# Exterior Set in Soft Biminimal Spaces 

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#### Abstract

The aim of this paper is to introduce the concept and some fundamental properties of exterior set in soft biminimal spaces.


Keywords: soft minimal, soft biminimal space, exterior set.

## 1 Introduction

In 2000, V. Popa and T.Noiri [14] introduced the concepts of minimal structure (briefly m-structure). They also introduced the concepts of $m_{X^{-}}$open set and $m_{X^{-}}$ closed set and characterize those sets using $m_{X}$-closure and $m_{X}$-interior operators respectively. J.C. Kelly [7] defined the study of bitopological spaces in 1963. In 2010, C. Boonpok [2] introduced the concept of biminimal structure space and studied $m_{X}^{1} m_{X}^{2}$-open sets and $m_{X}^{1} m_{X}^{2}$-closed sets in biminimal structure spaces. Russian researcher Molodtsov [5], initaited the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences in 1999. In 2015, R. Gowri and S. Vembu [11] introduced Soft minimal and soft biminimal spaces. The purpose of this paper is to introduce the concept of exterior set in soft biminimal spaces and their properties are studied.

## 2 Preliminaries

Definition 2.1 [11] Let $X$ be an initial universe set, $E$ be the set of parameters and $A \subseteq E$. Let $F_{A}$ be a nonempty soft set over $X$ and $\tilde{P}\left(F_{A}\right)$ is the soft power set of $F_{A}$. A subfamily $\tilde{m}$ of $\tilde{P}\left(F_{A}\right)$ is called a soft minimal set over $X$ if $F_{\emptyset} \in \tilde{m}$ and $F_{A} \in \tilde{m}$.
$\left(F_{A}, \tilde{m}\right)$ or $(X, \tilde{m}, E)$ is called a soft minimal space over $X$. Each member of $\tilde{m}$ is said to be $\tilde{m}$-soft open set and the complement of an $\tilde{m}$-soft open set is said to be $\tilde{m}$-soft closed set over $X$.

Definition 2.2 [11] Let $X$ be an initial universe set and $E$ be the set of parameters. Let $\left(X, \tilde{m}_{1}, E\right)$ and $\left(X, \tilde{m}_{2}, E\right)$ be the two different soft minimals over $X$. Then $\left(X, \tilde{m}_{1}, \tilde{m_{2}}, E\right)$ or $\left(F_{A}, \tilde{m_{1}}, \tilde{m_{2}}\right)$ is called a soft biminimal spaces.

Definition 2.3 [11] $A$ soft subset $F_{B}$ of a soft biminimal space $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ is called $\tilde{m}_{1} \tilde{m}_{2}$-soft closed if $\tilde{m} c l_{1}\left(\tilde{m} c l_{2}\left(F_{B}\right)\right)=F_{B}$. The complement of $\tilde{m}_{1} \tilde{m}_{2}$-soft closed set is called $\tilde{m}_{1} \tilde{m}_{2}$-soft open.

Proposition 2.4 [11] Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space over $X$. Then $F_{B}$ is a $\tilde{m}_{1} \tilde{m}_{2}$-soft open soft subsets of $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ if and only if $F_{B}=\tilde{m} \operatorname{Int}_{1}\left(\tilde{m} \operatorname{Int}_{2}\left(F_{B}\right)\right)$.

Proposition 2.5 [11] Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space.If $F_{B}$ and $F_{C}$ are $\tilde{m}_{1} \tilde{m}_{2}$-soft closed soft subsets of $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ then $F_{B} \tilde{\cap} F_{C}$ is $\tilde{m}_{1} \tilde{m}_{2}$-soft closed.

Proposition 2.6 [11] Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space over $X$. If $F_{B}$ and $F_{C}$ are $\tilde{m}_{1} \tilde{m}_{2}$-soft open soft subsets of $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$, then $F_{B} \cup \tilde{U}_{C}$ is $\tilde{m}_{1} \tilde{m}_{2}$-soft open.

Definition 2.7 [5] Let $U$ be an initial universe and $E$ be a set of parameters. Let $P(U)$ denote the power set of $U$ and $A$ be a nonempty subset of $E$. A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F: A \rightarrow P(U)$.

