A study on fuzzy multi-criteria decision making in gunshot analysis

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Abstract: *It is observed that over the last few decades the rate of crime using fire arms is increasing very rapidly. The criminal activity where fire arm is used for committing murder or suicide is very critical to investigate in the absence of eye witness. When eye witness is not found the investigation is performed totally depending upon the evidences found on the crime scene. To carry out these types of investigation a model has been designed using fuzzy multi-criteria decision making (FMCDM) to reduce the uncertainty factor which comes into play most of the time. This paper highlights and discusses how to help reconstruction of crime scene using FMCDM depending upon physical properties of bloodstain evidences and wounds.*

Keywords: FMCDM, angle of impact, gunshot analysis, crime scene reconstruction

I. INTRODUCTION

To investigate a crime where murder or suicide was committed with the help of bloodstain evidences and properties of wounds is a sophisticated study. As a fluid, blood spatters of human always obey basic laws of physics and causes bloodstain evidences. In the absence of any eye witness an investigator has to determine the actions done by the offender depending upon the evidences found on the crime scene. During such type of derivation, an investigator has to cope with uncertainty because nor are the evidences, neither the derivations to be made clear enough. The investigator always has to make rational decisions even in the absence of evidences. For this reason decision making under uncertainty arises in gunshot investigations [20].

The concept of fuzzy set was introduced by Lotif A. Zadeh in 1965. In 1977 Bellman and Zadeh introduced the fuzzy decision making called Fuzzy Multicriteria Decision Making (FMCDM). Mainly there are two approaches to MCDM: - Multi-Objective Decision Making (MODM) which concentrates on continuous decision space aimed at the realization of the best solution in which several objective function are to be achieved simultaneously. The Multi-Attribute Decision Making (MADM) refers towards decision making under discrete decision spaces and focuses on how to select different alternatives from existing alternatives. Some important MADM approaches are:- analytical hierarchy process (AHP) [19], Analytical Network Process (ANP) [19], Technique for ordered preference by Similarity to Ideal Solution (TOPOSIS) [9], VIKOR[15].Some mathematical programming technique such as Linear Programming (LP), Goal Programming (GP) and Mixed Integer Programming (MIP) are typically associated with MODM approaches.

Many authors from backgrounds of forensic science, physics and mathematics studied about bloodstain pattern analysis, properties of wounds and ballistics to help crime scene reconstruction. In bloodstain pattern analysis to determine the approximate blood source location the string method, the tangent method and the virtual string method are used till now ([2], [5], [11], [12]) using computer software ([5], [16]). For determination of angle of impact ellipse fitting ([6], [18], [21]) is usually used. Till now there is no any model or decision support system for gunshot cases using FMCDM. So, in this paper an attempt has been made to construct a fuzzy MCDM approach where physical properties of bloodstain and properties of wounds are used to help crime scene reconstruction.

II. Representation of physical properties of bloodstain and wounds as fuzzy criteria:-

When blood leaves the body as a drop or spatter or gushing flow it always obey some basic laws of physics, which causes bloodstain evidences [12]. There are several techniques to collect evidences such as:-photography and collecting samples. In our study we consider the diameter of the drop lets and length to width ratio as the main physical properties of bloodstain. From these physical properties of bloodstains mainly we make an attempt to determine the following:-

1) The distance of shot taken
2) Height from where blood was fall
3) The angle of impact i.e. the acute angle formed between the direction of a blood drop and the plane of the surface it strikes.

Point of convergence (The common point (area), on a two dimensional surface, over which the directionality of several blood drops can be retraced) is determined from the angle of impacts of different

group of bloodstains. But we are not going to determine point of convergence in this paper.

The mathematical relationship between the angle of impact and of a blood droplet in a surface and the length to width ratio of the resultant bloodstain has been a long accepted principle of bloodstain pattern analysis ([16]-[18]) . Due to some basic properties of blood as a fluid the diameter of blood droplets are related to corresponding fall distances.

