Unsteady Magneto hydrodynamic Convection Flow through Porous Medium along a Vertical Plate in Presence of Radiation

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Abstract

In the present study we investigate the effectsofthermalradiationandmagneticfieldonhydro-magneticconvectiveflowofahighlyviscousfluidwithte mperature dependent is cosity and thermal conductivity at constant pressure through a porous medium under the influence of uniform applied magnetic field. The concerning governing partial differential equations have been transformedtonon-linear coupled or dinary differential equations by virtue of thesteadynature of the flow and are solved numerically by using finite difference technique and numerical solutions are obtained for the velocity and temperature profiles within the boundary layer on magnetic field, radiation and permeability parameters. The e□ect of various material parameters on flow, heat, and mass transfer variables are discussed and illustrated graphically.

Keywords: *finite difference technique, Tridiagonal matrix, Radiation, MHD, radiation parameter.*

INTRODUCTION

Boundary layer flow and heat transfer within fluid-saturated porous media has attracted considerable importance because of its applications in geophysics, oil recovery techniques, thermal insulation engineering, packed-bed catalytic reactors, and heat storage beds. A number of similarity solutions and numerical studies of magnetohydrodynamic (MHD) flow and heat transfer in porous media have been presented. A comprehensive survey of relevant papers may be found in the recent monograph by Nield and Bejan[1]. Natural convection heat transfer is inevitable phenomenon in engineering systems to be analyzed due to its diverse applications in electronic cooling, heat exchangers and thermal systems to name a few. Studies pertaining to coupled heat and mass transfer due to free convection has got wide applications in different realms, such as, mechanical, geothermal, chemical sciences etc. Many industrial and technological physical setups such as nuclear reactors, food processing, polymer production etc.experience not onlytemperature difference but concentration difference also.

There has been a renewed interest in studying magneto hydrodynamic (MHD) flow and heat transfer in porous and non –porous media due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. In addition, this type of flow has attracted the interest of many investigators, in the view of its applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, and geothermal energy extractions. Engineers are continuously taking the task to improve the efficiency of the MHD energy systems.Raptis and Perdikis [2] studied the effects of thermal radiation and freeconvection flow past a moving vertical plate. The governing equations were solved analytically.

Soundalgekar and Ganesan [3] studied transient free convective flow past a semi-infinite vertical flat plate with mass transfer by using Crank-Nicholson finite difference method. Raptis [4] studied mathematically the case of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Chamkha [5] investigated thermal radiation and buoyancy effects on hydromagnetic flow over anaccelerating permeable surface with heat source or sink. Rahman M.M. and Sattar M.A.[6] investigated Magnetohydrodynamic convective flow of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption. Prasad et al. [7] analysed the radiation effects on an unsteady two dimensional hydromagnetic free convective boundary layer flow of a viscous incompressible fluid past a semi-infinite vertical plate with mass transfer in the presence of heat source or sink. Rajesh and Varma [8] investigated heat source effects on MHD flow past an exponentially accelerated vertical plate with variable temperature through a porous medium.Sharma and Singh [9]analysed effects of variable thermal conductivity, heat source/sink on flow of a viscous incompressible electrically conducting fluid in the presence of uniform transverse magnetic field and variable free stream near a stagnation point on a non-conducting stretching sheet. They discussed the influence of the flow parameters used and their physical implications. Das et al. [10] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. [11]. The dimensionless governing equations were solved by the usual Laplace-trans form technique and the solutions are valid only at lower time level. Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion has been studied byManivannan et al. [12]. Recently Mukesh Kumar Singh and A. K. Shukla [13] numericallyinvestigated the Soret-Dufour and Radiation Effects on Unsteady MHD Flow of Dusty Fluid over Inclined Porous Plate Embedded in Porous Medium.However, little attention has been paid to investigating the coupling influences of MHD and radiation on an unsteady convective heat and mass transfer in porous medium governed by B**oussinesq's** approximations. Hence, this is objective of the presentinvestigation.

1. MATHEMATICAL ANALYSIS

An unsteady two dimensional convective heats and mass transfer flow of a viscous, incompressible, electrically conducting optically thin fluid in a vertical plate filled with saturated porous medium in the presence of a uniform applied homogeneous magnetic field B_o is considered in the transverse direction. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. A Cartesian coordinate system (x' ,y') is assumed, where x' - axis is taken along a vertical plate and y' - axis in the normal to it. It is also assumed that radiation heat flux is in x-direction is small as compared to that of y-direction. Then, under the usual Boussinesq'sapproximation, the equations governing flow field under consideration are

$$\frac{\partial u_1}{\partial t_1} = \frac{\partial^2 u_1}{\partial {y_1}^2} + g\beta\left(T_1 - T_\infty\right) - \frac{\sigma B_0^2 u_1}{\rho} - \frac{\nu u_1}{K_1} \quad \text{............ (1)}$$

$$\rho c_p \frac{\partial T_1}{\partial t_1} = k\frac{\partial^2 T_1}{\partial {y_1}^2} - \frac{\partial q_r}{\partial y_1} \text{........ (2)}$$