In other words, a soft set over $U$ is a parametrized family of subsets of the universe $U$. For $\epsilon \in A . F(\epsilon)$ may be considered as the set of $\epsilon$ - approximate elements of the soft set $(F, A)$. Clearly, a soft set is not a set.

Example 2.8 [11] Let $U=\left\{u_{1}, u_{2}\right\}, E=\left\{x_{1}, x_{2}, x_{3}\right\}, A=\left\{x_{1}, x_{2}\right\} \subseteq E$ and $F_{A}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right),\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\}$. Then

$$
F_{A_{1}}=\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}
$$

$$
F_{A_{2}}=\left\{\left(x_{1},\left\{u_{2}\right\}\right)\right\}
$$

$$
F_{A_{3}}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right)\right\}
$$

$$
F_{A_{4}}=\left\{\left(x_{2},\left\{u_{1}\right\}\right)\right\}
$$

$$
F_{A_{5}}=\left\{\left(x_{2},\left\{u_{2}\right\}\right)\right\}
$$

$$
F_{A_{6}}=\left\{\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\}
$$

$$
F_{A_{7}}=\left\{\left(x_{1},\left\{u_{1}\right\}\right),\left(x_{2},\left\{u_{1}\right\}\right)\right\}
$$

$$
F_{A_{8}}=\left\{\left(x_{1},\left\{u_{1}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\}
$$

$$
F_{A_{9}}=\left\{\left(x_{1},\left\{u_{1}\right\}\right),\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\}
$$

$$
F_{A_{10}}=\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{1}\right\}\right)\right\}
$$

$$
F_{A_{11}}=\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\}
$$

$$
F_{A_{12}}=\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\}
$$

$$
F_{A_{13}}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right),\left(x_{2},\left\{u_{1}\right\}\right)\right\}
$$

$$
F_{A_{14}}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\}
$$

$$
F_{A_{15}}=F_{A}
$$

$$
F_{A_{16}}=F_{\emptyset}
$$

are all soft subsets of $F_{A}$. so $\left|\tilde{P}\left(F_{A}\right)\right|=2^{4}=16$.
$\tilde{m}=\left\{F_{\emptyset}, F_{A}, F_{A_{4}}, F_{A_{7}} F_{A_{11}} F_{A_{13}}\right\}$

## 3 Exterior set in soft biminimal spaces

In this section, we introduce the concept and study some fundamental properties of exterior set in soft biminimal spaces.
Definition 3.1 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS), $F_{B}$ be a soft subset of $F_{A}$ and $x \in F_{A}$. Then $x$ is called $\tilde{m}_{i} \tilde{m}_{j}$-exterior point of $F_{B}$ if
$x \in \tilde{m}_{i} \operatorname{Int}\left(\tilde{m}_{j} \operatorname{Int}\left(F_{A} \backslash F_{B}\right)\right)$. We denote the set of all $\tilde{m}_{i} \tilde{m}_{j}$-exterior point of $F_{B}$ by $\tilde{m} \operatorname{Ext}_{i j}\left(F_{B}\right)$ where $i, j=1,2$, and $i \neq j$.
From definition we have $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)$.

Example 3.2 Let $X=\left\{u_{1}, u_{2}\right\}, E=\left\{x_{1}, x_{2}, x_{3}\right\}, A=\left\{x_{1}, x_{2}\right\} \subseteq E$ and $F_{A}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right),\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\}$. Then

$$
F_{A_{1}}=\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\},
$$

$$
F_{A_{2}}=\left\{\left(x_{1},\left\{u_{2}\right\}\right)\right\},
$$

$$
F_{A_{3}}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right)\right\},
$$

$$
F_{A_{4}}=\left\{\left(x_{2},\left\{u_{1}\right\}\right)\right\},
$$

$$
F_{A_{5}}=\left\{\left(x_{2},\left\{u_{2}\right\}\right)\right\},
$$

$$
F_{A_{6}}=\left\{\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\},
$$

$$
F_{A_{7}}=\left\{\left(x_{1},\left\{u_{1}\right\}\right),\left(x_{2},\left\{u_{1}\right\}\right)\right\},
$$

$$
F_{A_{8}}=\left\{\left(x_{1},\left\{u_{1}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\},
$$

$$
F_{A_{9}}=\left\{\left(x_{1},\left\{u_{1}\right\}\right),\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\},
$$

$$
F_{A_{10}}=\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{1}\right\}\right)\right\},
$$

$$
F_{A_{11}}=\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\},
$$

$$
F_{A_{12}}=\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\},
$$

$$
F_{A_{13}}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right),\left(x_{2},\left\{u_{1}\right\}\right)\right\},
$$

$$
F_{A_{14}}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\},
$$

$$
F_{A_{15}}=F_{A},
$$

$$
F_{A_{16}}=F_{\emptyset} \text { are all soft subsets of } F_{A} \text {. }
$$