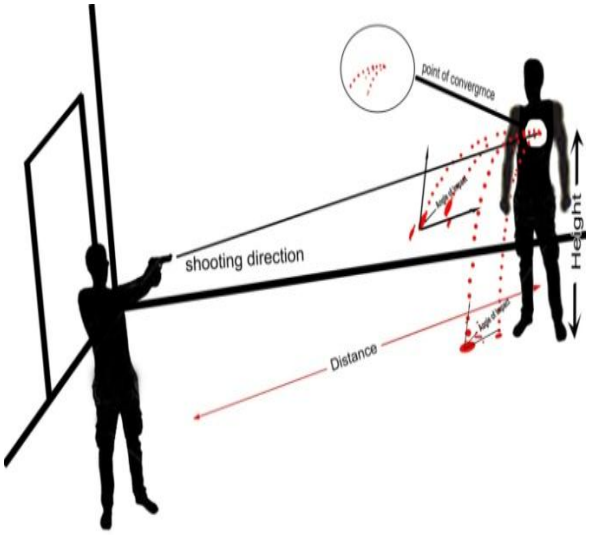

Fig.1: Diagram showing angle of impact, point of convergence and height from where blood fall

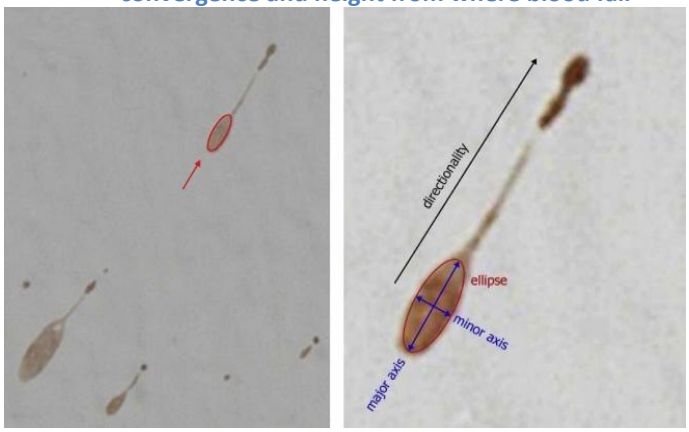

Fig 2: Directionality of bloodstains

For interpretation of bloodstain we have some standard results corresponding to standard measurements [7]:

(Diameter of a single drop let)	height
6.0-11 mm	0-0.4 feet (H_1)
11-13 mm	0.4-0.8 feet(H_2)
13-15 m	0.8-1.6 feet(H_3)
15-16 m	1.6-2.4 feet(H_4)
16-17 mm	2.4-4.0 feet(H_5)
17-18 mm	4.0-8.0 feet(H_6)
18-19 mm	8.0-17.6 feet(H_7)
19- mm	17.6- feet(H_8)

Length to width ratio	angle of impact
1.00-1.02 mm	80-90 degree(A_1)
1.02-1.09 mm	70-80 degree(A_2)
1.09-1.19 mm	60-70 degree(A_3)
1.19-1.39 mm	50-60 degree(A_4)
1.39-1.61 mm	40-50 degree(A_5)
1.61-1.95 mm	30-40 degree(A_6)
1.95-2.83 mm	20-30 degree(A_7)
2.83-6.28 mm	10-20 degree(A_8)
6.28- mm	00-10 degree(A_9)

As given, our standard physical properties are in ranges not in discrete values. Also there may be several drop lets of blood and results several measurements and hence the properties are fuzzy in nature.

Another important thing is physical properties of wounds. When a bullet strikes the skin, it first produces simply an indentation of the skin due to the fact skin is both tough and elastic. Typical wounds of entrance are neat round holes with an even gray ring around them and from which emerges comparatively small quantities of blood much greater than in wounds entrance and size of the entrance appear to be smaller than the bullet. The physical properties of wounds differ by the distance from which shot was taken [8].

Mainly there are three zones of distance from which fire arm was discharged [7]:-

1) Muzzle is placed contact with the skin 0-2 inches (D_1)
1) Muzzle was held at a distance about a distance 2-18 inches (D_2)
2) Muzzle was held at a distance about a distance more than 18 inches(D_3)

Rules to determine the distance zones [7]:-

Flames and expanding gases produced by the burning powder on the skin, wound is larger than the diameter of bullet, skin edges are ragged and torn, and there is actual charring of tissues due to the tremendous heat from muzzle blast. Then it is a contact wound (0-2 inches shot).

