A Study of Trans-Sasakian Manifold Admitting Semi-Symmetric Non-Metric Connection With $\tilde{R}.\tilde{S} = 0$

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Abstract: *In a trans-Sasakian manifold with $\tilde{R}.\tilde{S} = 0$, the Ricci tensors admitting semi-symmetric non-metric connection and admitting Levi-civita connection has been found. It has been proved that a β −Kenmotsu manifold admitting semi-symmetric non-metric connection satisfying $\tilde{R}.\tilde{S} = 0$ is an Einstein manifold with constant negative scalar curvature $-2n(2n+1)\beta^2$.*

Keywords: *semi-symmetric non-metric connection, Levi-civita connection, trans-Sasakian manifold, Ricci tensor, Einstein manifold.*

I. Introduction

Systematic study of semi-symmetric connection in a Riemannian manifold was initiated by Yano [16]. In 1992, Agashe and Chafle [1] introduced the notion of semi-symmetric non-metric connection. Later on it was studied by De and Kamilya [5], De and Biswas [2], Singh and Pandey [15] and others. On the other hand there is a class of almost contact metric manifold, namely trans-Sasakian manifold [11], which generalizes both α −Sasakian [14] and β-Kenmotsu [10] structure. It was also studied by several geometers [6, 9, 12]. In this paper, we study some properties of conformal curvature tensor on a trans-Sasakian manifold admitting the semi-symmetric non-metric connection. The conformal curvature tensor C on a $(2n+1)$ −dimensional Riemannian manifold is defined as follows [7].

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{(2n-1)}[S(Y,Z)X - S(X,Z)Y + \{g(Y,Z)QX - g(X,Z)QY + \frac{r}{2n(2n-1)}[g(Y,Z)X - g(X,Z)Y], \qquad (1.1)$$

where S and Q are Ricci-tensor and Ricci-operator respectively. The paper is organized as under. Section-**2** contains some preliminaries. In Section-**3**, the Ricci tensors with respect to Levi-civita connection and semi-symmetric non-metric connection are found in a tans-Sasakian manifold admitting semi-symmetric non-metric connection with $\tilde{R}.\tilde{S} = 0$. In this section, we deduce that a β −Kenmotsu manifold admitting semi-symmetric non-metric connection satisfying $\tilde{R}.\tilde{S} = 0$ is an Einstein manifold with constant scalar curvature $-2n(2n+1)\beta^2$. The scalar curvature is also found on a trans-Sasakian manifold with $\tilde{R}.\tilde{S} = 0$.

II. Preliminaries

In this section, we recall some general definitions and basic formulas which we will use later. For this, we recommend the reference [3], [4], [7] and [8].Let M be a$(2n+1)-$ dimensional almost contact metric manifold equipped with almost contact metric structure (φ, ξ, η, g), where φ is $(1, 1)$ tensor field, $\xi$is a vector field, η **is** $1-$ form and g is compatible Riemannian metric such that

$$\varphi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \varphi\xi = 0, \eta o\varphi = 0 \qquad (2.1)$$

$$g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y), \qquad (2.2)$$

$$g(\varphi X, Y) = -g(X, \varphi Y), \quad g(X, \xi) = \eta(X), \tag{2.3}$$

for all $X, Y \in TM$. An almost contact metric manifold M is called trans-Sasakian manifold if

$$(\nabla_X \varphi) Y = \alpha\{g(X, Y)\xi - \eta(Y)X\} + \beta\{g(\varphi X, Y)\xi - \eta(Y)\varphi X\} \tag{2.4}$$

where ∇ is Levi-civita connection of Riemannian metric g and α and β are smooth functions on M. From equation (2.4) and equations (2.1), (2.2) and (2.3), we have

$$\nabla_X \xi = -\alpha\varphi X + \beta[X - \eta(X)\xi], \tag{2.5}$$

$$(\nabla_X \eta)(Y) = -\alpha g(\varphi X, Y) + \beta g(\varphi X, \varphi Y) \tag{2.6}$$