$\tilde{m_{1}}=\left\{F_{\emptyset}, F_{A}, F_{A_{8}}, F_{A_{10}}\right\}$ and $\tilde{m_{2}}=\left\{F_{\emptyset}, F_{A}, F_{A_{1}}, F_{A_{12}}\right\}$.
Hence, $\tilde{m} \operatorname{Ext}_{12}\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}\right)=F_{A} \backslash\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}\right)=\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{1}\right\}\right)\right\}$,
$\tilde{m} \operatorname{Ext}_{21}\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}\right)=F_{A} \backslash\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}\right)=F_{\emptyset}$

Lemma 3.3 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}$ be a soft subset of $F_{A}$. Then for any $i, j=1,2$, and $i \neq j$, we have:
a) $\tilde{m} E x t_{i j}\left(F_{B}\right) \cap F_{B}=F_{\emptyset}$,
b) $\tilde{m} E x t_{i j}\left(F_{\emptyset}\right)=F_{A}$,
c) $\tilde{m} \operatorname{Ext}_{i j}\left(F_{A}\right)=F_{\emptyset}$

Proof: a) Assume that ( $F_{A}, \tilde{m}_{1}, \tilde{m}_{2}$ ) be a soft biminimal space (SBMS) and $F_{B}$ be a soft subset of $F_{A}$.
Since $F_{B} \subset \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)$
We have $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)$.
Now, $\tilde{m} E x t_{i j}\left(F_{B}\right) \cap F_{B}$
$=F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right) \cap F_{B}$
$=\left(F_{A} \backslash F_{B}\right) \cap F_{B}$
$=F_{\emptyset}$
Hence $\tilde{m} E x t_{i j}\left(F_{B}\right) \cap F_{B}=F_{\emptyset}$
b) $\tilde{m} E x t_{i j}\left(F_{\emptyset}\right)=F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{\emptyset}\right)\right)$

$$
\begin{aligned}
& =F_{A} \backslash F_{\emptyset} \\
& =F_{A}
\end{aligned}
$$

Hence $\tilde{m} E x t_{i j}\left(F_{\emptyset}\right)=F_{A}$
c) $\tilde{m} \operatorname{Ext}_{i j}\left(F_{A}\right)=F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{A}\right)\right)$

$$
=F_{A} \backslash F_{A}
$$

$$
=F_{\emptyset}
$$

Hence $\tilde{m} E x t_{i j}\left(F_{A}\right)=F_{\emptyset}$

Theorem 3.4 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}, F_{C}$ be a soft subset of $F_{A}$. If $F_{B} \tilde{\subseteq} F_{C}$, then $\tilde{m} E x t_{i j}\left(F_{C}\right) \widetilde{\subseteq} \tilde{m} E x t{ }_{i j}\left(F_{B}\right)$ Where $i, j=1,2$, and $i \neq j$.

Proof: Assume that $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}, F_{C}$ be a soft subset of $F_{A}$.
Let $F_{B} \tilde{\subseteq} F_{C}$
Thus $\tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right) \subseteq \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{C}\right)\right)$
Then $F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{C}\right)\right) \subseteq \tilde{F}_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)$
Hence, $\tilde{m} E x t_{i j}\left(F_{C}\right) \subseteq \tilde{m} E x t_{i j}\left(F_{B}\right)$ for any $i, j=1,2$, and $i \neq j$.
Theorem 3.5 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}$ be a soft subset of $F_{A}$. Then for any $i, j=1,2$, and $i \neq j, F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft closed if and only if $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash F_{B}$