1) The smoke and the soot from the burned powder were deposited around the wound of entrance producing a dirty gemmy appearance, simply deposited on the surface of the skin and can be wiped with cloth. Than fire arm was released at a distance 2-18 inches.
2) Otherwise fire arm was released from distance more than 18 inches

Most of the time the physical properties of wounds are not clear enough neither the distance of shot taken to be determined is not clear enough or we say fuzzy in nature.

In our study for construction of fuzzy decision situation the physical properties of bloodstain and wounds are considered as fuzzy criterion. Then there are

different types of results to be determined. Each result has its own fuzzy forms. From the criterion we rank all the fuzzy alternatives for each type of alternatives. For example: - from the physical properties of bloodstain and wounds we have to determine the angle of impact and the alternatives are as: - 80-90 degree, 70-80 degree, 60-70 degree… which are to be ranked.

III. Construction of fuzzy decision situation

Let $C_1, C_2, \dots C_m$ be the properties of evidences found on the crime scene and $A_1, A_2, \dots, A_p$ be the type of actions done by the offender. For each action of type A_j we have its fuzzy forms as $\tilde{A}_j^1, \tilde{A}_j^2, \dots, \tilde{A}_j^n$,

Consider the physical properties of evidences as criterion $C_1, C_2, \dots C_m$. For each alternative of the each type we construct the following decision situation [14]:-

$$\begin{array}{c} \\ C_1 \\ \vdots \\ C_m \end{array} \begin{array}{c} \begin{array}{cccccccccc} \tilde{A}_p^1 & \cdots & \tilde{A}_p^{n_k} & \tilde{A}_1^1 & \cdots & \tilde{A}_1^{n_1} & \tilde{A}_2^1 & \cdots & \tilde{A}_2^{n_2} \end{array} \\ \begin{bmatrix} r_1^{11} & \cdots & r_1^{1n_1} & r_2^{11} & \cdots & r_2^{1n_2} & \cdots & r_p^{11} & \cdots & r_p^{1n_k} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ r_1^{m1} & \cdots & r_1^{mn_1} & r_1^{m1} & \cdots & r_1^{mn_2} & \cdots & r_p^{m1} & \cdots & r_p^{mn_k} \end{bmatrix} \end{array}$$

Where each element $r_j^{it} \in [0,1]$ is the degree to which the evidence C_i is related to the action of the offender $\tilde{A}_j^t$. It is not the case that all r_j^{it} exists since some criteria have nothing to do with some alternatives, or maybe could not be found from the crime scene.

IV. Ranking of alternatives:-

In this section, a variation function has been defined to evaluate ranking of alternatives. For alternatives of the type A_j we take the variance of the degrees $r_j^{1i}, r_j^{2i}, \dots r_j^{mi}$ as:-

$$Var_j\left(r_j^{1i}, r_j^{2i}, \dots r_j^{mi}\right) = \sqrt{\sum_{k=1}^{m} \frac{\left(1-r_j^{ki}\right)^2}{\mathrm{m}}} = v_j^i$$

Variation of some numerical values from mean gives us the degree to which the values vary from the mean. In this method we take the variance from 1. In our fuzzy decision matrix each $r_j^{it} \in [0,1]$ and $r_j^{it} \to 0$ gives us the lower degree of relationship of the criteria C_i to the alternative $\tilde{A}_j^t$ and $r_j^{it} \to 1$ gives us the higher degree of relationship of the criteria C_i to the alternative $\tilde{A}_j^t$. Thus higher the variance gives lower the degree of preference of the alternatives and similarly lower the variance gives higher the degree of preference of the alternatives. Since $r_j^{it} \in [0,1]$, therefore $v_j^i \in [0,1]$.