In a trans-Sasakian manifold, we also have [9, 12]

$$R(X, Y)\xi = (\alpha^2 - \beta^2)(\eta(Y)X - \eta(X)Y) + 2\alpha\beta(\eta(Y)\varphi X - \eta(X)\varphi Y)$$

$$+(Y\alpha)\varphi X - (X\alpha)\varphi Y + (Y\beta)\varphi^2 X - (X\beta)\varphi^2 Y \tag{2.7}$$

$$R(\xi, Y)X = (\alpha^2 - \beta^2)(g(X, Y)\xi - \eta(X)Y) + 2\alpha\beta(g(\varphi X, Y)\xi + \eta(X)\varphi Y)$$

$$+(X\alpha)\varphi Y + g(\varphi X, Y)(\mathrm{grad}\alpha) + X\beta(Y - \eta(Y)\xi) - g(\varphi X, \varphi Y)(\mathrm{grad}\beta), \tag{2.8}$$

$$R(\xi, X)\xi = (\alpha^2 - \beta^2 - \xi\beta)(\eta(X)\xi - X) \tag{2.9}$$

and

$$2\alpha\beta + \xi\alpha = 0, \tag{2.10}$$

where R is the curvature tensor.

$$S(X, \xi) = (2n(\alpha^2 - \beta^2) - \xi\beta)\eta(X) - (2n - 1)X\beta - (\varphi X)\alpha, \tag{2.11}$$

$$Q\xi = (2n(\alpha^2 - \beta^2) - \xi\beta)\xi - (2n - 1)\mathrm{grad}\beta + \varphi(\mathrm{grad}\alpha), \tag{2.12}$$

where S is the Ricci-curvature and Q is the Ricci-operator of trans-Sasakian manifold of type (α, β). S and Q are related to each other by

$$S(X, Y) = g(QX, Y)$$

Under the condition $\varphi(\mathrm{grad}\alpha) = (2n - 1)(\mathrm{grad}\beta)$, we have

$$\xi\beta = 0. \tag{2.13}$$

Hence

$$S(X, \xi) = (2n(\alpha^2 - \beta^2) - \xi\beta)\eta(X), \tag{2.14}$$

$$Q\xi = (2n(\alpha^2 - \beta^2) - \xi)\xi. \tag{2.15}$$

An almost contact metric manifold M is said to be $\eta -$ Einstein if its Ricci-tensor S is of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y),$$

where a and b are smooth functions on M. A $\eta -$Einstein manifold becomes Einstein if $b = 0$.

If $\{e_1, e_2, \ldots\ldots\ldots e_n = \xi\}$ is a local orthonormal basis of vector fields in an $n -$dimensional almost contact manifold M, then $\{\varphi e_1, \varphi e_2, \ldots\ldots\ldots \varphi e_{n-1}, \xi\}$ is also a local orthonormal basis. It is easy to verify that

$$\sum_{i=1}^{n} g(e_i, e_i) = \sum_{i=1}^{n-1} g(\varphi e_i, \varphi e_i) + g(\xi, \xi) = n,$$

A linear connection $\tilde{\nabla}$ in an almost contact metric manifold M is said to be

- semi-symmetric connection [16] if its torsion tensor $T(X, Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X, Y]$ satisfies

$$T(X, Y) = \eta(Y)X - \eta(X)Y$$

- non-metric connection [1] if

$$(\tilde{\nabla})g \neq 0.$$

A semi-symmetric non-metric connection $\tilde{\nabla}$ [1] in an almost contact metric manifold M can be defined as

$$\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)X. \tag{2.16}$$

Let $\tilde{R}$ and R be the curvature tensors of the semi-symmetric non-metric connection $\tilde{\nabla}$ and the Levi-civita connection ∇ respectively. Then it is known that

$$\tilde{R}(X, Y)Z = R(X, Y)Z + A(X, Z)Y - A(Y, Z)X, \tag{2.17}$$

where A is a tensor field of type $(0, 2)$ given by

$$A(X, Y) = (\tilde{\nabla}_X \eta)Y = (\nabla_X \eta)Y - \eta(X)\eta(Y) \tag{2.18}$$

