Conclusions

In this paper the effect of a temperature dependent viscosity, thermal radiation, hall current parameter, the particle concentration parameter, Suction parameter, suction and injection velocity and an external uniform magnetic field on the unsteady laminar flow and temperature distributions of an electrically conducting viscous incompressible dusty fluid between two parallel porous plate through porous medium has been studied. The viscosity was assumed to vary exponentially with temperature and the joule and viscous dissipations were taken into consideration. Also, changing the viscosity parameter *a* leads to asymmetric velocity profiles about the central plane of the channel $(y = 0)$, which is similar to the effect of variable percolation perpendicular to the plates. The effect of the suction velocity on both the velocity and temperature of the fluid and particles is more pronounced for higher values of the parameter *a* and porosity parameter $K_0$.

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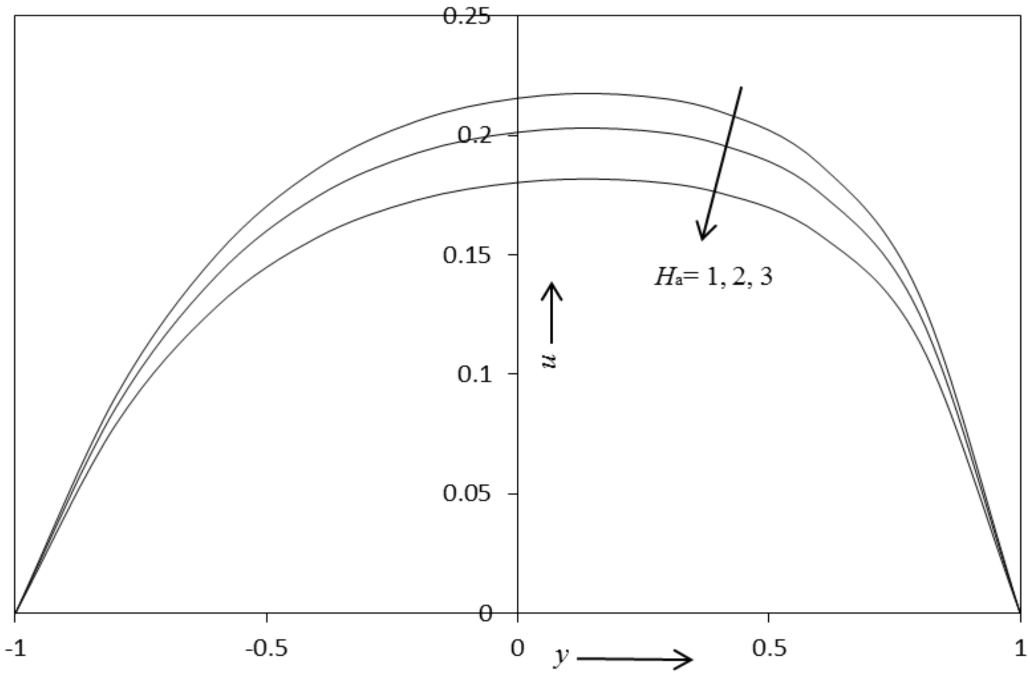


Fig. - 1: Velocity profile of fluid for different values of $H_0$.

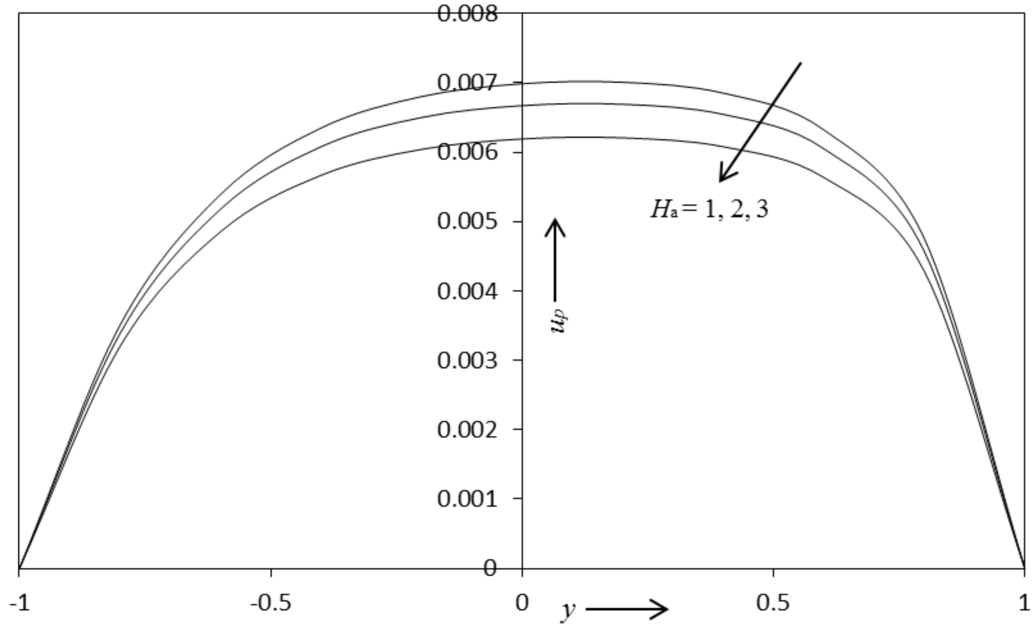


Fig. - 2: Velocity profile of dust particle for different values of $H_0$.