The initial and boundary conditions are

$t_1 \le 0, u_1(y_1, t_1) = 0 \ , T_1(y_1, t_1) = T_\infty$ .... (3)

$t_1 > 0, u_1(0, t_1) = u_0 \ , T_1(0, t_1) = T_\infty \ \ at \ \ y_1 = 0$ .... (4)

$t_1 > 0, u_1(\infty, t_1) = 0 \ , T_1(\infty, t_1) = T_\infty \ \ at \ \ y_1 \to \infty$ ………… (5)

By using the local Rosseland approximation, the radiative heat flux is given by

$$q_r = -\frac{4\sigma^*}{3\beta_r}\frac{\partial {T_1}^4}{\partial y_1} \text{.......... (6)}$$

where σ^* is the Stephen Boltzmann constant and the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids .If the temperature differences withinhe flow are sufficiently small, then equation (6) can be linearized by expanding $T_1^{\,4}$ into the Taylor series about $T\infty$, which after neglecting higher order terms takes the form

$T_1^{\,4} = 4T_\infty^3 T - 3T_\infty^4$ …… (7)

Using (6) and (7), equation (2) can be written as

$$\rho c_p \frac{\partial T_1}{\partial t_1} = \left[\kappa + \frac{16\sigma^* T_\infty^3}{3\beta_r}\right]\frac{\partial^2 T_1}{\partial {y_1}^2} \quad .(8)$$

Under the usual Boussinesq's approximation, the flow is governed by the following equations.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K_r}\right)u + G_r\theta \quad \text{.... (9)}$$

$$3N_a P_r \frac{\partial \theta}{\partial t} = (3N_a + 4)\frac{\partial^2 \theta}{\partial y^2} \quad \text{………….. (10)}$$

The equations (9) and (10) are obtained by using the nondimensioned form of (1) and (8).In nondimensionalization following quantities are used

$$\left.\begin{array}{l} y = \dfrac{v_0 y_1}{\nu}, \quad u = \dfrac{u_1}{U_0}, \quad \theta = \dfrac{T_1 - T_\infty}{T_w - T_\infty} \\ G_r = \dfrac{\nu g\beta(T_w - T_\infty)}{u_0^3}, \ P_r = \dfrac{\rho\nu C_p}{\nu}, \ t = \dfrac{u_0^2 t_1}{\nu} \\ N_a = \dfrac{K\beta_r}{4\sigma^* T_\infty^3}, \quad k_r = \dfrac{u_0^2 k}{\nu^2}, \ M = \dfrac{\sigma B_0^2 \nu}{\rho u_0^2} \end{array}\right\} \text{… (11)}$$

The corresponding boundary and initial conditions are

$$\begin{array}{l} t \le 0, u(y, t) = 0 \ , T(y, t) = 0 \\ t > 0, u(0, t) = 0, \theta(0, t) = 1 \ at \ \ y = 0 \\ t > 0, u(\infty, t) = 0 \ , \theta(\infty, t) = 0 \ as \ \ y \to \infty \end{array}$$

…….(12)

2. METHOD OF SOLUTION

Here we sought a solution by finite difference technique of implicit type namely Crank- Nicolson implicit finite difference method which is always convergent and stable. This method has been used to solve Equations (9) and (10) subject to the conditions given by (12) by following as in [14]. To obtain the difference equations, the region of the flow is divided into a gird or mesh of lines parallel to y and t axes. Solutions of difference equations are obtained at the intersection of these mesh lines called nodes. The values of the dependent variables u and θ at the nodal points along the plane y = 0 are given by u(0,t) and θ(0,t) which from the boundary conditions. Let Δy and Δt are constant mesh sizes along y and t directions respectively. We need a scheme to find single values at next time level in terms of known values at an earlier time level. A forward difference approximation for the first order partial derivatives of u and θ w.r.t. t and y and a central difference approximation for the second order partial derivative of u and θ w.r.t. y are used. On introducing finite difference approximations the corresponding equations becomes

$$\frac{1}{\Delta t}\left(u_{i,j+1} - u_{i,j}\right) = \frac{1}{2(\Delta y)^2}$$
$$\left(u_{i+1,j} + u_{i-1,j} - 2u_{i,j} + u_{i+1,j+1} + u_{i-1,j+1} - 2u_{i,j+1}\right)$$
$$-\left(M + \frac{1}{K_r}\right)u_{i,j} + G_r\theta_{i,j}$$
……. (13)

$$3N_a P_r\left(\frac{\theta_{i,j+1} - \theta_{i,j}}{4\Delta t}\right) = (3N_a + 4)$$
$$\left(\frac{\theta_{i+1,j} + \theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j+1} + \theta_{i-1,j+1} - 2\theta_{i,j+1}}{2(\Delta t)^2}\right)$$