From (2.17), we deduce that

$$\tilde{S}(X, Y) = S(X, Y) - 2nA(X, Y), \tag{2.19}$$

$$\tilde{r} = r - 2n\,\mathrm{trace}A, \tag{2.20}$$

where $\tilde{S}$ and S are Ricci-tensors and $\tilde{r}$ and r are scalar curvatures of the semi-symmetric non-metric connection $\tilde{\nabla}$ and the Levi-civita connection ∇ respectively.
On a trans-Sasakian manifold with respect to semi symmetric non-metric connection, we have [13]

Lemma 2.1 Let M be a trans-Sasakian manifold with respect to semi-symmetric non-metric connection, then

$$(\tilde{\nabla}_X \varphi)(Y) = \alpha\{g(X, Y)\xi - \eta(Y)X\} + \beta\{g(\varphi X, Y)\xi - \eta(Y)\varphi X\} - \eta(Y)\varphi X, \tag{2.21}$$

$$\tilde{\nabla}_X \xi = X - \alpha\varphi X + \beta\{X - \eta(X)\xi\}, \tag{2.22}$$

$$(\tilde{\nabla}_X \eta)Y = -\alpha g(\varphi X, Y) + \beta g(\varphi X, \varphi Y) - \eta(X)\eta(Y), \tag{2.23}$$

$$\begin{aligned}\tilde{R}(X, Y)Z = R(X, Y)Z &+ \alpha\{g(\varphi Y, Z)X - g(\varphi X, Z)Y\} - \beta\{g(Y, Z)X - g(X, Z)Y\}\\ &+(\beta + 1)\eta(Z)\{\eta(Y)X - \eta(X)Y\}\end{aligned} \tag{2.24}$$

We also have the following theorem [13].

Theorem 2.2 In an $(2n + 1)$ $-$dimensional trans-Sasakian manifold, the Ricci-tensor$\tilde{S}$and the scalar curvature $\tilde{r}$with respect tosemi-symmetric non-metric connection$\tilde{\nabla}$ are given by

$$S(X, Y) = S(X, Y) + 2n[\alpha g(\varphi X, Y)] - \beta g(X, Y) + (\beta + 1)\eta(X)\eta(Y)], \tag{2.25}$$

$$\tilde{r} = r - 2n(2n\beta - 1). \tag{2.26}$$

III. Trans-Sasakian manifold admitting semi-symmetric non-metric connection with $\tilde{R}.\tilde{S} = 0$

The relation between the conformal curvature tensor with respect to semi-symmetric non-metric connection and the conformal curvature tensor with respect to Levi-civita connection on a trans-Sasakian manifold is as follows [13]

$$\begin{aligned}\tilde{C}(X, Y)Z = C(X, Y)Z &- \frac{\alpha}{(2n-1)}[\{g(\varphi Y, Z)X - g(\varphi X, Z)Y\}\\ &+2n\{g(Y, Z)\varphi X - g(X, Z)\varphi Y\}] + \frac{(1+\beta)}{(2n-1)}[\{g(Y, Z)X - g(X, Z)Y\}\\ &+\eta(Z)\{\eta(X)Y - \eta(Y)X\} + 2n\{\eta(Y)g(X, Z) - \eta(X)g(Y, Z)\}\xi],\end{aligned}$$

where $\tilde{C}$ and C are the conformal curvature tensor admitting semi-symmetric non-metric connection and the conformal curvature tensor admitting Levi-civita connection respectively.
Let us take a trans-Sasakian manifold admitting semi-symmetric non-metric connection satisfying $\tilde{R}.\tilde{S} = 0$
From above equation, we have

$$\tilde{S}(\tilde{R}(X, Y)Z, W) + \tilde{S}(Z, \tilde{R}(X, Y)W) = 0. \tag{3.1}$$

Taking $X = W = \xi$, We get

$$\tilde{S}(\tilde{R}(\xi, Y)Z, \xi) + \tilde{S}(Z, \tilde{R}(\xi, Y)\xi) = 0. \tag{3.2}$$