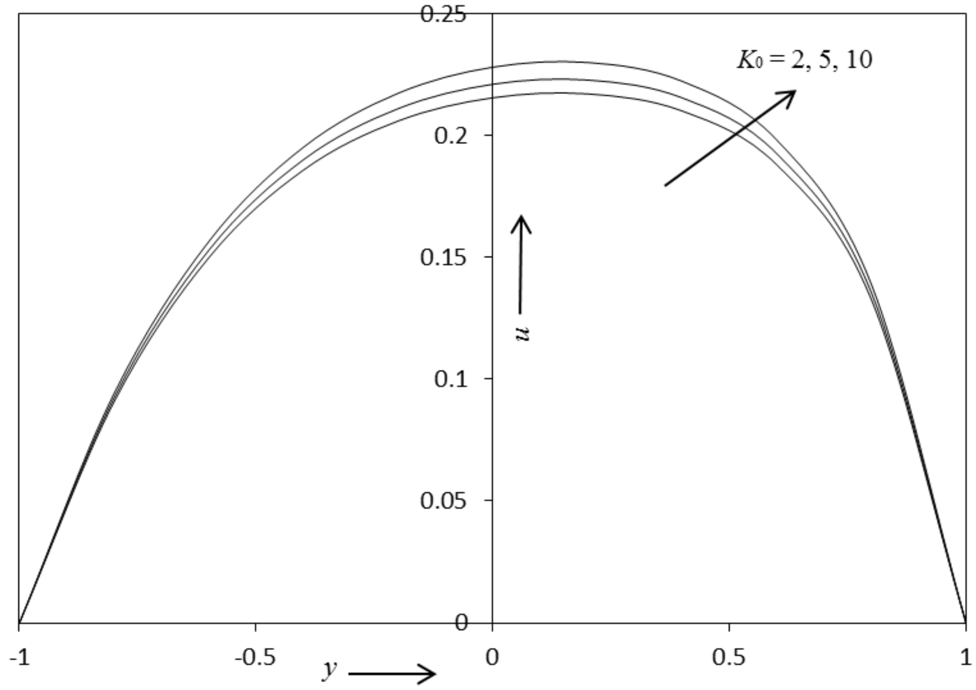


Fig. - 3: Velocity profile of fluid for different values of $K_0$.

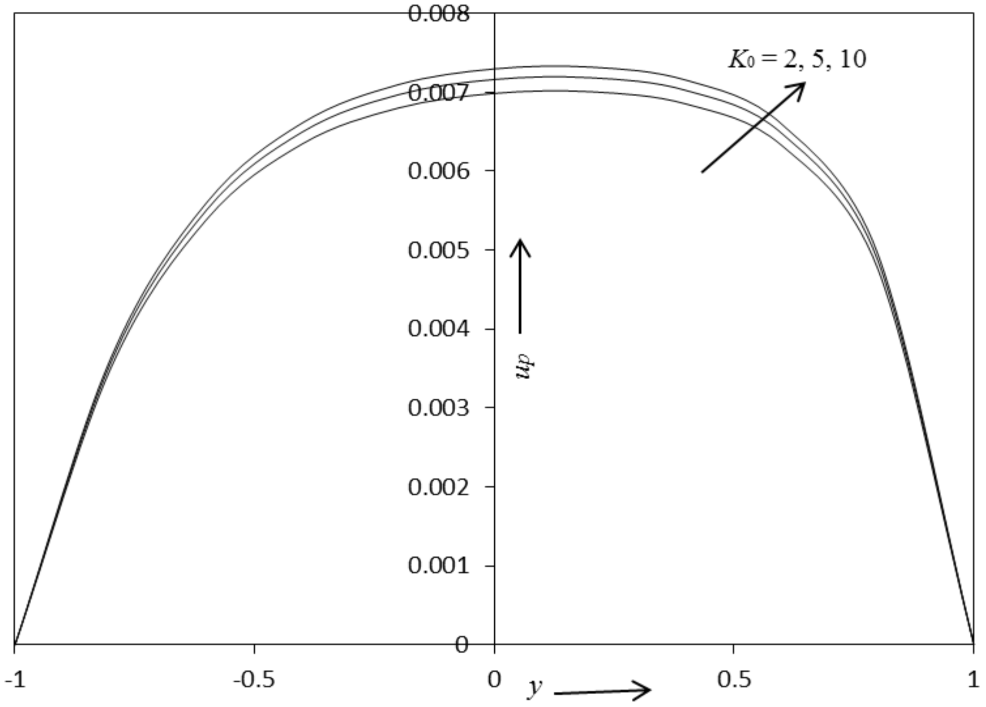


Fig. - 4: Velocity profile of dust particle for different values of $K_0$.

