Barla's Formula

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Introduction

The complex and complicated expansions such as Binomial expansions can be easily computed using derivatives by the formula named by me as "Barla's Formula". The use of derivatives in expansions simplifies computations and understanding of the process of computation of expansion.

1. Barla's Formula and its Applications :-

1.1. Evaluating Expansions by Derivatives:-

$$(x+k)^n = x^n + \sum_{i=1}^{n} \left(\frac{k^i D^i x^n}{i!} \right) \text{ ... Barla's Formula}$$

$$= x^n + \frac{k^1 D^1 x^n}{1!} + \frac{k^2 D^2 x^n}{2!} + \ldots + \frac{k^n D^n x^n}{n!}$$

$$= x^n + \frac{k^1 D^1 x^n}{1!} + \frac{k^2 D^2 x^n}{2!} + \ldots + \frac{k^n n!}{n!}$$

$$= x^n + \frac{k^1 D^1 x^n}{1!} + \frac{k^2 D^2 x^n}{2!} + \frac{k^3 D^3 x^n}{3!} + \ldots + k^n$$

... .(Expanded form of Barla's Formula)

Where x is a variable; K is any constant; n is any natural number; $\sum$ = symbol of summation; D denotes operator "d/dx" and further $D^i = d^i/dx^i$ i.e. if i = 2, $D^i = d^2/dx^2$ which means differentiation twice and so on, as the term may be; and i! = 1X 2 X 3... ... .X i;

Let K = 1 in abovesaid Barla's Formula to consider the following illustrations explaining expansions by said Formula :-

A. Illustration No.1- Computation of $(x+1)^2$

$$(x+1)^n = x^n + \sum_{i=1}^{n} \left(\frac{1^i D^i x^n}{i!} \right)$$

By putting n=2 in above equation we get-

$$(x+1)^2 = x^2 + \sum_{i=1}^{2} \left(\frac{1^i D^i x^2}{i!} \right)$$

$$= x^2 + \frac{D^1 x^2}{1!} + \frac{D^2 x^2}{2!} \quad (\text{Where } D = d/dx)$$

$$= x^2 + 2x + 2/2$$

$$= x^2 + 2x + 1$$

B. Illustration No.2 - Computation of $(x+1)^3$

$$(x+1)^n = x^n + \sum_{i=1}^{n} \left(\frac{1^i D^i x^n}{i!} \right)$$

By putting n=3 in above equation, we get-

$$(x+1)^3 = x^3 + \sum_{i=1}^{3} \left(\frac{1^i D^i x^3}{i!} \right)$$

$$= x^3 + \frac{D^1 x^3}{1!} + \frac{D^2 x^3}{2!} + \frac{D^3 x^3}{3!} \quad (\text{Where } D = d/dx)$$

$$= x^3 + 3x^2 + 3x + 6/6$$

$$= x^3 + 3x^2 + 3x + 1$$

C. Illustration No.3 - Computation of $(x+1)^4$

$$(x+1)^n = x^n + \sum_{i=1}^{n} \left(\frac{1^i D^i x^n}{i!} \right)$$

By putting n=4 in above equation, we get-

$$(x+1)^4 = x^4 + \sum_{i=1}^{4} \left(\frac{1^i D^i x^4}{i!} \right)$$

$= x^4 + \frac{D^1x^4}{1!} + \frac{D^2x^4}{2!} + \frac{D^3x^4}{3!} + \frac{D^3x^4}{4!}$

(Where D = d/dx)

$= x^4 + 4x^3 + \frac{12x^2}{2} + \frac{24x}{6} + \frac{24}{24}$

$= x^4 + 4x^3 + 6x^2 + 4x + 1$

D. Illustration No.4 - Computation of $(x+1)^5$

$$(x+1)^n = x^n + \sum_{i=1}^{n} \left(\frac{1^i D^i x^n}{i!}\right)$$

By putting n=5 in above equation, we get-

$$(x+1)^5 = x^5 + \sum_{i=1}^{5} \left(\frac{1^i D^i x^5}{i!}\right)$$

$= x^5 + \frac{D^1x^5}{1!} + \frac{D^2x^5}{2!} + \frac{D^3x^5}{3!} + \frac{D^4x^5}{4!} + \frac{D^5x^5}{5!}$

(Where D=d/dx)

$= x^5 + 5x^4 + \frac{20x^3}{2} + \frac{60x^2}{6} + \frac{120x}{24} + \frac{120}{120}$

$= x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

and similarly any expansion upto nth level, as herein below submitted, can be calculated.