…………. (14)

Now multiply both sides of equations (13) and (14) by Δt and after simplification let $\frac{\Delta t}{(\Delta y)^2}$ =r'=1

Under this condition Crank- Nicolson method is always stable and convergent. The region of integration is considered as a rectangle with 0 ≤x≤1 and y max- =14, where y max corresponds to ∞ which lies very well outside both the momentum and energy boundary layers. The maximum of y was chosen as 14 so that the last two of the boundary conditions (5) are satisfied within the 10E-5 tolerance limit. After experimenting with a few set of mesh sizes, the mesh sizes have been fixed at the level Δy =0.25 with time step Δt= 0.01.To get the numerical solutions of the temperature θ and flow velocity u , necessary code is developed in in Mathematica5.0. Initially numerical solutions of Velocity and Temperature, w.r.t. time are calculated. At every time step i the finite difference approximation of equation (14) gives a linear system of equations for j=0 and i= 1,2,3,…..,n-1 we get n-1 equations in n-1unkowns of known initial and boundary values resulting in Tridiagonal matrix which is solved by using Gauss-Elimination method.

3. Results and Discussion

In order to illustrate the influence of the various parameters M, Na ,Pr, Kr and time t on the velocity profile, temperature profile graphically refer the Fig.1 to Fig. 6 for various values of parameters depicted in graphs. Gr=5 indicates strong thermal buoyancy force. The Pr=0.71 Prandtl number for air at 20^o C, 0.025 for liquid metal and 0.7 for water. Fig.1 illustrates the effect of radiation parameter Na on temperature profile for the conducting air,Pr= 0.71 All profiles decay exponentially from extreme value θ=1 to zero in the free stream.

Fig. 2 illustrates the impact of Na in presence of conducting air on velocity profile. As Na increments, impressive diminishment is observed in velocity from the peak value at the wall (y = 0) across the boundary layer regime to the free stream, at which the velocity is negligible for any value of Na. All profiles decay asymptotically to zero in the free stream.

Fig. 3 illustrates the effects of porosity, Kr, on the velocity profiles. The presence of a permeable medium increases the resistance to flow resulting in decrease in the flow velocity. This behaviour is depicted by the decline in the velocity as Kr decreases for air.

Fig. 4 illustrates the variation of transient velocity with different Prandtlnumbers Pr as well as time parameter t. As Pr increased from 0.025 (liquid metal i.e. very high thermal conductivity), through 0.71 (air), to 7.0 (sea water), there is a remarkable decrease in flow velocity i.e. the stream is decelerated through the boundary layer transverse to the plate when the plate is cooled by the free convection currents Gr > 0 . Pr encapsulates the ratio of momentum diffusivity to thermal diffusivity for a given fluid. It is also the product of dynamic viscosity and specific heat capacity divided by thermal conductivity. Higher Pr fluids will therefore possess higher viscosities (and lower thermal conductivities) inferring that such fluids will flow slower than lower Pr fluids. As a result the velocity will be decreased significantly with increasing Prandtl number.Additionally the flow velocity is diminished with time parameter.

Fig. 5 illustrates that the fluid temperature is decreased monotonically when the Prandtl number is increased. As the smaller value of Prandtl number are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Prandtl number.

Fig. 6 illustrates the transient velocity profiles in the boundary layer for various values of Gr. The thermal Grashof number Gr implies the relative effect of the thermal buoyancy (due to density differences) force to the viscous hydrodynamic force in the boundary layer flow. The positive values of Gr correspond to cooling of theplate by natural convection. Heat is therefore conducted away from the vertical plate into the fluid which increases temperature and thereby enhances the buoyancy force. It is observed that the transient velocity accelerates due to increase in the thermal buoyancy force, i.e., free convection effects. The maximum flow velocity occurs at the plate. However, as the buoyancy affects gets considerably large, a visible peak in the velocity profiles, occurs in the fluid adjacent to the wall and this peak more distinctive as Gr increase further.