The ranking for the alternatives are introduced as $R = \sum_i (1 - v_j^i)\,/\tilde{A}_j^i$

Since, high variance gives low degree of preference of the alternatives and similarly low variance gives high degree of preference of the alternatives, for the final ranking of alternatives we subtract the variances from 1 which will give us the higher ranking for the preferable alternatives and lower ranking for not preferable alternatives.

Later we used trapezoidal fuzzy number for the degree of membership to the alternatives so we will use fuzzy trapezoidal number and introduce the linguistic variables Very low probable (VLP), Low Probable (LP), Medium probable (MP), highly probable (HP) and Very highly probable (VHP) for the rating of alternatives, as shown in the following table:-

Table 1: Linguistic variables for the consequence ratings

Linguistic variable	Trapezoidal fuzzy number
Very low possibility (VLP)	(0,0,0.2,0.4)
Low possibility (LP)	(0,0.2,0.4,0.6)
Medium possibility (MP)	(0.2,0.4,0.6,0.8)
Highly possibility (HP)	(0.4,0.6,0.8,1)
Very highly possibility (VHP)	(0.6,0.8,1,1)

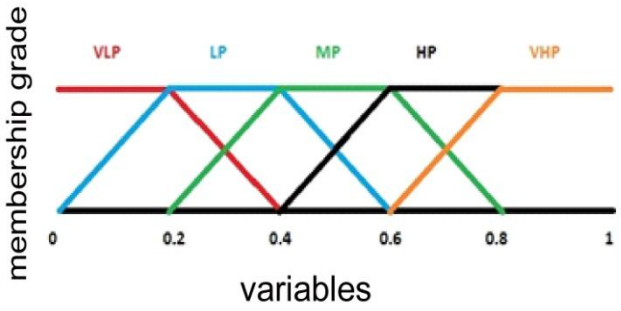

Fig 3: Linguistic variables for corresponding rating

V. Determination of degrees to which the bloodstain evidences and properties wounds are satisfied by the alternatives:-

As seen earlier the standard properties of bloodstain evidences and their corresponding results both are in certain ranges not in discrete values so trapezoidal fuzzy numbers will be more appropriate than other fuzzy numbers for determination of degrees to which the evidences are related to the alternative

As evidences vary in different crime scenes, investigators point of view plays an important role in any investigation. In our hypothetical case study we consider the investigators preference as one of the common criteria for ranking of all alternatives.

Since the diameter of wound increases as the distance of shot taken decreases, we take the ratio of bullet diameter to wound diameter for determination of degree to which a distance zone is related.

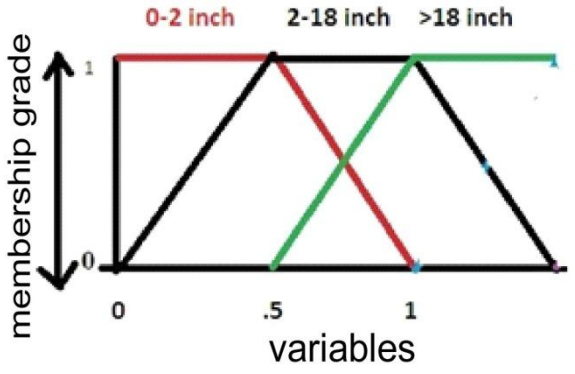

Fig 4: Degree to which ratio of bullet diameter to wound diameter corresponds to the distance from where shot was taken

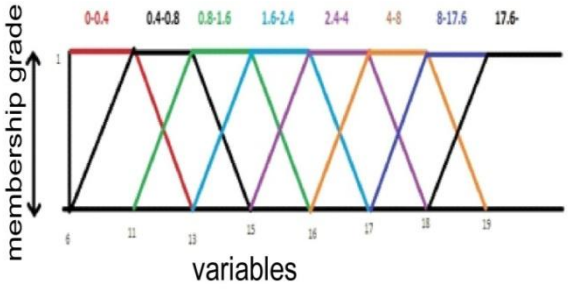

Fig 5: Degree to which the diameter of a single blood droplet corresponds to height from where blood fall