1.2 nth level computation and its application :

$$(x+k)^n = x^n + \sum_{i=1}^{n} \left(\frac{k^i D^i x^n}{i!}\right)$$

$$(x+1)^n = x^n + \sum_{i=1}^{n} \left(\frac{1^i D^i x^n}{i!}\right) \quad (\text{Where } k=1);$$

$$= x^n + \sum_{i=1}^{n} \left(\frac{D^i x^n}{i!}\right)$$

$= x^n + \frac{D^1x^n}{1!} + \frac{D^2x^n}{2!} + \frac{D^3x^n}{3!} + \frac{D^4x^n}{4!} + \ldots + \frac{D^nx^n}{n!}$

(Where D = d/dx)

$= x^n + \frac{nx^{n-1}}{1!} + \frac{n(n-1)x^{n-2}}{2!} + \frac{n(n-1)(n-2)x^{n-3}}{3!} +$

$\frac{n(n-1)(n-2)(n-3)x^{n-4}}{4!} + \ldots \ldots \ldots \ldots \ldots \ldots +$

$\frac{n(n-1)(n-2)(n-3)(n-4)\ldots\ldots..1}{n!}$

$= x^n + \frac{nx^{n-1}}{1!} + \frac{n(n-1)x^{n-2}}{2!} + \frac{n(n-1)(n-2)x^{n-3}}{3!} +$

$\frac{n(n-1)(n-2)(n-3)x^{n-4}}{4!} + \ldots\ldots\ldots.+ \frac{n!}{n!}$

$= x^n + \frac{nx^{n-1}}{1!} + \frac{n(n-1)x^{n-2}}{2!} + \frac{n(n-1)(n-2)x^{n-3}}{3!} +$

$\frac{n(n-1)(n-2)(n-3)x^{n-4}}{4!} + \ldots\ldots\ldots.+ 1 \quad \ldots..\text{Exp.}(1)$

By binomial theorem, we also know-

$(x+1)^n = x^n + {}^nC_1x^{n-1} + {}^nC_2x^{n-2} + {}^nC_3x^{n-3} + {}^nC_4x^{n-4}$

$+\ldots\ldots\ldots\ldots.+ {}^nC_{n-1}x^{n-(n-1)} + {}^nC_nx^0$

$= x^n + {}^nC_1x^{n-1} + {}^nC_2x^{n-2} + {}^nC_3x^{n-3} + {}^nC_4x^{n-4}$

$+\ldots\ldots\ldots\ldots.+ {}^nC_{n-1}x^{n-(n-1)} + {}^nC_n \quad \ldots\ldots...\text{Exp.}(2)$

On Comparing Exp.(1) with Exp.(2) we get—

Exp.(1) = Exp.(2)

By Comparing last term in Exp.(1) and Exp.(2)—

${}^nC_n = 1$

ie. $\frac{n!}{(n-n)!n!} = 1$

Thus $\frac{1}{0!} = 1$

ie. $0! = 1$

Thus factorial of Zero is 1.

1.3. Manner of application of this formula –

Putting k = 1 in Barla's Formula, we get---

$$(x+1)^n = x^n + \sum_{i=1}^{n} \left(\frac{D^i x^n}{i!}\right) \quad (\text{Where } k=1);$$

As an illustration ---- evaluate $(e^x + 1)^2$

Replace e^x by x in above evaluation ---

Thus $(e^x + 1)^2 = (x+1)^2$

Now we have---

$(x+1)^2 = x^2 + 2x + 1$ ... ... Exp.(i)

Again replacing x by e^x in abve expression, we get

$\underline{(e^x + 1)^2 = (e^x)^2 + 2e^x + 1}$

In this manner, any expansion ie. logarithmic, trigonometric or exponential , must be calculated by using Barla's Formula on expansions in aforesaid manner.

Conclusion

It is submitted that this formula facilitates easier solution to complex expansions. This formula as evolved, opens the door for calculus to field of expansions and hence introduces calculus to the field of elementary mathematics..

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