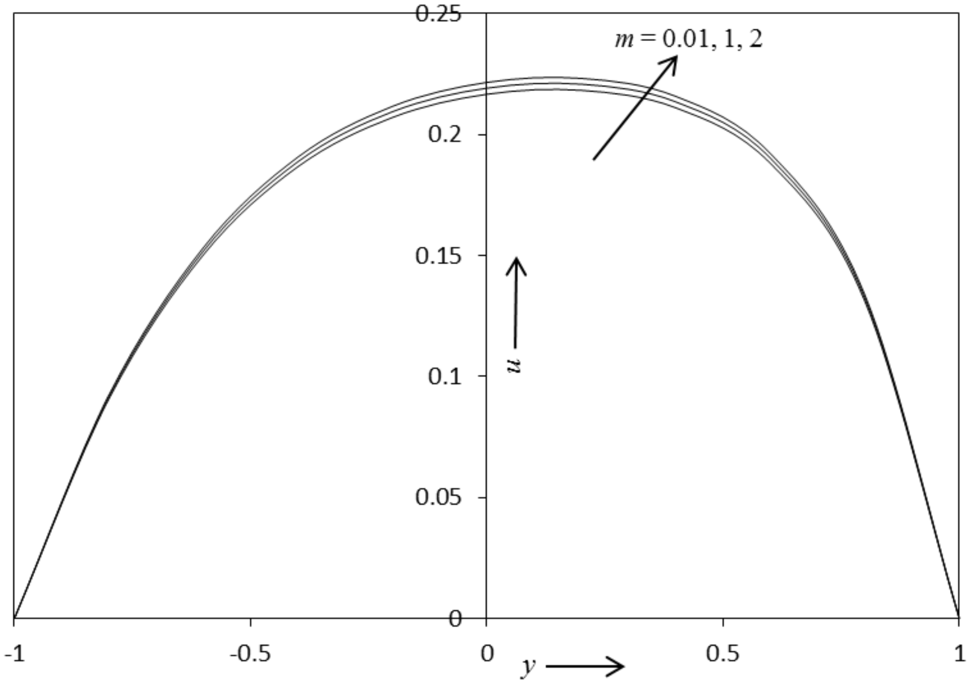


Fig. - 5: Velocity profile of fluid for different values of $m$.

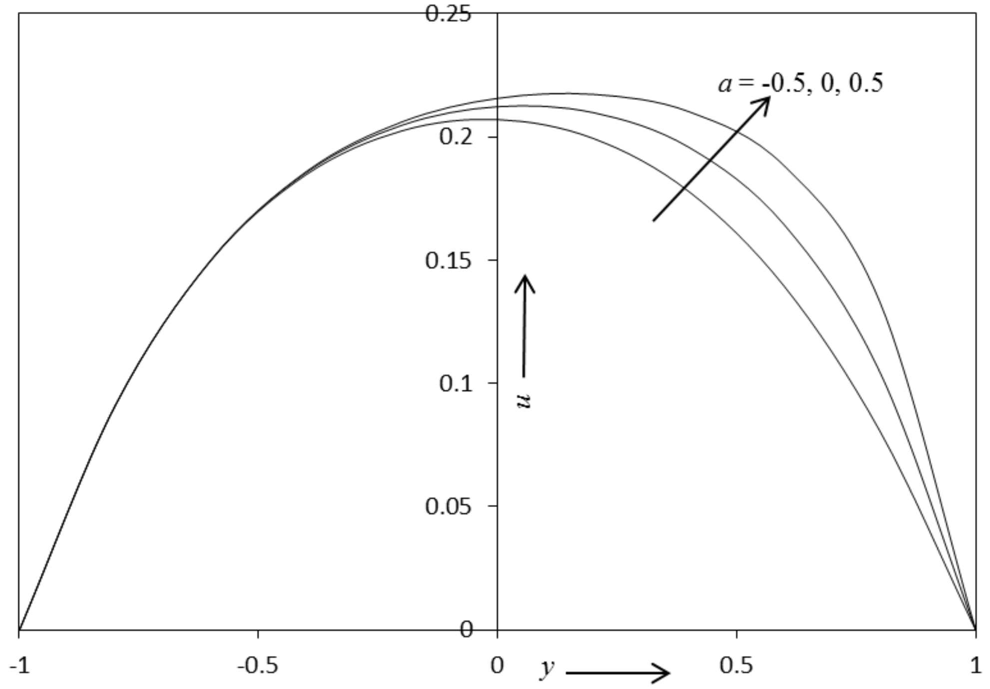


Fig. - 6: Velocity profile of fluid for different values of *a*.

