Numerical Simulation of Cylindrical Shock Wave in inhomogeneous medium

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***Abstract* -** The propagation of a cylindrical shock wave in an ideal gas with exponentially increasing density. The shock wave is driven out by a piston moving with time according to power law. The solution is applicable for any arbitrary ratio of specific heats and valid even for large time.

***Keywords* -** Shock wave, inhomogeneous medium, specific heat

I. INTRODUCTION

Marshak (1958) studied the effects of radiation on the shock propagation by introducing the radiation diffusion approximation numerical solutions for self self-similar adiabatic flows in self-gravitating gas were obtained by Sedov (1) and Carrus et al (2) independently Purohit (3) and Singh and Vishwakarma (4) have discussed homo-thermal flows behind a spherical shock wave in a self-gravitating gas using similarity method. Nath (5) have studied the above problem assuming the flow to be adiabatic and self-similar and obtained the effects of the presence of a magnetic field.

Our physical situation is that if we have strong shock wave created by any, means, travelling in a medium of increasing density then we should like to know the nature of the flow field behind the shock. The shock situation is time dependent, and we assume the density to increase as power law. Grover Hardy (1), Hayes (2) has also studied the propagation of shock waves in an exponential medium.

Sakarai (3) has considered the problem of a shock wave arriving at the edge of a gas medium in which density varies as power law Ojha and Onkar (4) discussed the problem of explosion in non uniform self gravitating medium. Deb Ray and Bhomick (5) found the, solution of gas dynamics shock wave. The problem of shock propagation in a self gravitating mass of gas has been discussed by Sedov (8).

In the present work, I have discussed the strong cylindrical shocks in a medium of exponentially increasing density. The solution is applicable for any arbitrary ratio of specific heats and valid even for large time.

II. EQUATION OF MOTION AND BOUNDARY CONDITIONS

The fundamental equation governing the motion of the fluid in gas is

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\,\frac{\partial u}{\partial r} = 0 \qquad [1]$$

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \qquad [2]$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + \gamma p\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = 0 \qquad [3]$$

where ρ, u, P are density, velocity and pressure of the gas at a radial distance r from the centre at time t, r is the ratio of specific heats.

Assuming local thermodynamics equilibrium and taking Roseland's approximation, we have

$$q = -\frac{Cv}{3}\frac{\partial(\sigma T^4)}{\partial r} \qquad [4]$$

Where $\frac{1}{3}\boldsymbol{\sigma C}$ is the Stefan Boltzmann constant, C the velocity of light and ν the mean free path of radiation is a function of density and temperature following wang (6), we take

$$\upsilon = \nu_0\, \rho^{\alpha} T^{\beta} \qquad [5]$$

Where ν_0, α and β being constants

We assume that a cylindrical shock is propagating in the medium and the flow variables immediately ahead of the shock froud are

$$u = 0 \qquad [6]$$

$$\rho = \rho_1 = \text{constant} \qquad [7]$$

$$p = p_1 = -\frac{(n+1)}{2n} A^2 R^{2n} \qquad [8]$$

where R is the shock radius and A and n are constants, due to passes of the shock, the gas is highly ionized and its electrical conductivity becomes infinitely large. The conditions across such a gas ionizing shock are (Singh and Srivastava and Vishwakarma and Pandey)

$$\rho_2 (V-u_2) = \rho_1 v = m_s \qquad [9]$$

$$P_2 - P_1 = m_s u_2 \qquad [10]$$

$$e_2 + \frac{P_2}{\rho_2} + \frac{1}{2}(v-u_2)^2 - \frac{q_2}{m_s} = e_1 + \frac{P_1}{\rho_1} + \frac{1}{2}V^2 \qquad [11]$$

where subscripts 2 and 1 are for the regions just behind and just ahead of the shock surface respectively, and v denotes the shock velocity. The shock front is assumed to be opaque and it does not receive any heat flux from external sources. Therefore, the heat flux q_2 is the heat flux exchanged between the flow-field and the shock front. The jump conditions (9), (10), (11) are not sufficient to determine all the flow variables at the shock front

The strong shock conditions are (by Widham (7))

$$\frac{u_1}{V} = \frac{2}{\gamma+1} \qquad [12]$$

$$\frac{\rho_1}{\rho_0} = \frac{\gamma+1}{\gamma-1} \qquad [13]$$

$$\frac{p_1}{\rho_0} = \frac{\gamma+1}{\gamma-1} \qquad [14]$$

Where u_1, ρ_1 and p_1 are the velocity, density and pressure just behind the shock and ρ_0 the undistributed density just ahead of the shock, respectively and V is the shock velocity.