Now in virtue of equation (2.8) and (2.24), we have

$$\tilde{R}(\xi, Y)Z = R(\xi, Y)Z + \alpha g(\varphi Y, Z)\xi - \beta(g(Y, Z)\xi - \eta(Z)Y) + (1+\beta)\eta(Z)\{\eta(Y)\xi - Y\},$$

$$\begin{aligned}\tilde{R}(\xi, Y)Z = (\alpha^2 - \beta^2 - \beta)\,g(Y, Z)\,\xi &- (1 + \alpha^2 - \beta^2)\,\eta(Z)\,Y + (2\alpha\beta - \alpha)\,g(\varphi Z, Y)\xi\\ &- 2a\beta\eta(Z)\,\varphi Y + (Za)\,\varphi Y + g(\varphi Z, Y)\,\mathrm{grad}\alpha + (Z\beta)\,Y - (Z\beta)\eta(Y)\,\xi\\ &-g(Y, Z)\mathrm{grad}\,\beta - \eta(Y)\,\eta(Z)\,\mathrm{grad}\,\beta + (1+\beta)\eta(Y)\eta(Z)\xi.\end{aligned} \tag{3.3}$$

Putting $Z = \xi$ In the equation (3.3), we have

$$\tilde{R}(\xi, Y)\xi = (\alpha^2 - \beta^2 - \beta)\,\eta(Y)\xi - (\alpha^2 - \beta^2 - \beta)\,Y - 4a\beta\varphi Y. \tag{3.4}$$

In virtue of equation (3.3), we have

$$\begin{aligned}\tilde{S}(\tilde{R}(\xi, Y)Z, \xi) = {} & (\alpha^2 - \beta^2 - \beta)\,g(Y,Z)\,\tilde{S}(\xi, \xi)\\ & -(1 + \alpha^2 - \beta^2)\,\eta(Z)\,\tilde{S}(Y, \xi) + (2\alpha\beta - \alpha)\,g(\varphi Z, Y)\,\tilde{S}(\xi, \xi)\\ & - 2a\beta\eta(Z)\,\tilde{S}(\varphi Y, \xi) + (Z\alpha)\,\tilde{S}(\varphi Y, \xi) + g(\varphi Z, Y)\tilde{S}(\mathrm{grad}\,\alpha, \xi)\\ & +(Z\beta)\tilde{S}(Y, \xi) - (Z\beta)\eta(Y)\tilde{S}(\xi, \xi) - g(Y, Z)\tilde{S}(\mathrm{grad}\,\beta, \xi)\\ & +\eta(Y)\eta(Z)\tilde{S}(\mathrm{grad}\,\beta, \xi) + (1 + \beta)\eta(Y)\eta(Z)\tilde{S}(\xi, \xi).\end{aligned} \tag{3.5}$$

Now from equation (2.25), we have

$$\tilde{S}(Y, \xi) = S(Y, \xi) + 2n(-\beta\eta(Y) + (1 + \beta)\eta(Y)).$$

In virtue of equation (2.11), above equation reduces to

$$\tilde{S}(Y, \xi) = (2n(\alpha^2 - \beta^2 + 1) - \xi\beta)\eta(Y) - ((2n - 1)Y\beta + (\varphi Y)\alpha). \tag{3.6}$$

Taking $Y = \xi$ in the equation (3.6), we get

$$\tilde{S}(\xi, \xi) = 2n(\alpha^2 - \beta^2 + 1 - \xi\beta). \tag{3.7}$$

Now replacing Y by φY in the equation (3.6), we get

$$\tilde{S}(\varphi Y, \xi) = -(2n - 1)(\varphi Y)\,\beta + (Y\alpha) - \eta(Y)(\xi\alpha). \tag{3.8}$$

Again replacing Y by grad α in the equation (3.6), we get

$$\tilde{S}(\mathrm{grad}\,\alpha, \xi) = (2n(\alpha^2 - \beta^2 + 1) - \xi\beta)\,(\xi\alpha) - (2n - 1)g(\mathrm{grad}\,\alpha, \mathrm{grad}\,\beta) \tag{3.9}$$

and on replacing Y by grad β in the equation (3.6), we get

$$\begin{aligned}\tilde{S}(\mathrm{grad}\,\beta, \xi) = {} & (2n(\alpha^2 - \beta^2 + 1) - \xi\beta)\,(\xi\beta)\\ & -(2n - 1)g(\mathrm{grad}\,\beta, \mathrm{grad}\,\beta) - g(\varphi\mathrm{grad}\,\beta, \mathrm{grad}\,\alpha).\end{aligned} \tag{3.10}$$

