Super Root Square Mean Labeling of Some Graphs

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Abstract : *Let G be a (p, q) graph and $f:V(G)\rightarrow\{1,2,...p+q\}$ be an injective function. For a vertex labeling f, the induced edge labeling $f^*(e=uv)$ is defined by* $f^*(e)=\left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil$ *or* $\left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$ *, then f is called a super root square mean labeling if $f(V)\cup\{f^*(e):e\in E(G)\} = \{1,2,3,...p+q\}$. A graph which admits super root square mean labeling is called super root square mean graph. In this paper we investigate super root square mean labeling of some graphs.*

Key words: *Root square mean graph, Super root square mean graph, nP_m , nK_3 , $P_n\odot\overline{K_2}$, middle graph of path P_n , dragon $C_n\odot P_m$.*

I. INTRODUCTION

All graphs in this paper are finite, simple and undirected graph G = (V, E) with p vertices and q edges. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices(edges) then the labeling is called a vertex labelling(an edges labeling). Several types of graph labeling and a detailed survey is available in [1]. For all other standard terminology and notations we follow Harary[2].

Mean labeling was introduced by S. Somasundaram and R. Ponraj[3],[4]. Root square mean labeling was introduced by S.S. Sandhya, S. Somasundaram and S. Anusa in[5]. In this paper, we introduce super root square mean labeling of graphs and investigate super root square mean labeling of nP_m , nK_3 , $P_n\odot\overline{K_2}$, middle graph of path P_n , dragon $C_n\odot P_m$. The following definitions are useful for the present study.

Definition 1.1: [6] Let $f : V(G)\rightarrow\{1,2,\ldots p+q\}$ be an injective function. For a vertex labeling f, the induced edge labeling $f^*(e)$ is defined by $f^*(e)=\left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$, then f is called a **super root square mean labeling** if $f(V)\cup\{f^*(e):e\in E(G)\}=\{1,2,3,\ldots p+q\}$. A graph which admits super root square mean labeling is called **super root square mean graph**.

Definition 1.2: A Walk in which $u_1u_2\ldots u_n$ are distinct is called a **path**. A path on n vertices is denoted by P_n .

Definition 1.3: A closed path is called a **cycle**. A cycle on n vertices is denoted by C_n .

Definition 1.4: The corona $G_1\odot G_2$ of two graph G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1, (which has P_1 vertices) and P_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2 .

Definition 1.5: The middle graph M(G) of a path G is the graph whose vertex set is $V(G)\cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

II. Main Results

Theorem 2.1: nP_m is a super root square mean graph.

Proof: Let the vertices of nP_m be
$\{v_{i,j} : 1\leq i\leq n , 1\leq j\leq m\}$ and the edge of nP_m be
$\{e_i : (v_{i,j}v_{i,j+1}: 1\leq i\leq n, 1\leq j\leq m-1\}$.
Define a function $f : V(nP_m) \rightarrow\{1,2,\ldots p+q\}$ by
$f(v_{i,j})=(2m-1)(i-1)+2j-1, 1\leq i\leq n , 1\leq j\leq m$.
Then the induced edge labels of nP_m is
$f^*(v_{i,j}\ v_{i,j+1})=(2m-1)(i-1)+2j, 1\leq i\leq n , 1\leq j\leq m-1$.
Thus the vertices and edges together get distinct labels.
Hence nP_m is a super root square mean graph.

Example 2.2: super root square mean labeling of $4P_5$ is shown in figure 2.1

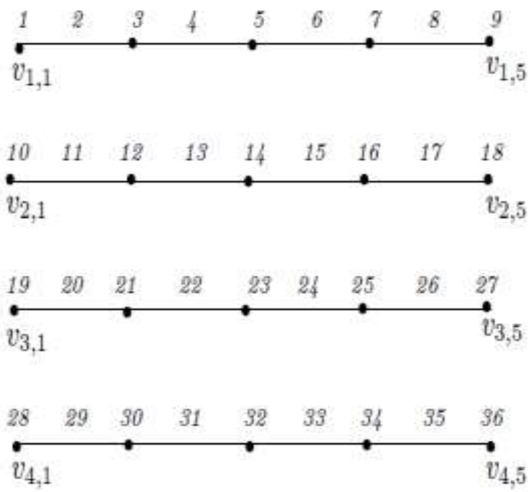

Figure 2.1

Theorem 2.3: nK_3 is a super root square mean graph.