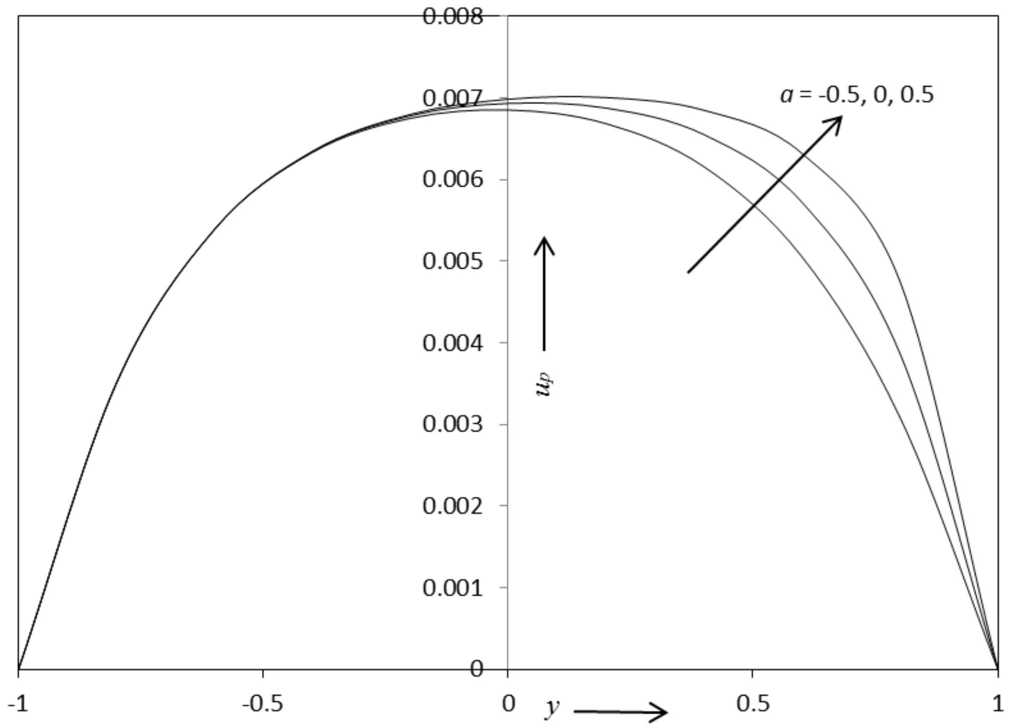


Fig. - 7: Velocity profile of dust particle for different values of *a*.

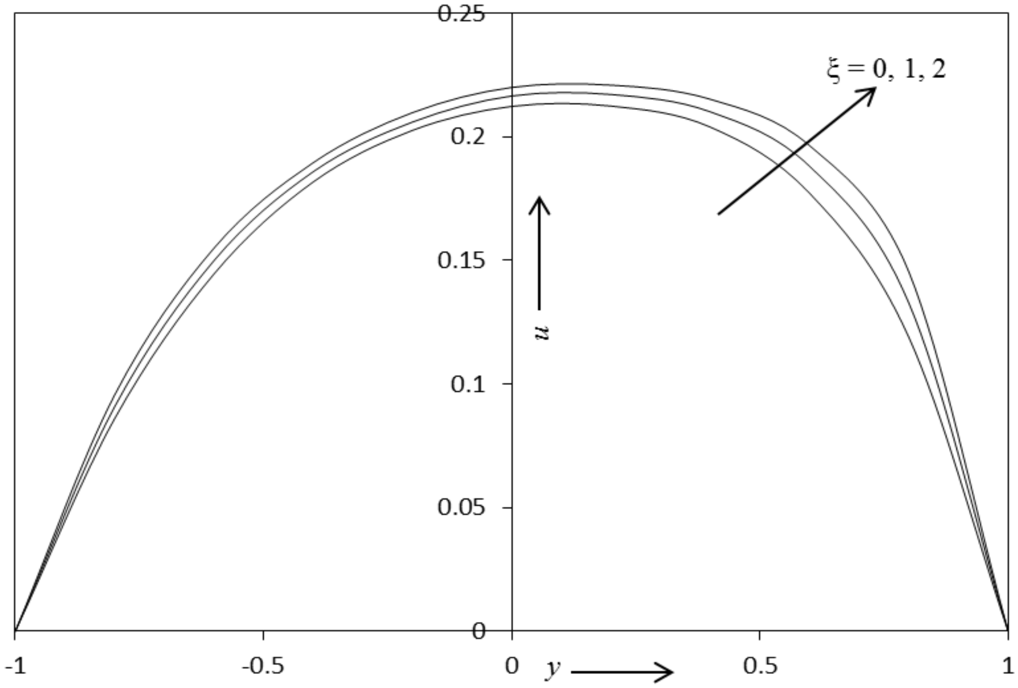


Fig. - 8: Velocity profile of fluid for different values of $\xi$.

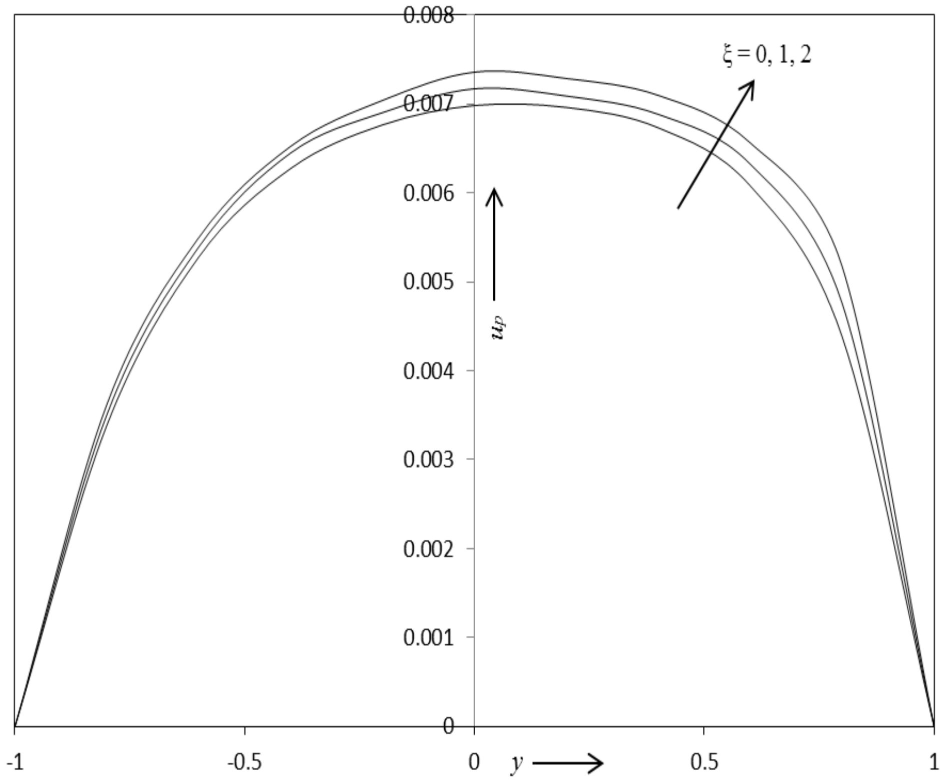


Fig. - 9: Velocity profile of dust particle for different values of $\xi$.