Hence, from equation (3.5), we have

$$\begin{aligned}\tilde{S}(\tilde{R}(\xi, Y)Z, \xi) = {} & \{2n(\alpha^2 - \beta^2 + 1 - \xi\beta)\,(\alpha^2 - \beta^2 - \beta) + 2n(\alpha^2 - \beta^2 + 1 - \xi\beta)\xi\beta\\ & + g((2n - 1)\mathrm{grad}\,\beta - \varphi\mathrm{grad}\,\alpha), \mathrm{grad}\,\beta\}g(Y, Z\\ & -\{g(((2n - 1)\mathrm{grad}\,\beta - \varphi\mathrm{grad}\,\alpha), \mathrm{grad}\,\beta) - (2\alpha\beta)^2\\ & +2n(\alpha^2 - \beta^2 + 1 - \xi\beta)\,(\alpha^2 - \beta^2 + 1) - 2n(a^2 - \beta^2 + 1 - \xi\beta)\,\xi\beta\\ & -2n(\alpha^2 - \beta^2 + 1 - \xi\beta)\,(1 + \beta)\}\eta(Y)\eta(Z) + \{2n(\alpha^2 - \beta^2 + 1 - \xi\beta)\,\alpha\\ & +2(2n - 1)\alpha\beta(\xi\beta) + (2n - 1)\,g(\mathrm{grad}\alpha, \mathrm{grad}\beta)\}\,g(\varphi Y, Z)\\ & +\{(2n - 1)\xi\beta(Z\beta) + 2\,\alpha\beta(Z\alpha)\}\eta(Y) + \{(\alpha^2 - \beta^2 + 1)\,((2n - 1)\,(Y\beta)\\ & +(\varphi Y)\,\alpha) - 2\alpha\beta((2n - 1)\,(\varphi Y)\,\beta - (Y\alpha))\}\,\eta(Z)\\ & -(Z\alpha)\,((2n - 1)\,(\varphi Y)\beta - (Y\alpha))\\ & -(Z\beta)\,((2n - 1)\,(Y\beta) + (\varphi Y)\,\alpha).\end{aligned} \tag{3.11}$$

In the similar manner, we get

$$\begin{aligned}\tilde{S}(Z, \tilde{R}(\xi, Y)\xi) = {} & -(\alpha^2 - \beta^2 + 1 - \xi\beta)\,\tilde{S}(Y, Z)\\ & +(2n - (\alpha^2 - \beta^2 + 1) - \xi\beta)\,(\alpha^2 - \beta^2 + 1 - \xi\beta)\,\eta(Y)\,\eta(Z)\\ & -(\alpha^2 - \beta^2 + 1 - \xi\beta)\,((2n - 1)\,(Z\beta) + (\varphi Z)\alpha)\,\eta(Y).\end{aligned} \tag{3.12}$$

On adding equations (3.11) and (3.12) and taking account of (3.1), we get

$$\begin{aligned}& \{2n(\alpha^2 - \beta^2 + 1 - \xi\beta)(\alpha^2 - \beta^2 - \beta)\\ & -(2n(\alpha^2 - \beta^2 + 1) - \xi\beta)\,\xi\beta\\ & +g(((2n - 1)\,\mathrm{grad}\,\beta - \varphi\mathrm{grad}\,\alpha)\,, \mathrm{grad}\,\beta\}\,g(Y, Z)\\ & -\{g(((2n - 1)\,\mathrm{grad}\,\beta - \varphi\mathrm{grad}\,\alpha),\ \mathrm{grad}\,\beta) - (2\alpha\beta)^2\\ & +2n(\alpha^2 - \beta^2 + 1 - \xi\beta)\,(\alpha^2 - \beta^2 + \beta)\\ & -(2n(\alpha^2 - \beta^2 + 1) - \xi\beta)\,(1 + \alpha^2 - \beta^2)\}\,\eta(Y)\,\eta(Z)\\ & +\{2n(\alpha^2 - \beta^2 + 1 - \xi\beta)\,\alpha + 2(2n - 1)\,a\beta\,(\xi\beta)\\ & +(2n - 1)\,g(\mathrm{grad}\,\alpha, \mathrm{grad}\,\beta)\}\,g(\varphi Y, Z)\\ & +\{(2n - 1)\,(\xi\beta)\,(Z\beta) + 2\alpha\beta\,(Z\alpha)\\ & -(\alpha^2 - \beta^2 + 1 - \xi\beta)\,((2n - 1)\,(Z\beta) + (\varphi Z)\,\alpha)\}\,\eta(Y)\\ & +\{(\alpha^2 - \beta^2 + 1)\,((2n - 1)\,(Y\beta) + (\varphi Y)\,\alpha)\end{aligned}$$