Proof: Let G_1, G_2, … G_n be n copies of K_3.
Let v_1, v_2, v_3 be the vertices of G_1,
v_4,v_5,v_6 be the vertices of G_2 …v_{3n-2} ,v_{3n-1},v_{3n} be the vertices of G_n .
Here $|V(nK_3|$=3n, $|E(nK_3|$=3n.
Define a function f : V(nK_3) →{1,2,…p+q} by
v_{3i-2} = 6i-5, $1 \le i \le n$
v_{3i-1} = 6i-3, $1 \le i \le n$
v_{3i} = 6i, $1 \le i \le n$
Then the induced edge labels of nK_3 is
$f^*(v_{3i-2}v_{3i-1})$ = 6i-4, $1 \le i \le n$
$f^*(v_{3i-2}v_{3i})$ = 6i-2, $1 \le i \le n$
$f^*(v_{3i-1}v_{3i})$ = 6i-1, $1 \le i \le n$
Then the vertices and edges together get distinct labels.
Hence nK_3 is a super root square mean graph.

Example 2.4: super root square mean labeling of $4K_3$ is shown in figure 2.2

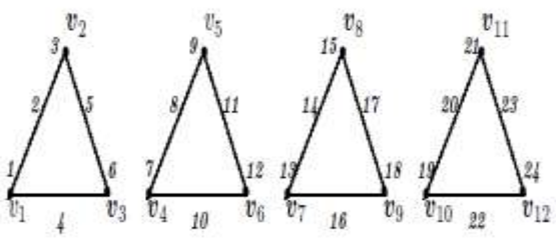

Figure 2.2

Theorem 2.5: $P_n \odot \overline{K_2}$ is a super root square mean graph.

Proof: Let P_n be the path u_1,u_2, …u_n and v_i , w_i be the vertices of $\overline{K_2}$ which are attached to the vertex u_i of P_n .
Let G = $P_n \odot \overline{K_2}$.
Here $|V(G)|$=3n , $|E(G)|$=3n-1.
Define a function f : V(G) →{1,2,…p+q} by
$f(u_i)$=6i-3, $1 \le i \le n$
$f(v_i)$=6i-5, $1 \le i \le n$
$f(w_i)$=6i-1, $1 \le i \le n$
Then the induced edge labels of G as follows,
$f^*(u_iv_i)$=6i-4, $1 \le i \le n$
$f^*(u_iw_i)$=6i-2, $1 \le i \le n$
$f^*(u_iu_{i+1})$=6i, $1 \le i \le n-1$
Thus the vertices and edges together get distinct labels.
Hence $P_n \odot \overline{K_2}$ is super root square mean graph.

Example 2.6: super root square mean labeling of $P_n \odot \overline{K_2}$ is shown in figure 2.3

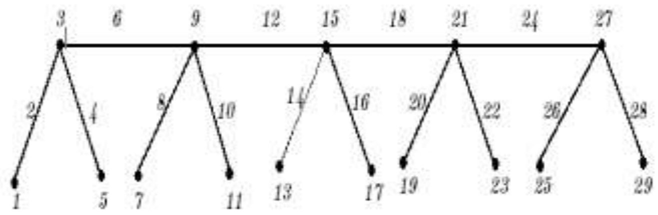

Figure 2.3

Theorem 2.7: The middle graph $M(P_n)$ of a path P_n is a super root square mean graph.

Proof: Let $M(P_n)$= (V, E) , where
$V(M(P_n))$ = {u_i : $1 \le i \le n$, v_j : $1 \le i \le n-1$}
$E(M(P_n))$={v_jv_{j+1}, u_iv_i ,v_iu_{i+1} / $1 \le i \le n-1$, $1 \le j \le n-2$}.
Here $|V(M(P_n))|$=2n-1 and $|E(M(P_n))|$=3n-4.
Define a function f : $V(M(P_n))$ →{1,2,…p+q} by
$f(u_1)$=1
$f(u_i)$=5i-5, $2 \le i \le n$
$f(v_j)$= 5j-2, $1 \le j \le n-1$
Then the induced edge labels of $M(P_n)$ as follows,
$f^*(u_iv_i)$=5i-3, $1 \le i \le n-1$
$f^*(v_iu_{i+1})$=5i-1, $1 \le i \le n-1$
$f^*(v_jv_{j+1})$=5i+1, $1 \le j \le n-2$
Then the vertices and edges together get distinct labels.
Hence $M(P_n)$ is a super root square mean graph.