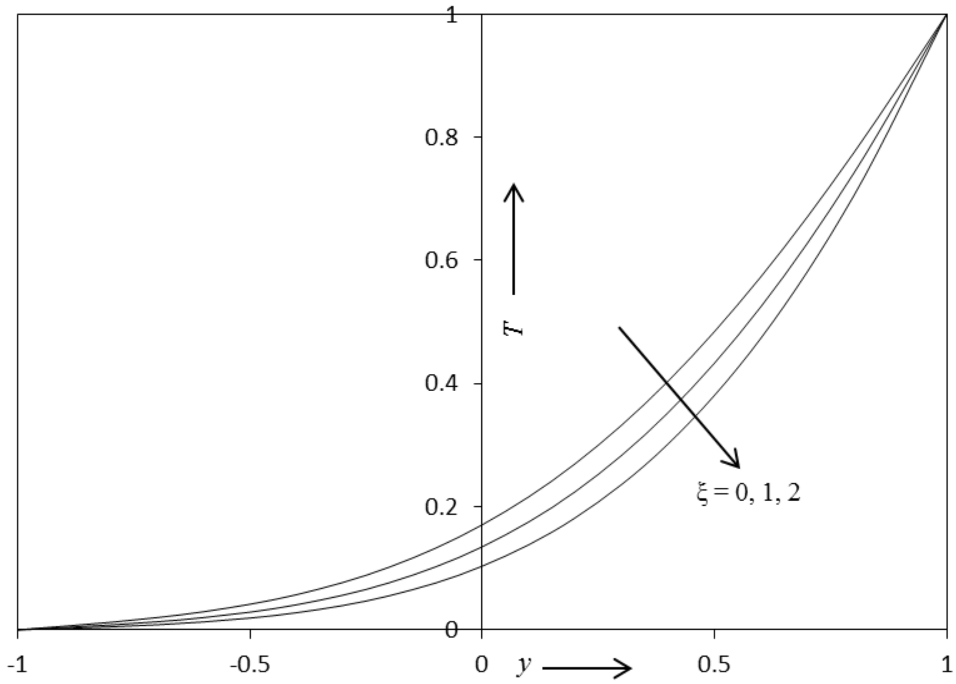


Fig. - 10: Temperature profile of fluid for different values of $\xi$.

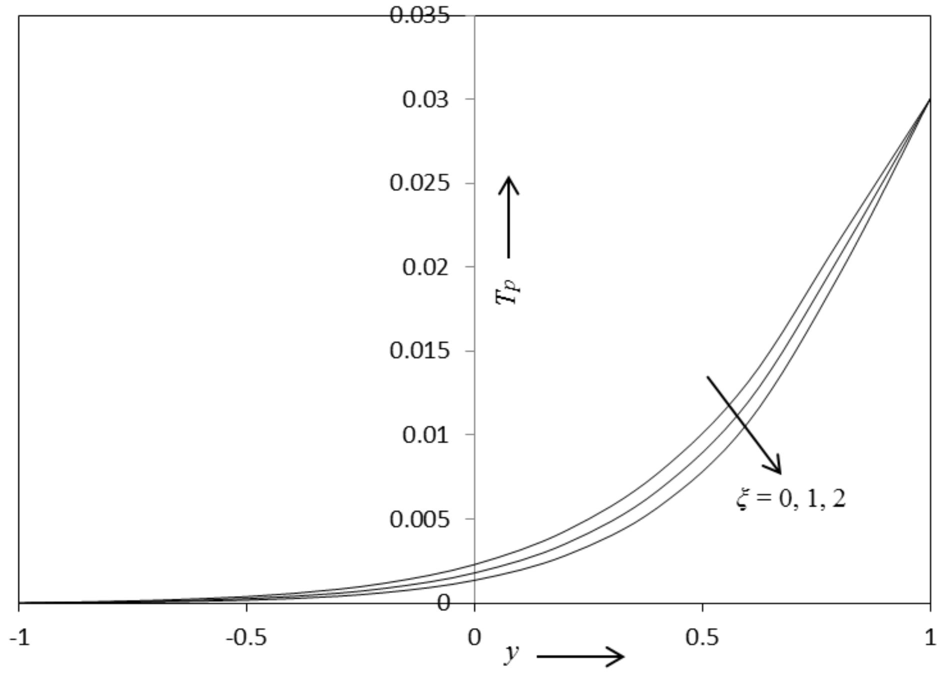


Fig. - 11: Temperature profile of dust particle for different values of $\xi$.