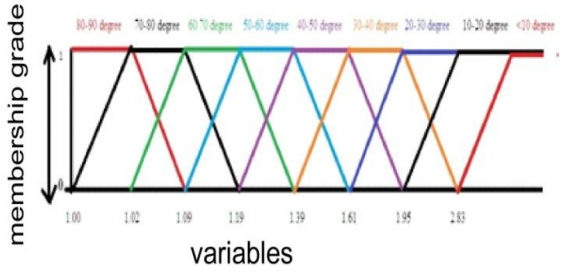

Fig 6: Degree to which length to width ratio of blood spatters corresponds to angle of impact

VI. Hypothetical case study:-

Let us assume that a man of height 6 feet shot at his chest. The diameter $(DMR)$of blood droplets from single drops measures average of 17.6mm. The length to width ratio (LWR) of blood spatter around the wound is 1.42, at the floor is 1.2 and at the vertical wall is 1.68. Ratio to wound diameter and bullet diameter (WBR) is .6. A little gun powder (GPD) was deposited at the skin and the skin was burned (BRN) a little. The stroke of bullet (SOB) was found on the wall after penetrating victims body at a height 5.2 feet from the floor. According to an eye witness (EWT) victim was seen standing when shot and other information was found from the crime scene as well as eye witness.

It is to be noted that particular evidence used to evaluate only a particular type of alternatives i.e. each criteria is not used to determine each alternatives.

Now, we construct the following fuzzy decision situation:-

A. For height of blood fall:-

$$\begin{array}{c} \\ DMR \\ SOB \\ INP \\ EWT \end{array}\begin{array}{c} \begin{array}{cccccccc} H_1 & H_2 & H_3 & H_4 & H_5 & H_6 & H_7 & H_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & .6 & 1 & .4 & 0 \\ 0 & 0 & 0 & .5 & .8 & 1 & .4 & 0 \\ 0 & 0 & 0 & 0 & .3 & .8 & .4 & 0 \\ 0 & 0 & 0 & 0 & .8 & .9 & .5 & 0 \end{bmatrix} \end{array}$$

Now:-

$v_H^1 = 1, v_H^2 = 1, v_H^3 = 1, v_H^4 = .5, v_H^5 = .42, v_H^6 = .11, v_H^7 = .57, v_H^8 = 1,$

Ranking for heights of blood fallen:-

$$R_H = \sum_i \left(1 - v_H^i\right) / H_i$$
$$= 0/H_1 + 0/H_2 + 0/H_3 + .5/H_4 + .58/H_5 + .89/H_6 + .43/H_7 + 0 / H_8$$

So we can conclude that there is a very high possibility that blood fall from a height 4.0-8.0 feet(H_6), medium possibility that the blood fall from height 2.40-4.0 feet (H_5) and 8.0-17.6 (H_7) .Which suggest most probably the victim was in standing position.

B. Angle of impact for the group of bloods around the wound:-

$$\begin{array}{c} \\ LWR \\ INP \\ EWT \end{array}\begin{array}{c} \begin{array}{ccccccccc} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & .84 & 1 & .16 & 0 & 0 & 0 \\ 0 & 0 & .3 & .7 & .8 & .4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now:-

$v_A^1 = 1, v_A^2 = 1, v_A^3 = .7, v_A^4 = .2, v_A^5 = .11, v_A^6 = .60, v_A^7 = 1, v_A^8 = 1, v_A^9 = 1$

Ranking for angle of impact for the group of bloods around the wound:-

$$R_A = \sum_i \left(1 - v_A^i\right) / A_i$$
$$= 0/A_1 + 0/A_2 + .3/A_3 + .8/A_4 + .89/A_5 + .40 / A_6 + 0/A_7 + 0/A_8 + 0/A_9$$

So we can conclude that there is a very high possibility that the blood flow strike around the wound at an angle of 40-50 degree, low possibility that the blood flow strike around the wound at an angle 30-40 degree, high possibility that the blood flow strike around the wound at an angle 50-60 degree and low possibility that the blood flow strike around the wound at an angle 60-70 degree.