Example 2.8: super root square mean labeling of the middle graph of a path P_5 ie) $M(P_4)$ is shown in figure 2.4

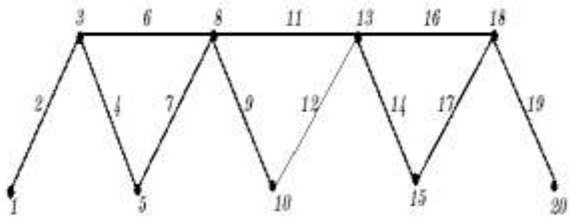

Figure 2.4

Theorem 2.9: Dragon $C_n \odot P_m$ is a super root square mean graph.

Proof: Let $u_1, u_2, \ldots u_n$ be the vertices of the cycle C_n and $v_1, v_2, \ldots v_m$ be the vertices of the path P_m.
Here $u_n = v_1$.
Let $G = C_n \odot P_m$, where $V(G) = \{u_i : 1 \le i \le n, v_i : 1 \le i \le m\}$ $E(G) = \{e_i = u_i u_{i+1}, 1 \le i \le n-1, e_n = u_1 u_n, e_j = v_j v_{j+1}, 1 \le j \le m-1\}$
Define a function $f : V(C_n \odot P_m) \to \{1,2,\ldots p+q\}$ by

$$f(u_i)=\begin{cases} 2i-1; & 1 \le i \le \frac{k}{2} \\ 2i; & \frac{k+2}{2} \le i \le n \end{cases} \text{ if } k=\left\lfloor \sqrt{\frac{4n^2+1}{2}} \right\rfloor \text{ is even}$$

$$f(u_i)=\begin{cases} 2i-1; & 1 \le i \le \frac{k-1}{2} \\ 2i; & \frac{k+1}{2} \le i \le n \end{cases} \text{ if } k=\left\lfloor \sqrt{\frac{4n^2+1}{2}} \right\rfloor \text{ is odd}$$

$f(v_i) = 2n+2i-2, \quad 1 \le i \le m$

Then the induced edge labeling of $C_n \odot P_m$ is as follows

$$f^*(u_i u_{i+1})=\begin{cases} k; & i = n \\ 2i; & 1 \le i \le \frac{k-1}{2} \\ 2i+1; & \frac{k+1}{2} \le i \le n-1 \end{cases} \text{ if } k=\left\lfloor \sqrt{\frac{4n^2+1}{2}} \right\rfloor$$

is odd

$$f^*(u_i u_{i+1})=\begin{cases} k; & i = n \\ 2i; & 1 \le i \le \frac{k}{2}-1 \\ 2i+1; & \frac{k}{2} \le i \le n-1 \end{cases} \text{ if } k=\left\lfloor \sqrt{\frac{4n^2+1}{2}} \right\rfloor$$

is even

$f^*(e_j = v_j v_{j+1}) = 2n + 2j - 1, \quad 1 \le j \le m-1$

Thus the vertices and edges together get distinct labels.
Hence dragon $C_n \odot P_m$ is a super root square mean graph.

Example 2.10: super root square mean labeling of $C_5 \odot P_6$ is shown in figure 2.5

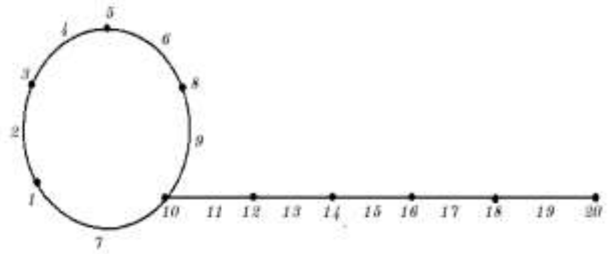

Figure 2.5

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