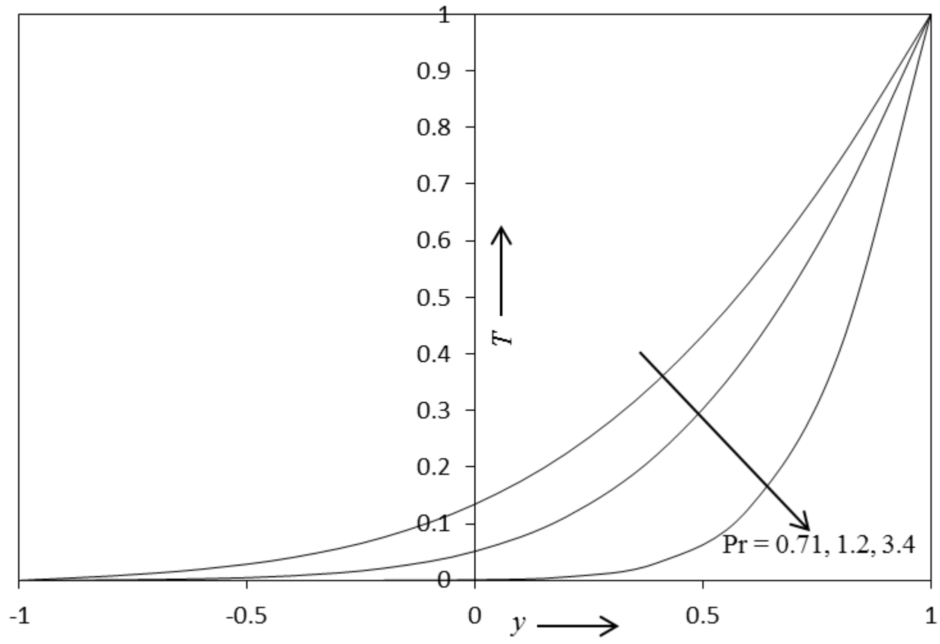


Fig. -12: Temperature profile of fluid for different values of Pr.

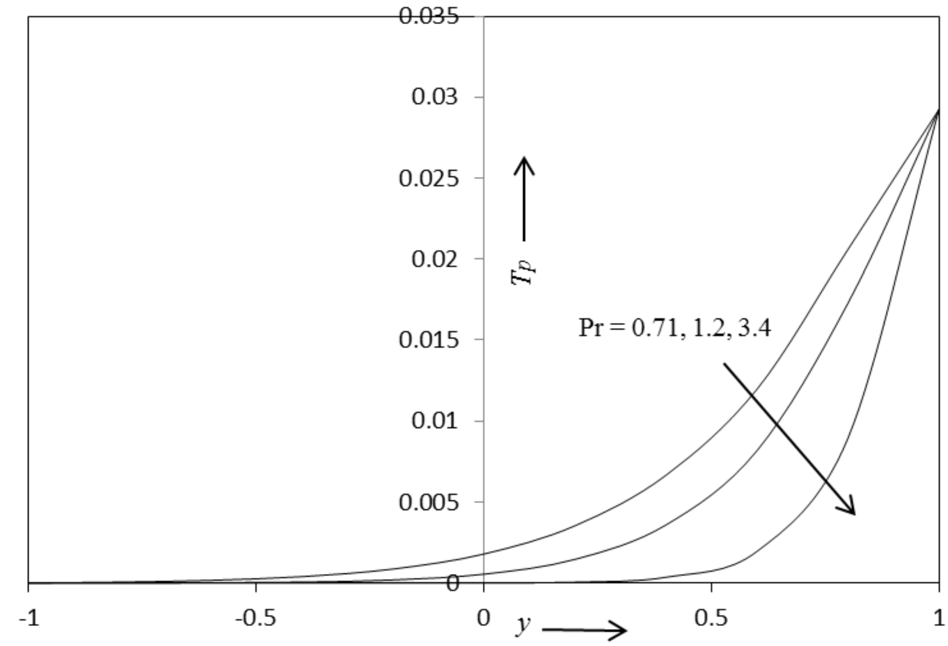


Fig. - 13: Temperature profile of dust particle for different values of Pr.

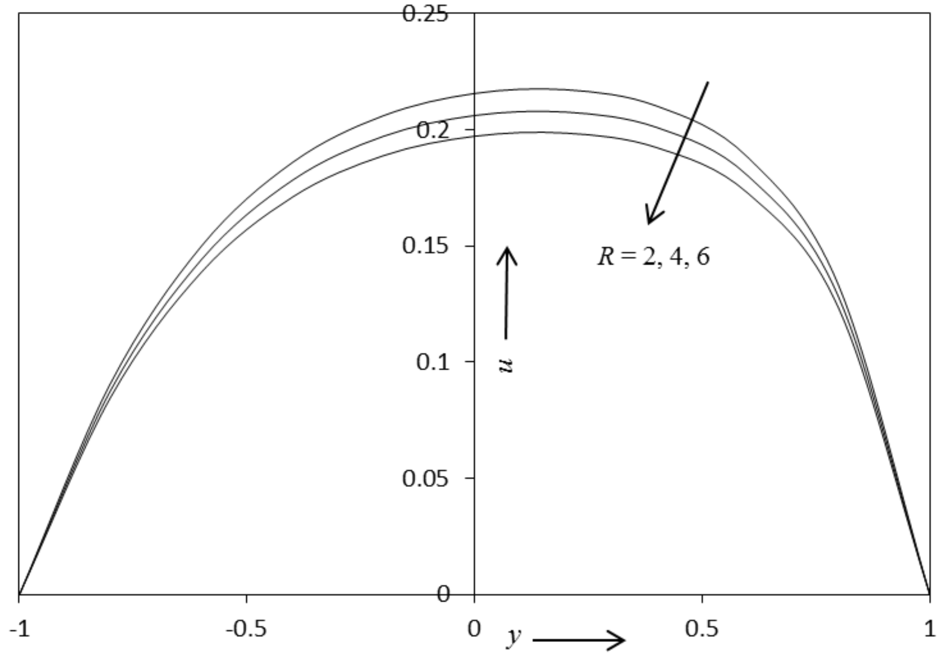


Fig. - 14: Velocity profile of fluid for different values of $R$.

