H.C.F and L.C.M – A Case Study of Completeness Property of Real Numbers

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Abstract: *In the basic mathematics H.C.F and L.C.M are an important concept related to factors or multiples of numbers . The calculation of H.C.F or L.C.M mainly follows the factorization of given numbers into prime numbers. The largest common factor of two or more numbers is called the highest common factor (HCF). Lowest Common Multiple is the smallest positive number that is a multiple of two or more numbers. In the present paper it is tried to show that the whole theory follows the completeness property of real numbers.*

Keywords: H.C.F , L.C.M ,completeness ,real numbers;

I. INTRODUCTION

This article attempts to provide an important use of completeness property of real numbers . It is tried to elaborate the functional concepts working behind the calculation of H.C.F (highest common factor) and L.C.M (least common multiple) of two or more numbers [14].

The simplest way to represent a real number is as a decimal of the form $a.d_1d_2d_3$……. where a is an integer (the integral part) and d_1, d_2, d_3,…… being the digits 0,1,2,……,9.If the decimal is terminating or non-terminating but recurring, the decimal represents a real rational or simply a rational number and if the decimal is non-terminating and non-recurring , it represents a real irrational or simply an irrational numbers. The set R of all real numbers consists of all the rational numbers as well as all the irrational numbers[10]. Generally , in basic mathematics ,H.C.F or L.C.M of positive integers or positive fraction are calculated and all these numbers are members of real numbers. Concept of LCM, HCF important for number theory and remainder based problems. Set of natural numbers N, set of integers Z ,set of rational numbers Q etc. are subsets of set of real numbers R .i.e.,

$$N \subset Z \subset Q \subset R$$

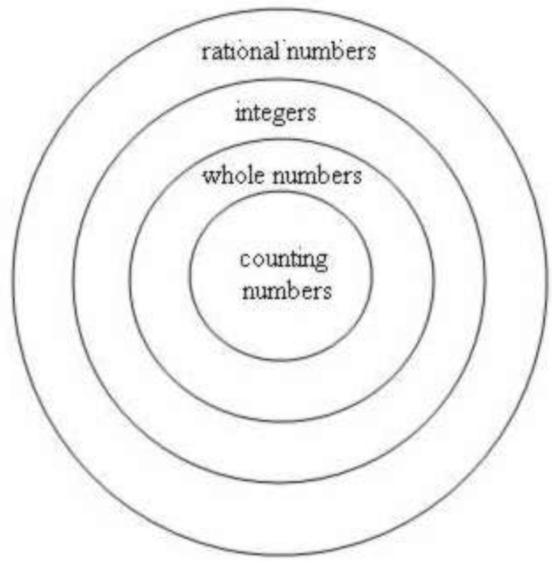


Figure1 [4] :To understand Sets of numbers by a rough diagram

So , completeness property of real numbers may be apply on these numbers. The methods available for calculation of H.C.F or L.C.M generally hides the rules of completeness property of real numbers for learners. The methods of calculation and concepts behind the theory of H.C.F and L.C.M are looking different. So, in the present paper it is tried to explain the concept behind H.C.F or L.C.M of numbers.

II. MATHEMATICAL DEFINITION

A. *Definition of Set:*

A set is any well defined collection of objects. For an example a collection, class and aggregate are used synonymously for the term Set. Here “well defined” means that it is possible to determine readily whether an object is member of a given set or not. The object that belongs to a set is called its element (or point or member). Now let us describe the two important methods in connection to a set. [12]-[13].

1) *Tabulation Method*

The tabulation method enumerates or list individual elements separated by commas and enclosed in braces. Example: English alphabet is written as{a, e, i, o, u}

2) *Defining property Method*

This method is often more compact and convenient. A defining property of a set is property which is satisfied by each element of that set and nothing by else. A set can be expressed as: {x|defing property} Or
{x: defing property}. x is a dummy symbols.
Examples of set:
N=set of natural numbers.
Z=set of integers.
Q=set of rational numbers.
R=set of real numbers

B. Definition of upper bound and lower bound

Let S be a subset of set of real numbers R. A real number u is said to be an upper bound of S if $x \varepsilon S$ which implies that $x \leq u$.
A real number l is said to be an lower bound of S if $x \varepsilon S$ which implies that $x \geq l$.

Let S be a subset of set of real numbers R. S is said to be bounded above if S has an upper bound. S is said to be bounded below if S has a lower bound.
In other words , a set S which is subset of R is said to be bounded above if there exist a real number u such that $x \varepsilon S$ which implies that $x \leq u$; S is said to be bounded below if there exists a real number l such that $x \varepsilon S$ which implies that $x \geq l$.