C. **Angle of impact for the group of bloods found on the floor:-**

$$\begin{array}{c} \\ LWR \\ INP \\ EWT \end{array}\begin{array}{c} \begin{array}{ccccccccc} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 \end{array} \\ \begin{bmatrix} 0 & 0 & .05 & 1 & .95 & 0 & 0 & 0 & 0 \\ 0 & 0 & .3 & .7 & .4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now:-

$v_A^1 = 1, v_A^2 = 1, v_A^3 = .67, v_A^4 = .1, v_A^5 = .35, v_A^6 = 1, v_A^7 = 1, v_A^8 = 1, v_A^9 = 1$

Ranking for angle of impact for the group of bloods found on the floor:-

$R_A = \sum_i \left(1 - v_A^i\right) / A_i$

$= 0/A_1 + 0/A_2 + .33/A_3 + .90/A_4 + .65/A_5 + 0 / A_6 + 0/A_7 + 0/A_8 + 0/A_9$

Similarly there is a very high possibility that the blood flow strike the floor at an angle of 50-60 degree(A_4), low possibility that the blood flow strike around the wound at an angle 40-50 degree and medium possibility that the blood flow strike around the wound at an angle 60-70 degree

D. Angle of impact for the group of bloods found on the wall:-

$$\begin{array}{c} \\ LWR \\ INP \\ EWT \end{array}\begin{array}{c} \begin{array}{ccccccccc} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & .75 & 1 & .25 & 0 & 0 \\ 0 & 0 & 0 & .3 & .4 & .8 & .2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now:-

$v_A^1 = 1, v_A^2 = 1, v_A^3 = 1, v_A^4 = .49, v_A^5 = .37, v_A^6 = .11, v_A^7 = .25, v_A^8 = 1, v_A^9 = 1$

Ranking for angle of impact for the group of bloods found on the wall:-

$R_A = \sum_i \left(1 - v_A^i\right) / A_i$

$= 0/A_1 + 0/A_2 + 0/A_3 + .51/A_4 + .63/A_5 + .89 / A_6 + .75/A_7 + 0/A_8 + 0/A_9$

Thus, there is a very high possibility that the blood flow strike the wall at an angle of 30-40 degree, low possibility that the blood flow strike the wall at an angle 20-30 degree and medium possibility that the blood flow strike around the wall at an angle 40-50 degree.

E. For distance of shot taken

$$\begin{array}{c} \\ GPD \\ BRN \\ WBR \\ INP \\ EWT \end{array}\begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} .7 & .9 & .1 \\ .4 & .7 & .1 \\ .2 & 1 & .8 \\ 0 & .7 & .6 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$v_D^1 = .21, v_D^2 = .16, v_D^3 = .6$

$R_A = \sum_i \left(1 - v_D^i\right) / D_i$

$= .79/A_1 + .84/A_2 + .40/A_3$

So we can conclude that there is a very high possibility that shot was shot was taken at a distance from 2-18 inches and high possibility that shot was taken from more than 18 inches and medium possibility that shot was taken at a distance from 0-2 inches.

VII. Conclusion and future work

As the crime increased in last few years, before one case is solved another cases gathered at investigators desk and results huge work load on the investigator and resulting in slowdowns the investigation process. That is why it is necessary to have efficient mathematical models or decision support system to help investigators.

In our study we applied a fuzzy MCDM approach to make decisions from a criminal investigators perspective in gunshot cases where someone was shot. In this paper, we propose to use trapezoidal fuzzy number for determination of degrees to which our standard physical properties of wounds and bloodstains are related to the particular derivations already mentioned, since the standard properties of bloodstain evidences and their corresponding results both are in certain ranges not in discrete values. Also, using variance of degrees a ranking method is introduced for the ranking of alternatives. To check the validity of the propped approach a hypothetical case study has been carried out.

However, one limitation of this current study is that the origin of bloodshed or point of convergence using the angle of impacts from different group of blood spatters cannot be determined as the methodology here proposed will not be applicable for determination of origin of bloodshed or point of convergence. So, future aim could be attempt to determination of point of convergence of bloodshed.