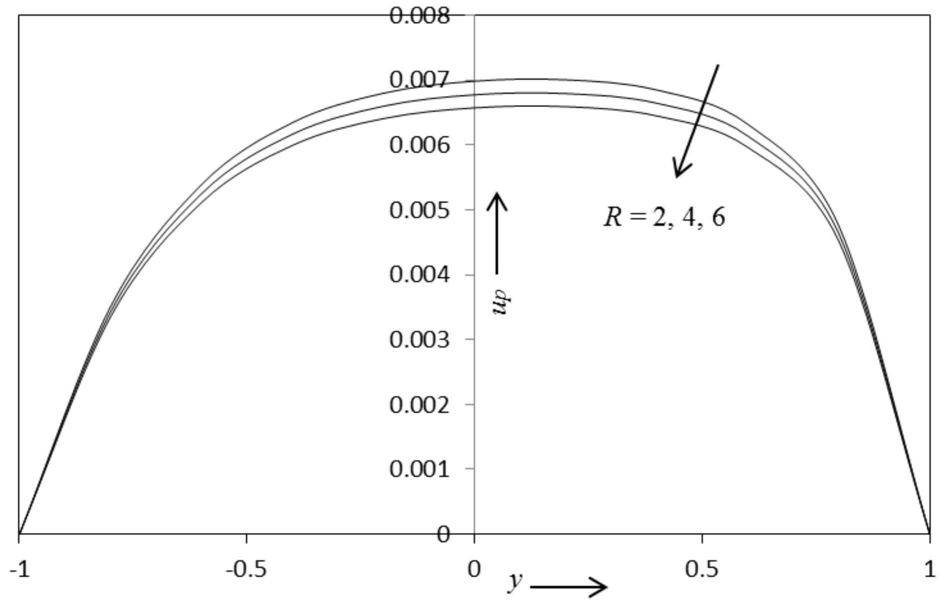


Fig. - 15: Velocity profile of dust particle for different values of $R$.

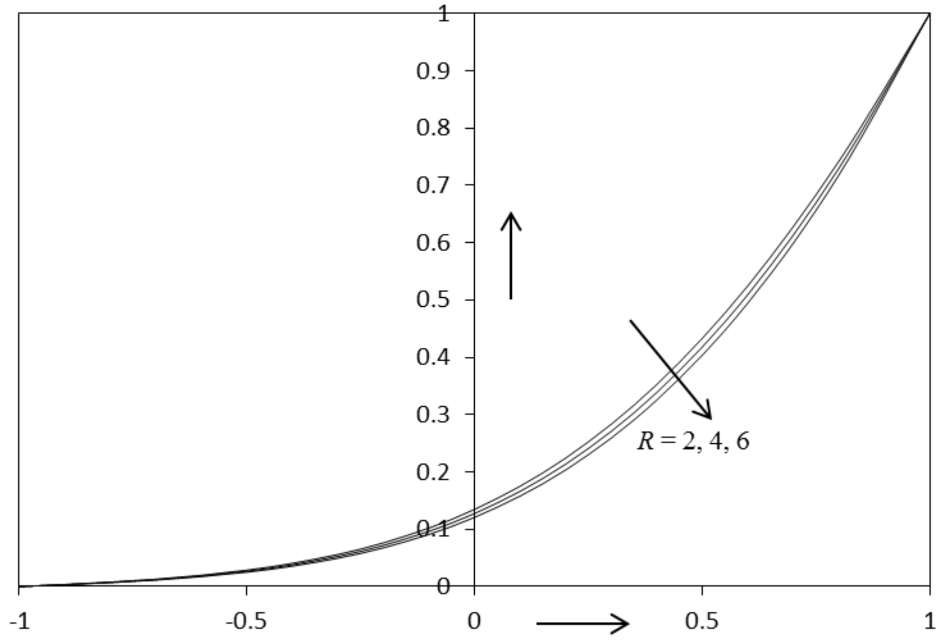


Fig. -16: Temperature profile of fluid for different values of $R$.