Proof: Let $F_{B}$ be a soft subset of $F_{A}$.
Assume that $F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft closed
Since $F_{B}=\tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)$
By Definition (3.1) in SBMS, $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)$
Therefore $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)=F_{A} \backslash F_{B}$
Hence, $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash F_{B}$
conversely, $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash F_{B}$
Since, $F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)=F_{A} \backslash F_{B}$
That implies $\tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)=F_{B}$
Hence, $F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft closed.
Theorem 3.6 $\operatorname{Let}\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}$ be a soft subset of $F_{A}$. Then for any $i, j=1,2$, and $i \neq j, F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft open if and only if $\tilde{m} E x t_{i j}\left(F_{A} \backslash F_{B}\right)=F_{B}$

Proof: Let $F_{B}$ be a soft subset of $F_{A}$.
Assume that $F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft open
Since $F_{A} \backslash F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft closed.
By Definition (3.1) $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash \tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{B}\right)\right)$.
Therefore $\tilde{m} E x t_{i j}\left(F_{A} \backslash F_{B}\right)=F_{A} \backslash\left(\tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{A} \backslash F_{B}\right)\right)=F_{B}\right.$.
Hence, $\tilde{m} E x t_{i j}\left(F_{A} \backslash F_{B}\right)=F_{B}$
conversely, $\tilde{m} E x t_{i j}\left(F_{A} \backslash F_{B}\right)=F_{B}$
Since, $F_{B}=\tilde{m} E x t_{i j}\left(F_{A} \backslash F_{B}\right)=F_{A} \backslash\left(\tilde{m}_{i} C l\left(\tilde{m}_{j} C l\left(F_{A} \backslash F_{B}\right)\right)=\tilde{m}_{i} \operatorname{Int}\left(\tilde{m}_{j} \operatorname{Int}\left(F_{B}\right)\right)\right.$.
Hence, $F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft open.
Theorem 3.7 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a $S B M S$ and $F_{B}$ be a soft subset of $F_{A}$. If $F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft closed, then $\tilde{m} E x t_{i j}\left(F_{A} \backslash \tilde{m} E x t_{i j}\left(F_{B}\right)\right)=\tilde{m} E x t_{i j}\left(F_{B}\right)$. Then for any $i, j=1,2$, and $i \neq j$.

Proof: Assume that $F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft closed
By Theorem 3.5, $F_{B}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft closed if and only if $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash F_{B}$
That implies, $\tilde{m} E x t_{i j}\left(F_{A} \backslash \tilde{m} E x t_{i j}\left(F_{B}\right)\right)$

$$
\begin{aligned}
& =\tilde{m} E x t_{i j}\left(F_{A} \backslash\left(F_{A} \backslash F_{B}\right)\right) \\
& =\tilde{m} E x t_{i j}\left(F_{B}\right) .
\end{aligned}
$$

Corollary 3.8 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}, F_{C}$ be a soft subset of $F_{A}$. Then for any $i, j=1,2$, and $i \neq j$. If $F_{B}$ and $F_{C}$ are $\tilde{m}_{i} \tilde{m}_{j}$-soft open, then $\tilde{m} \operatorname{Ext}_{i j}\left(F_{A} \backslash\left(F_{B} \cup F_{C}\right)\right)=F_{B} \cup F_{C}$.

Proof: Assume that $F_{B}$ and $F_{C}$ are $\tilde{m}_{i} \tilde{m}_{j}$-soft open, then $F_{B} \cup F_{C}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft open.
It follows from Theorem 3.6 that $\tilde{m} E x t_{i j}\left(F_{A} \backslash\left(F_{B} \cup F_{C}\right)\right)=F_{B} \cup F_{C}$.
Corollary 3.9 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}, F_{C}$ be a soft subset of $F_{A}$. Then for any $i, j=1,2$, and $i \neq j$. If $F_{B}$ and $F_{C}$ are $\tilde{m}_{i} \tilde{m}_{j}$-soft closed, then $\tilde{m} E x t_{i j}\left(F_{A} \backslash\left(F_{B} \cap F_{C}\right)\right)=F_{B} \cap F_{C}$.

Proof: The proof is obivious.
Theorem 3.10 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}, F_{C}$ be a soft subset of $F_{A}$, then $\tilde{m} E x t_{i j}\left(F_{B}\right) \cup \tilde{m} E x t_{i j}\left(F_{C}\right) \widetilde{\subseteq} \tilde{m} E x t_{i j}\left(F_{B} \cap F_{C}\right)$ where $i, j=1,2$, and $i \neq j$.