REFERENCES

[1] Bellman, R. Zadeh, L.A. (1970). Decision-making in a fuzzy environment. *Management Science*.

[2] Bevel, T., Gardner, R.M. (2002). *Bloodstain Pattern Analysis*. CRC, Boca Raton, FL, 159–192.

[3] Camana, F., "Determining the area of convergence in Bloodstain Pattern Analysis: a probabilistic approach," Interregional forensic sciene office (G.I.P.S.), Piazzetta Palatucci, 5 – 35123, Padova (ITALY), 2013.

[4] Carter, A.L.(2001). The directional analysis of bloodstain patterns theory and experimental validation. *Journal of the Canadian Society of Forensic Science*. 34. 173–189.

[5] Carter, A.L., Illes, M., Maloney, K.(2005). Further validation of the BackTrackTM computer programe for bloodstain pattern analysis—precision and accuracy. *IABPA Newslett*.15–22,. http://www.iabpa.org/newslett.htm.

[6] Chafe, F.C.(2003). Determination of Impact angle using mathematical properties of ellipse. *International association of Bloodstain Pattern analysts News. 19(1)*. 5-9.

[7] Chenuyan K., Chi L. and Fredrick S.(1993). An expert system on Crime Investigation Based on Mark- Theory to Reconstruct Crime Scene. *Pan Pacific Conference on Information System*.

[8] DiMaio, J. M.V.(1999). *Practical Aspects of Firearms, Ballistics and Forensic Techniques.* 2nd edition, CRC Press, Boca Raton, New York,.

[9] Hwang, C.L. Yoon, K. (1981). *Multiple Attribute Decision Making: Methods and Application.* Springer, New York.

[10] International Association of Bloodstain Pattern Analysts (IABPA) (**http://iabpa.org/**)

[11] James, H.S. and Eckert, G.W. (1999). *Interpretation of Bloodstain Evidence at Crime Scenes*. New York, 2nd edition, CRC Press, Boca Raton.

[12] James, S.H. (1999). *Scientific and Legal Applications of Bloodstain Pattern Interpretation.* CRC, Boca Raton, FL, New York.

[13] James, S.H. Kish, P.E. & Sutton, T.P. (2005). *Principles of Bloodstain Pattern Analysis: Theory and Practice*. CRC Taylor & Francis, Boca Raton, FL, 226–234.

[14] Klir, J.G. & Yuan, B. (2009). *Fuzzy Sets and Fuzzy Logic*. Pearson Education, 399-401.

[15] Opricovic, S., and Tzeng, G.H. (2004). "Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS", *European Journal of Operational Research, 156(2).*445–455.

[16] Pace, A. (2005) . The Relationship between Errors in Ellipse Fitting and the Increasing Degree of Error in Angle of Impact Calculations," *International Association of Bloodstain Pattern Analysts News, 21(3),*12-14.

[17] Pace, A., Carter, A.L., Moor,e C. & Yamashita, B.(2006). *Another treatment of three-dimensional bloodstain pattern analysis*," IABPA Newslett. 4–11,. http://www.iabpa.org/newslett.htm.

[18] Reynolds, M., D. Franklin, Raymond, M. A. & Dadour, I. (2008). Bloodstain measurement using computer-fitted theoretical ellipses: *A study in accuracy and precision." Journal of Forensic Identification, 58(4),* 469-484.

[19] Saaty, T.L. (1980). The Analytic Hierarchy Process, *McGraw-Hill, New York*.

[20] Taroni, F., Bozza, S. & Aitken, (2005). Decision analysis in forensic science. *Journal of Forensic Sciences. 50(4),* 894–905.

[21] Willis,C., Piranian, A. K., Donaggio, J. R., Barnett, R. J. & Rowe, W. F. (2001). Errors in the Estimation of the Distance of fall and Angles of Impact Blood Drops. *Forensic Science International*. *123(1),* 1-4.

[22] Zadeh, L.A.(1965) . *Fuzzy sets, Information and Control*. 8(3), 338–353.

[23] Zimmermann, H.J. (1985) "*Fuzzy Set Theory and Its Applications*. Dordrecht, Boston.