Fuzzy Semimodular Lattice

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Abstract *- In this Paper, Fuzzy Semimodular Lattice – Definition of Fuzzy semimodular Lattice-Characterization theorem are given.*

Keywords *- Fuzzy Lattice, Fuzzy Modular Lattice, Fuzzy Distributive Lattice, Fuzzy Semimodular Lattice.*

I. INTRODUCTION

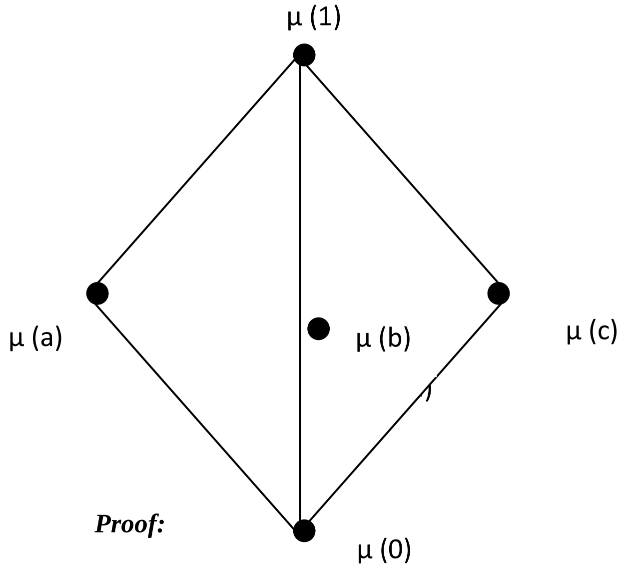
The Concept of Fuzzy Lattice was already introduced by Ajmal,N[1], S.Nanda[4] and WilCox,L.R [5] explained modularity in the theory of Lattices, G.Gratzer[2], BarDalo, G.H and Rodrigues,E[3] Stern,m[6] explained semimodular Lattices, M.Mullai and B.Chellappa[7] explained Fuzzy L-ideal. A few definitions and results are listed that the fuzzy semimodular lattice using in this paper we explain fuzzy semimodular lattice, Definition of fuzzy semimodular lattice, Characterization theorem of Fuzzy semimodular lattice and some examples are given, If L is a fuzzy semimodular lattice then μ (a) < μ (b) implies that μ (a ∨ c) < μ (b ∨ c) or μ (a∨c) = μ (b ∨ c).

Definition: 1.1

A fuzzy lattice L is called Fuzzy Semimodular lattice if it satisfies the upper covering condition. That is μ (a) < μ (b) implies that μ (a ∨ c) < μ (b ∨ c) or μ (a∨c) = μ (b ∨ c) for all μ (a), μ (b), μ (c) in L.

Example: 1.1

Consider the Fuzzy lattice of figure



Proof:

μ (a v c) ≥ min{ μ (a) , μ (c) }
≥ min{ μ (b) , μ (c) },
Since μ (a) < μ (b)
≥ μ (b v c)

This is a Fuzzy Semimodular lattice.

Theorem: 1.1

Every Fuzzy modular lattice is Fuzzy Semimodular and the coverse is not true.

Proof:

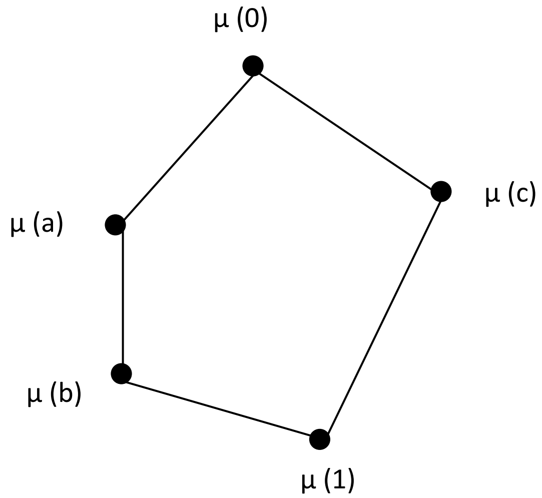
Given L is a Fuzzy modular lattice
To Prove L is Fuzzy Semimodular
Suppose μ (a) < μ (b)

⇒ μ (a) < μ (b) and there is no element between μ (a) and μ (b)

⇒ μ (a v c) < μ (b v c), no element between then

⇒ μ (a v c) < μ (b v c)

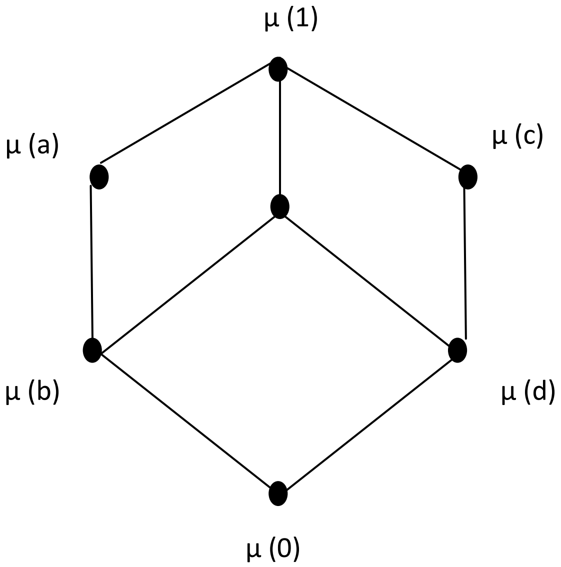
otherwise, we get N_5



⇒ L is not Fuzzy modular
This is a Contradiction
Hence L is a Fuzzy Semimodular Lattices.