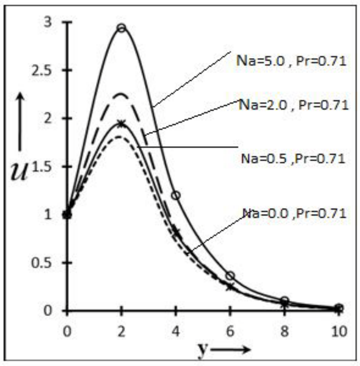

Fig 1. Velocity profiles for different values of Na when Gr=5, M=5, Kr=0.5, t=0.5

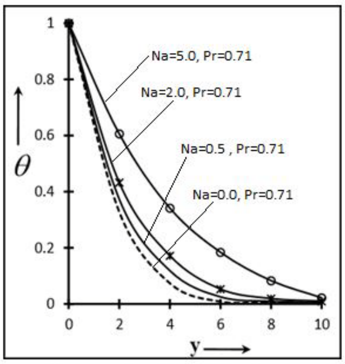

Fig 2. Temperature profiles for different values of Na

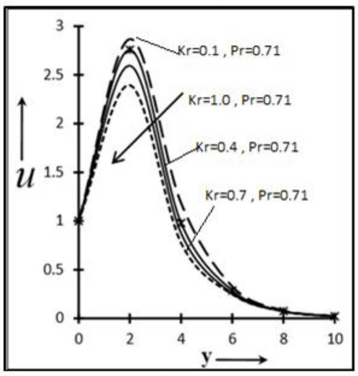

Fig 3. Velocity profiles for different values of Kr when Na=2 Gr=5, M=5, , t=0.5

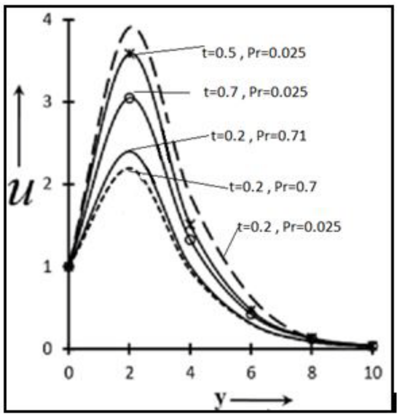

Fig 4. Velocity profiles for different values Pr and t when Gr=5, M=5, Kr=0.5, Na=5

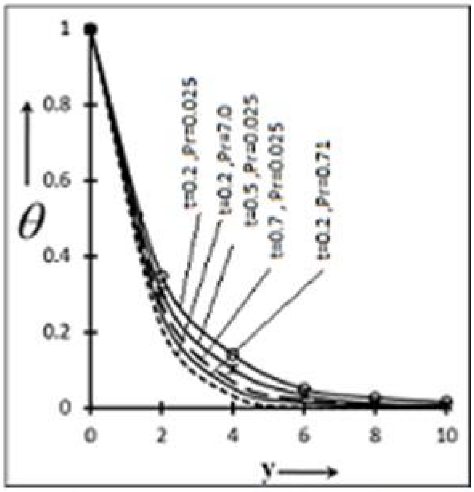

Fig 5. Temperature profiles for different values Pr and t when Na=3

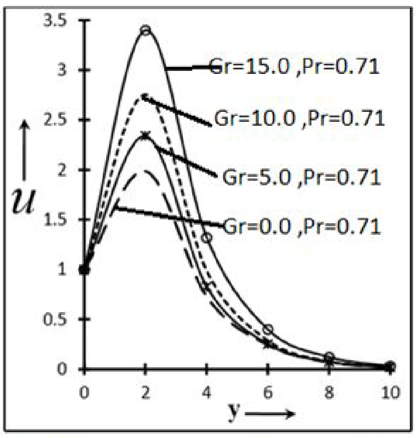

Fig 6. Velocity profiles for different values Gr when t =0.5, M=5, Kr=0.5, Na=3

4. CONCLUSIONS

A numerical analysis is performed to examine the transient free convection-radiation magnetohydrodynamic viscous flow along an impulsively moving infinite vertical plate inundated in a porous medium under a transverse magnetic field. A flux model has been utilised to simulate thermal radiation effects, valid for optically thick gases. The important conclusions of the study are as follows:

1. It is likewise observed that diminishment in velocity and temperatures are joined by synchronous decreases in both velocity and thermal boundary layers.
2. The flow is generally decelerated with the increase of porosity parameter Kr for the conducting air.
3. Velocity and temperature were diminished with an expansion in free convection-radiation Na.
4. Increasing porosity contribution Kr or magnetic field M serves to depress shear stress significantly in the regime for the cases of air.
5. With an increase in time parameter t , both the flow velocity and temperature is depressed.
6. With an increase in free convection parameter Gr, the flow velocity is accelerated due to enhancement in the thermal buoyancy force.

The present study has utilised Newtonian viscous model.

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