Proof: Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}, F_{C}$ be a soft subset of $F_{A}$.
Since, $F_{B} \cap F_{C} \tilde{\subseteq} F_{B}$ and $F_{B} \cap F_{C} \tilde{\subseteq} F_{C}$.
Then $\tilde{m} E x t_{i j}\left(F_{B}\right) \subseteq \tilde{\subseteq} \tilde{m} E x t_{i j}\left(F_{B} \cap F_{C}\right)$ and $\tilde{m} E x t_{i j}\left(F_{C}\right) \subseteq \tilde{m} E x t_{i j}\left(F_{B} \cap F_{C}\right)$.
It follows that $\tilde{m} E x t_{i j}\left(F_{B}\right) \cup \tilde{m} E x t_{i j}\left(F_{C}\right) \widetilde{\subseteq} \tilde{m} E x t_{i j}\left(F_{B} \cap F_{C}\right)$.
Theorem 3.11 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}, F_{C}$ be a soft subset of $F_{A}$. Then for any $i, j=1,2$, and $i \neq j$, If $F_{B}, F_{C}$ are $\tilde{m}_{i} \tilde{m}_{j}$-soft closed, then $\tilde{m} E x t_{i j}\left(F_{B}\right) \cup \tilde{m} E x t_{i j}\left(F_{C}\right)=\tilde{m} E x t_{i j}\left(F_{B} \cap F_{C}\right)$

Proof: Assume that $F_{B}$ and $F_{C}$ are $\tilde{m}_{i} \tilde{m}_{j}$-soft closed. Thus $F_{B} \cap F_{C}$ is $\tilde{m}_{i} \tilde{m}_{j}$-soft closed.
It follows from Theorem 3.5 that $\tilde{m} E x t_{i j}\left(F_{B}\right)=F_{A} \backslash F_{B}$
Thus $\tilde{m} E x t_{i j}\left(F_{B} \cap F_{C}\right)=F_{A} \backslash\left(F_{B} \cap F_{C}\right)$

$$
\begin{aligned}
& =\left(F_{A} \backslash F_{B}\right) \cup\left(F_{A} \backslash F_{C}\right) \\
& =\tilde{m} E x t_{i j}\left(F_{B}\right) \cup \tilde{m} E x t_{i j}\left(F_{C}\right)
\end{aligned}
$$

Hence $\tilde{m} E x t_{i j}\left(F_{B}\right) \cup \tilde{m} E x t_{i j}\left(F_{C}\right)=\tilde{m} E x t_{i j}\left(F_{B} \cap F_{C}\right)$
Example 3.12 Let $X=\left\{u_{1}, u_{2}\right\}, E=\left\{x_{1}, x_{2}, x_{3}\right\}, A=\left\{x_{1}, x_{2}\right\} \subseteq E$ and $F_{A}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right),\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\}$. Then $\tilde{m}_{1}=\left\{F_{\emptyset}, F_{A}, F_{A_{7}}, F_{A_{11}}\right\}, \tilde{m}_{2}=\left\{F_{\emptyset}, F_{A}, F_{A_{1}}, F_{A_{2}}\right\}$ $\tilde{m} \operatorname{Ext}_{i j}\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}=F_{A} \backslash \tilde{m}_{1} C l\left(\tilde{m}_{2} C l\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}\right)$, $\tilde{m} \operatorname{Ext}_{i j}\left\{\left(x_{2},\left\{u_{1}\right\}\right)\right\}=F_{A} \backslash \tilde{m}_{1} C l\left(\tilde{m}_{2} C l\left\{\left(x_{2},\left\{u_{1}\right\}\right)\right\}\right.$, and $\tilde{m} \operatorname{Ext}_{i j}\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\} \cap\left\{\left(x_{2},\left\{u_{1}\right\}\right)\right\}\right)=F_{A} \backslash \tilde{m}_{1} C l\left(\tilde{m}_{2} C l\left(F_{\emptyset}\right)\right)$ Hence $\tilde{m} \operatorname{Ext}_{i j}\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}=\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\}$, $\tilde{m} \operatorname{Ext}_{i j}\left\{\left(x_{2},\left\{u_{1}\right\}\right)\right\}=F_{\emptyset}$ and $\tilde{m} \operatorname{Ext}_{i j}\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\} \cap\left\{\left(x_{2},\left\{u_{1}\right\}\right)\right\}\right)=F_{A}$
Therefore $\tilde{m} \operatorname{Ext}_{i j}\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}\right) \cup \tilde{m} E x t_{i j}\left(\left\{\left(x_{2},\left\{u_{1}\right\}\right)\right\}\right) \neq$ $\tilde{m} \operatorname{Ext}_{i j}\left(\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}\right) \cap\left\{\left(x_{2},\left\{u_{1}\right\}\right)\right\}\right)$