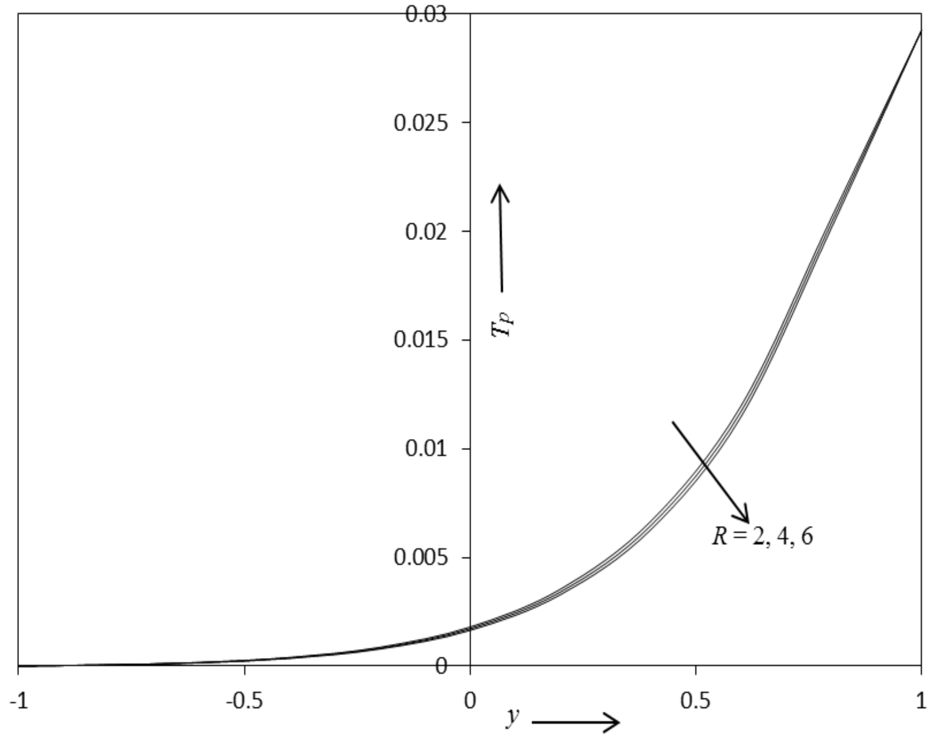


Fig. - 17: Temperature profile of dust particle for different values of R.

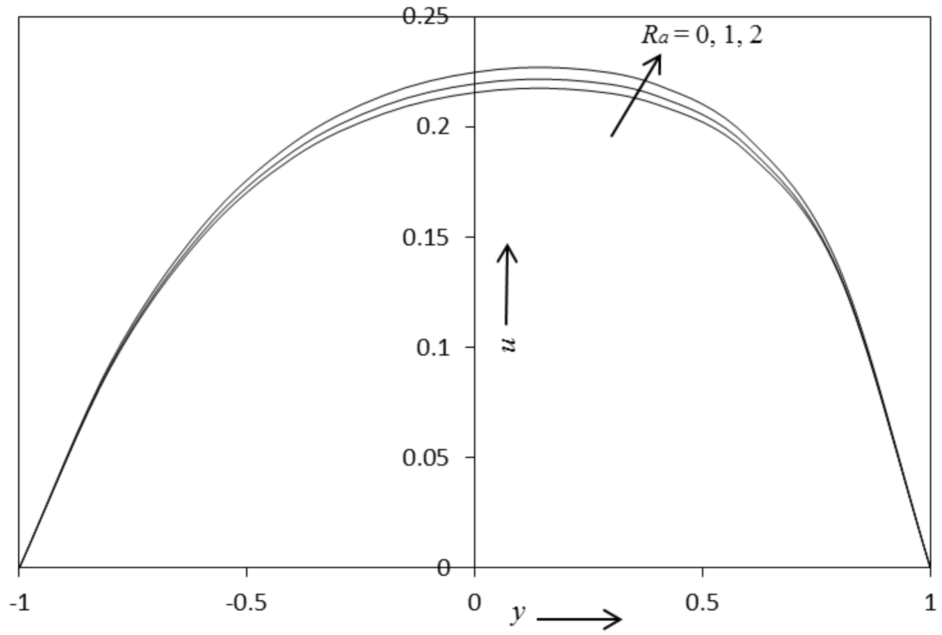


Fig. - 18: Velocity profile of fluid for different values of *Ra*.

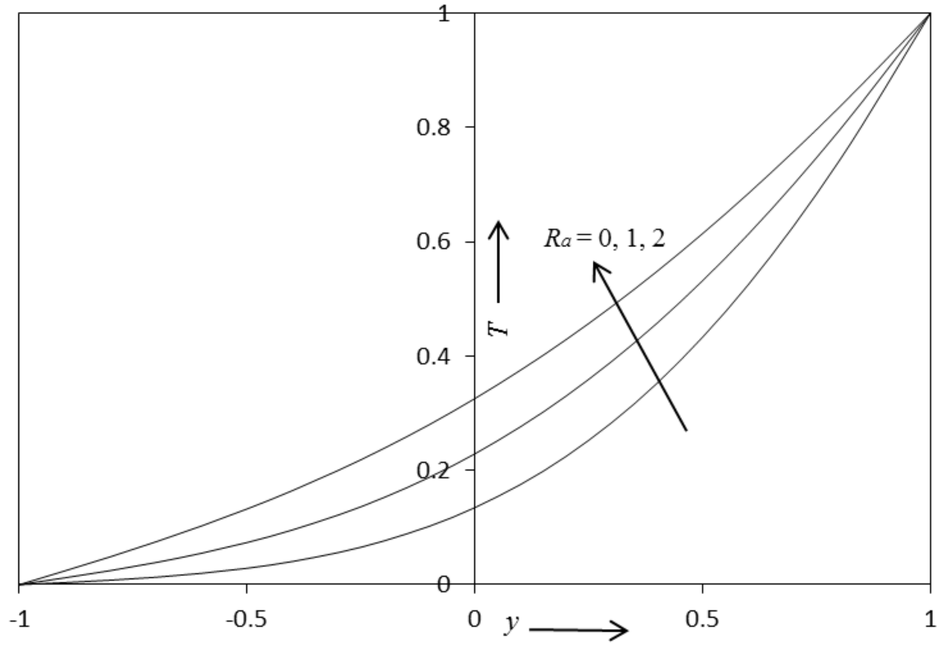


Fig. -19: Temperature profile of fluid for different values of *Ra*.