The converse need not be true.
(i.e) Every Fuzzy Semimodular lattice need not be Fuzzy modular.
We shall verify it by the following example.

Consider the Fuzzy lattice of following figure



This is a Fuzzy Semimodular lattice but not fuzzy modular

$\mu\,[(b\wedge d)\vee(b\wedge a)] \geq \min\{\,\mu\,(b \wedge d)\,,\,\mu\,(b \wedge a)\,\}$
$\geq \min\{\,\mu\,(0)\,,\,\mu\,(a)\,\}$
$= \mu\,(0\vee a)$
$= \mu\,(a)$

$\mu\,[b\wedge(d\vee(b\wedge a))] \geq \min\{\mu(b),\,\mu(d\vee(b \wedge a))\,\}$
$\geq \min\{\mu(b),\,\min\{\mu(d),\,\mu(b\wedge a)\,\}\}$
$\geq \min\{\mu(b),\min\{\,\mu(d),\,\mu(a)\,\}\}$
$\geq \min\{\,\mu\,(b),\,\mu\,(d \vee a)\,\}$
$\geq \min\{\,\mu\,(b),\,\mu\,(1)\,\}$
$= \mu\,(b)$

Therefore $\mu\,[(b\wedge d)\vee(b\wedge a)] \neq \mu\,[b\wedge(d\vee(b\wedge a))]$

Hence the Fuzzy lattice is not Fuzzy modular.

Theorem: 1.2

Every Fuzzy distributive lattice is Fuzzy Semimodular and the coverse is not true.

Proof

Given L is a Fuzzy distributive lattice.

To Prove L is a Fuzzy Semimodular lattice.

L is a Fuzzy distributive lattice.

$\Rightarrow$ L is a Fuzzy modular lattices.

$\Rightarrow$ L is a Fuzzy Semimodular lattices.

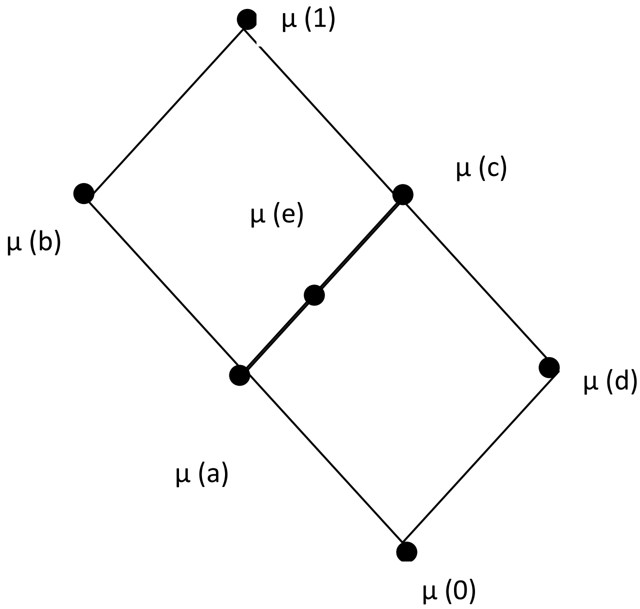
Hence every Fuzzy distributive lattice is Fuzzy Semimodular.

The converse need not be true.

(i.e) Every Fuzzy Semimodular lattice need not Fuzzy distributive.

We shall verify it by the following example.

consider the Fuzzy lattice of the following figure.



This is a Fuzzy Semimodular lattice but not Fuzzy distributive.

$\mu\,[a\vee(b \wedge c)] \geq \min\{\,\mu\,(a)\,,\,\mu\,(b \wedge c)\,\}$
$\geq \min\{\,\mu\,(a)\,,\,\mu\,(a)\,\}$
$=\mu\,(a\vee a)$
$= \mu\,(a)$

$\mu\,[(a\vee b) \wedge (a\vee c)] \geq \min\{\,\mu\,(a\vee b),\,\mu(a \vee c)\,\}$
$\geq \min\{\,\mu\,(b),\,\mu\,(1)\,\}$
$= \mu\,(b \wedge 1)$
$= \mu\,(b)$

Therefore $\mu\,[a\vee(b \wedge c)] \neq \mu\,[(a\vee b) \wedge (a\vee c)]$

Hence, the Fuzzy lattice is not Fuzzy distributive.

II. CONCLUSION

The paper is proved that Every Fuzzy modular lattice is Fuzzy Semimodular and the coverse is not true and Every Fuzzy distributive lattice is Fuzzy Semimodular and the coverse is not true.

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