Theorem 3.13 Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space (SBMS) and $F_{B}, F_{C}$ be a soft subset of $F_{A}$, then $\tilde{m} E x t_{i j}\left(F_{B} \cup F_{C}\right) \subseteq \tilde{m} E x t_{i j}\left(F_{B}\right) \cap \tilde{m} E x t_{i j}\left(F_{C}\right)$ where $i, j=1,2$, and $i \neq j$.

Proof: Let $\left(F_{A}, \tilde{m}_{1}, \tilde{m}_{2}\right)$ be a soft biminimal space $(\mathrm{SBMS})$ and $F_{B}, F_{C}$ be a soft subset of $F_{A}$.
Since $F_{B} \subseteq F_{B} \cup F_{C}$ and $F_{C} \subseteq F_{B} \cup F_{C}$.
Then $\tilde{m} E x t_{i j}\left(F_{B} \cup F_{C}\right) \subseteq \tilde{m} E x t_{i j}\left(F_{B}\right)$ and $\tilde{m} E x t_{i j}\left(F_{B} \cup F_{C}\right) \subseteq \tilde{m} E x t_{i j}\left(F_{C}\right)$.
It follows that $\tilde{m} E x t_{i j}\left(F_{B} \cup F_{C}\right) \subseteq \tilde{m} E x t_{i j}\left(F_{B}\right) \cap \tilde{m} E x t_{i j}\left(F_{C}\right)$.
Example 3.14 Let $X=\left\{u_{1}, u_{2}\right\}, E=\left\{x_{1}, x_{2}, x_{3}\right\}, A=\left\{x_{1}, x_{2}\right\} \subseteq E$ and $F_{A}=\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right),\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\}$. Then $\tilde{m}_{1}=\left\{F_{\emptyset}, F_{A}, F_{A_{1}}, F_{A_{2}}, F_{A_{7}}, F_{A_{11}}\right\}, \tilde{m}_{2}=\left\{F_{\emptyset}, F_{A}, F_{A_{1}}, F_{A_{2}}, F_{A_{7}}, F_{A_{11}}\right\}$ $\tilde{m} \operatorname{Ext}_{i j}\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}=F_{A} \backslash \tilde{m}_{1} C l\left(\tilde{m}_{2} C l\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}\right)$, $\tilde{m} E x t_{i j}\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\}=F_{A} \backslash \tilde{m}_{1} C l\left(\tilde{m}_{2} C l\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\}\right.$
Hence $\tilde{m} \operatorname{Ext}_{i j}\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\} \cup\left\{\left(x_{1},\left\{u_{1}, u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\}\right)=F_{\emptyset}$, $\tilde{m} \operatorname{Ext}_{i j}\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}=\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{1}, u_{2}\right\}\right)\right\}$, $\tilde{m} \operatorname{Ext}_{i j}\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\}=\left\{\left(x_{1},\left\{u_{1}\right\}\right),\left(x_{2},\left\{u_{1}\right\}\right)\right\}$
Therefore $\tilde{m} \operatorname{Ext}_{i j}\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\} \cup\left\{\left(x_{1},\left\{u_{2}\right\}\right)\right\} \neq\right.$ $\tilde{m} \operatorname{Ext}_{i j}\left(\left\{\left(x_{1},\left\{u_{1}\right\}\right)\right\}\right) \cap \tilde{m} \operatorname{Ext}_{i j}\left(\left\{\left(x_{1},\left\{u_{2}\right\}\right),\left(x_{2},\left\{u_{2}\right\}\right)\right\}\right)$

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