S is said to be a bounded set if S be bounded above as well as bounded below.[5]

C. Property of real numbers: Completeness property

(i) Every non–empty set of real numbers which is bounded above has the supremum or the least upper bound in R.
(ii) Every non–empty set of real numbers which is bounded below has the infimum or the greatest lower bound in R. [5]-[9]

D. Order Properties of Real Numbers

(i) If a,b ,then exactly one of the following statement hold : a<b,or a=b, or b<a
(ii) a<b and b<c => a<c for a,b,cε R
(iii) a<b => a+c <b+c for a,b,cε R
(iv) a<b and b<c => ac<bc for a,b,cεR . [5]

E. Factors and Multiples

If number *a* divided another number *b* exactly, we say that *a* is a **factor** of *b*.
In this case, *b* is called a **multiple** of *a*.[3]

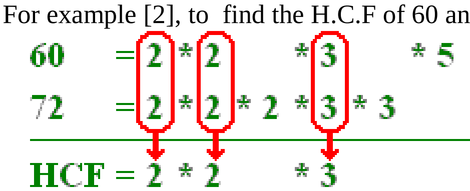
F. Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.)

The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.
A positive number a is said to be g.c.d of integers b and c if a/b and a/c and every common divisor of b and c also divide a, a≠0.
There are two methods of finding the H.C.F. of a given set of numbers:

1) **Factorization Method**

Express the each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.

For example [2], to find the H.C.F of 60 and 72



2) **Division Method**

Suppose we have to find the H.C.F. of two given numbers, divide the larger by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is required H.C.F.

- **Finding the H.C.F. of more than two numbers:**

Suppose we have to find the H.C.F. of three numbers, then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given number.
Similarly, the H.C.F. of more than three numbers may be obtained.

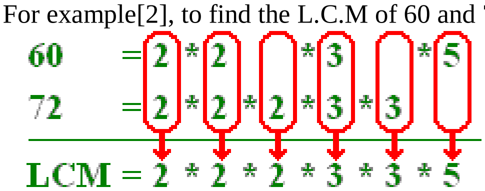
G. Least Common Multiple (L.C.M.)

The least number which is exactly divisible by each one of the given numbers is called their L.C.M.
A positive number a is called L.C.M of non zero integer b and c if b/a and c/a and every common multiple of b and c is also a multiple of a.
There are two methods of finding the L.C.M. of a given set of numbers:

1) **Factorization Method**

Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.[3]

For example[2], to find the L.C.M of 60 and 72



2) **Division Method (short-cut)**

Arrange the given numbers in a row in any order. Divide by a number which divided exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.[3]

H. Relationship among two positive numbers

Product of two numbers = Product of their H.C.F. and L.C.M. [3]

I. Co-primes

Two numbers are said to be co-primes if their H.C.F. is 1. [3]

J. H.C.F. and L.C.M. of Fractions

1. $\text{H.C.F.} = \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$

2. $\text{L.C.M.} = \dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$

K. H.C.F. and L.C.M. of Decimal Fractions

In a given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.[3]

L. Comparison of Fractions

Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.[3]

III. USE OF COMPLETENESS PROPERTY IN H.C.FAND L.C.M OF NUMBERS

The supremum principle is basic property of the Real number System in which every non –empty set of real numbers which is bounded above has its supremum .There is also the concept of infimum of a set exist in which it is considered that every non-empty set of real numbers which is bounded below , has its infimum.

Example(i): Find the H.C.F of 9 and 12

Factors of 9 : 1, 3, 9

Factors of 12 : 1, 2, 3, 4, 6, 12

Common Factors :1, 3

Greatest common factor : 3

Here take set S_1={1,3,9} , S_2={1,2,3,4,6,12} Set S_3={1,3}.Now set S_3 is subset of real number R so completeness property of real numbers holds here. In the set S_3 , $1 \leq 3$ and 3 is the greatest elements. Since, every non–empty set of real numbers which is bounded above has the supremum or the least upper bound in R.

Example 2: Find the L.C.M of 12 and 18.

Multiple of 12-

12, 24,36,48,60,72,84,96,108,120,…………

Multiple of 18-

18,36,54,72,90,108,126,…………………..

Common multiple:36,72,108,…….

Least common multiple: 36

Here all the numbers may be considered in the sets

S_1={12, 24,36,48,60,72,84,96,108,120,………}

S_2={18,36,54,72,90,108,126,…………………}

S_3 = {36,72,108,…….} and Now set S_3 is subset of real number R and least element in S_3 is 36. Since every non–empty set of real numbers which is bounded below has the infimum or the greatest lower bound in R.

Remarks : Is the greatest common factor of two negative numbers a negative?[1]

No. Consider two numbers a and b and a negative number c that divides both a and b and has the highest absolute value of all numbers that divide a and b. Regardless of a and b being positive or negative, -c which is positive will divide both a and b. -c > c since c is negative and thus -c will be the GCD and not c.

IV. CONCLUSION

(i)It is well known theorem that " Supremum of a set S ⊂ R , If it exists, is unique .Similarly if H.C.F of a two or more number if exist , it will be unique.

(ii)It is well known theorem that " Infimum of a set S ⊂ R , If it exists, is unique .Similarly if L.C.M of a two or more number if exist , it will be unique.

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