The density in undistributed gaseous medium is by our assumption

$$\rho_0 = A e^{\delta x}$$

A and δ being positive constants

III. SIMILARITY SOLUTIONS

For self-similar motions, the system of partial differential equations (1), (2), (3) reduces to a system of ordinary differential equations in new unknown functions of the similarity variable $\eta = te^{\lambda r}$. Let us derive these equations

$$u = \frac{1}{t} V(\eta) \qquad [15]$$

$$p = t^{(\delta-2)} f(\eta) \qquad [16]$$

$$\rho = t^{\delta} g(n) \qquad [17]$$

$$\eta = te^{\lambda r},\ \lambda \neq 0 \qquad [18]$$

where λ and δ are to be determined from the conditions of the problem transformation in terms η is necessarily a non similarly one the value of η is assumed constant at the shock surface

$$\text{hence} \qquad V = -\frac{1}{\lambda t},\ \lambda < 0 \qquad [19]$$

which represents an outgoing shock surface. The solution of eqn (1), (2), (3) are compatible with the shock conditions (12), (13), (14) in the form (15) – (18) if

$$\delta = 2,\ 2\eta = m = 2\lambda + k,\ \alpha\text{-}1 \text{ and } \beta = -\frac{5}{2} \qquad [20]$$

It can be easily seen that the strength of the shock, under these conditions, remains constant from eqn (19), we get

$$R = -\frac{1}{\lambda} \log \frac{t}{\tau} \qquad [21]$$

where τ is the duration of the almost instantaneous explosion

By the use of eqn (15) – (18), (1) to (3) can be transformed and simplified to

$$\frac{dG}{dx} = \frac{G\,[\frac{dW}{dx} + 2\log\frac{t}{\tau}]}{(1-W)} \qquad [22]$$

$$\frac{dP}{dx} = \frac{\gamma p\,(\frac{dW}{dx} + \frac{W}{x})}{(1-W)} \qquad [23]$$

$$\frac{dW}{dx} = \frac{W\left[\frac{\gamma LGP}{x} - log\frac{t}{\tau}\right](1-W)}{[(1-W)^2 - \gamma LGP]} \qquad [24]$$

use here the relation

$$P = P_1P,\ r = xR,\ u = VW$$

$$\partial P = P_1 dP,\ \partial r = R\partial x\ ,\ \partial u = VdW$$

$$v = -\frac{1}{\lambda t}\ ,\ R = -\frac{1}{\lambda} log\frac{t}{\tau}$$

using here shock condition eqn (13) – (14),

$$\frac{P_1}{\rho_1} = LV^2$$

here $L = 2(\gamma -1)/(\gamma +1)^2$

IV. RESULTS AND DISCUSSION

In this problem I have investigated the propagation of cylindrical shock wave in homogenous medium, from present investigation it is clear that the nature of shockwave is directly depending on particle loading and diameter of dust particle of inhomogeneous medium. A cylindrical wave propagation into a dust suspension can be described by exponential decay curve, which assume a general form for a given incident shock wave.

In order to exhibit the numerical solution it is convenient to write the field variable in the non-dimensional form as

$$X = 1,\ W = (1 - \frac{1}{N}),\ G = 1,\ P = 1$$

The numerical integration of equation (22) to (24) was carried out by using the well known RKGS programme. The numerical results for a certain choice of parameter are reproduced in tabular form. The nature of variation in field variables is illustrated through the table. I have calculated the result for $\gamma = 1.4$, L = 10

At the shock boundary we have

$$x = 1,\ W = (1-\frac{1}{N})\ ,\ G = 1, P = 1$$

using these initial values eqn (22) to (24) have been integrated for the values of r = 1.4, M^2 = 10, L = 10 the total energy of the wave is non constant and varies as the square of the shock radius. Nature of flow variables is seen through table

V. TABLE 1

x	W	G	P
1.00	1.000000	1.000000	1.000000
0.99	0.990761	1.08729	1.08739
0.98	0.982372	1.173134	1.735171
0.97	0.971471	1.265072	1.277321
0.96	0.961102	1.340721	1.312005
0.95	0.9543201	1.440322	1.463212
0.94	0.943210	1.60352	1.635098
0.93	0.937177	1.730991	1.739209
0.92	0.928719	1.845297	1.867912
0.91	0.912654	1.973292	1.913520
0.90	0.909408	2.012745	2.04832

From table we see that discontinuity in velocity distribution is large at shock front and decreases as we move towards the line of explosion where as distribution of density and pressure in small at shock surface and increases towards the line of explosion, But as temperature increases in the velocity of shock and after getting maximum value it decreases near the line of explosion. Thus the result shown by table represents the physical situation behind the cylindrical shock produce by line explosion.

VI. CONCLUSIONS

REFERENCES

[1] Hardy J. W. and Grover R Astrphys J 143 (1966) 48.
[2] Hayes W. D. J. fluid Mech. 32 (1968) 305.
[3] Sakurai A. Comm. Pure Applied Mathematics 13 (1960) 353.
[4] Ojha S. N. Actaciencia Indica 2(1)(1976)86.
[5] Ray G. Deb Proc. Nat. Inst. Science India 35(1967)86.
[6] Wang K. C. Physics fluid 9(1966)1922.
[7] Witham G. B. J. fluid Mech. 4(1958)337.
[8] Sedov L. I. Similarity and dimensional method in mechanics, Academic press, New York (1959).
[9] Carrus AP. J. (1951), 113(3), 496 – 518.
[10] Elliott L. A. 1960, similarity method in radiation hydrodynamics Proc R. Soc. Land A, 258(3) 287 – 301.
[11]Marshak R. E. 1958, Effect of radiation on shock wave behavior, Phys, Fluids, 1(1), 24 – 29.
[12] J. P. Vishwakarma and Arvind K. Singh, J. Astrophys. Astr. (2009)30, 53 – 69.
[13] J. P. Vishwakarma and Mahendra Singh, Applied Mathematics (2012), 2(1): 01 – 07.
[14] G. Nath and A. K. Sinha (2011), Phys. Research International