$$-2\alpha\beta((2n-1)\,(\varphi Y)\beta-(Y\alpha))\}\,\eta(Z)$$
$$-(Z\alpha)\,((2n-1)\,(\varphi Y)\,\beta-(Y\alpha))$$
$$-(Z\beta)\,((2n-1)\,(Y\beta)+(\varphi Y)\,\alpha).$$
$$-\left(\alpha^2-\beta^2+1-\xi\beta\right)\tilde{S}(Y,Z)=0. \tag{3.13}$$

Using equation (2.25) in the above equation, we have

$$\{2n(\alpha^2-\beta^2+1-\xi\beta(\alpha^2-\beta^2)$$
$$-(2n\left(\alpha^2-\beta^2+1\right)-\xi\beta)\,\xi\beta$$
$$+g(((2n-1)\,\mathrm{grad}\,\beta-\varphi\mathrm{grad}\,\alpha),\,\mathrm{grad}\,\beta\}\,g(Y,Z)$$
$$-\{g(((2n-1)\,\ \mathrm{grad}\,\beta-\varphi\mathrm{grad}\,\alpha),\,\mathrm{grad}\beta)-(2\alpha\beta)^2$$
$$+2n(\alpha^2-\beta^2+1-\mathcal{E}\beta)\,(\alpha^2-\beta^2+1)$$
$$-(2n(\alpha^2-\beta^2+1)\ -\mathcal{E}\beta)\,(1+\alpha^2-\beta^2)\}\,\eta(Y)\,\eta(Z)$$
$$+\{(2n-1)\,g(\mathrm{grad}\,\alpha,\,\mathrm{grad}\,\beta)-(2n-1)\,(\xi\alpha)\,(\xi\beta)\}\,g(\varphi Y,Z)$$
$$+\{(2n-1)\,(\xi\beta)\,(Z\beta)-(\xi\alpha)\,(Z\alpha)$$
$$-(\alpha^2-\beta^2+1-\xi\beta)\,((2n-1)\,(Z\beta)+(\varphi Z)\alpha)\}\eta(Y)$$
$$+\{\left(\alpha^2-\beta^2+1\right)((2n-1)\,(Y\beta)+(\varphi Y)\alpha)$$
$$+(\xi\alpha)\,((2n-1)\,(\varphi Y)\beta-(Y\alpha))\}\,\eta(Z)$$
$$-(Z\alpha)\,((2n-1)(\varphi Y)\beta-(Y\alpha))$$
$$-(Z\beta)\,((2n-1)\,(Y\beta)+(\varphi Y)\alpha).$$
$$-\left(\alpha^2-\beta^2+1-\xi\beta\right)\tilde{S}(Y,Z)=0. \tag{3.14}$$

Hence, we have

Theorem 3.1 On a trans-Sasakian manifold with $\tilde{R}.\tilde{S}=0$, the Ricci tensors admitting semi-symmetric non-metric connection and admitting Levi-civita connection are given by the equation (3.13) and (3.14).

For $\beta-$ Kenmotsu manifold, equation (3.14) takes the form

$$S(Y,Z)=-2n\,\beta^2 g(Y,Z). \tag{3.15}$$

Contracting the above equation, we get

$$r=-2n(2n+1)\beta^2. \tag{3.16}$$

Form equations (3.15) and (3.16), we conclude that

Corollary 3.2 A β −Kenmotsu manifold admitting semi-symmetric non-metric connection satisfying $\tilde{R}.\tilde{S}=0$ is an Einstein manifold with constant negative scalar curvature $-2n(2n+1)\beta^2$.

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