The integration of certains products of special functions and multivariable Aleph-functions

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ABSTRACT
The main of the paper is to obtain a finite integral involving a product of Fujiwara's polynomial [1], M-series [4], a class of polynomials and two Aleph-functions of several variables. The results are quite general in nature and hence encompass several cases of interest.

KEYWORDS : Aleph-function of several variables, M-series, finite integral, Fujiwara's polynomial, general class of polynomials.

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1.Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [2] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c|}\mathrm{y}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_r\end{array}\right.$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\cdots\cdots\cdots\cdots\ ,[\tau_i(b_{ji};\beta_j^{(1)},\cdots,\beta_j^{(r)})_{m+1,q_i}]:$

$\left.\begin{array}{c}[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)y_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$a_j, j = 1, \cdots, p; b_j, j = 1, \cdots, q;$

$c_j^{(k)}, j = 1, \cdots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R, i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1 \cdots, r, i = 1, \cdots, R, i^{(k)} = 1, \cdots, R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\alpha_1} \ldots |y_r|^{\alpha_r}), max(|y_1| \ldots |y_r|) \to 0$

$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\beta_1} \ldots |y_r|^{\beta_r}), min(|y_1| \ldots |y_r|) \to \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \cdots, m_k$ and

$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$

Serie representation of Aleph-function of several variables is given by

$$\aleph(y_1,\cdots,y_r)=\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\psi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})$$

$$\times\ \theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r})y_1^{-\eta_{G_1,g_1}}\cdots y_r^{-\eta_{G_r,g_r}} \tag{1.6}$$

Where $\psi(.,\cdots,.)$, $\theta_i(.)$,$i=1,\cdots,r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1}=\frac{d_{g_1}^{(1)}+G_1}{\delta_{g_1}^{(1)}},\cdots,\ \eta_{G_r,g_r}=\frac{d_{g_r}^{(r)}+G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[d_j^i+p_i]\neq\delta_j^{(i)}[d_{g_i}^i+G_i]$ (1.7)

for $j\neq m_i, m_i=1,\cdots\eta_{G_i,g_i};p_i,n_i=0,1,2,\cdots,;y_i\neq 0,i=1,\cdots,r$ (1.8)

Consider the Aleph-function of s variables

$$\aleph(z_1,\cdots,z_s)=\ \aleph^{0,N:M_1,N_1,\cdots,M_s,N_s}_{P_i,Q_i,\iota_i;r:P_{i^{(1)}},Q_{i^{(1)}},\iota_{i^{(1)}};r^{(1)};\cdots;P_{i^{(s)}},Q_{i^{(s)}};\iota_{i^{(s)}};r^{(s)}}\left(\begin{array}{c|}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_s\end{array}\right.$$

$$\begin{array}{ll}[(\mathrm{u}_j;\mu_j^{(1)},\cdots,\mu_j^{(r)})_{1,N}] & ,[\iota_i(u_{ji};\mu_j^{(1)},\cdots,\mu_j^{(r)})_{N+1,P_i}]:\\ \cdots\cdots\cdots\cdots\cdots\cdots & ,[\iota_i(v_{ji};v_j^{(1)},\cdots,v_j^{(r)})_{M+1,Q_i}]:\end{array}$$

$$\left.\begin{array}{l}[(\mathrm{a}_j^{(1)});\alpha_j^{(1)})_{1,N_1}],[\iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)};\alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}];\cdots;[(\mathrm{a}_j^{(s)});\alpha_j^{(s)})_{1,N_s}],[\iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)};\alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_i^{(s)}}]\\ [(\mathrm{b}_j^{(1)});\beta_j^{(1)})_{1,M_1}],[\iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)};\beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}];\cdots;[(\mathrm{b}_j^{(s)});\beta_j^{(s)})_{1,M_s}],[\iota_{i^{(s)}}(b_{ji^{(s)}}^{(s)};\beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_i^{(s)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_r}\zeta(t_1,\cdots,t_s)\prod_{k=1}^{s}\phi_k(t_k)z_k^{t_k}\,\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.9}$$

with $\omega=\sqrt{-1}$

$$\zeta(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{N}\Gamma(1-u_j+\sum_{k=1}^{s}\mu_j^{(k)}t_k)}{\sum_{i=1}^{r'}[\iota_i\prod_{j=N+1}^{P_i}\Gamma(u_{ji}-\sum_{k=1}^{s}\mu_{ji}^{(k)}t_k)\prod_{j=1}^{Q_i}\Gamma(1-v_{ji}+\sum_{k=1}^{s}v_{ji}^{(k)}t_k)]} \tag{1.10}$$

and $$\phi_k(t_k)=\frac{\prod_{j=1}^{M_k}\Gamma(b_j^{(k)}-\beta_j^{(k)}t_k)\prod_{j=1}^{N_k}\Gamma(1-a_j^{(k)}+\alpha_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{r^{(k)}}[\iota_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-b_{ji^{(k)}}^{(k)}+\beta_{ji^{(k)}}^{(k)}t_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(a_{ji^{(k)}}^{(k)}-\alpha_{ji^{(k)}}^{(k)}s_k)]} \tag{1.11}$$

Suppose , as usual , that the parameters

$u_j,j=1,\cdots,P;v_j,j=1,\cdots,Q;$

$a_j^{(k)}, j = 1, \cdots, N_k; a_{ji^{(k)}}^{(k)}, j = n_k+1, \cdots, P_{i^{(k)}};$

$b_{ji^{(k)}}^{(k)}, j = m_k+1, \cdots, Q_{i^{(k)}}; b_j^{(k)}, j = 1, \cdots, M_k;$

with $k = 1 \cdots, s, i = 1, \cdots, r', i^{(k)} = 1, \cdots, r^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{N} \mu_j^{(k)} + \iota_i \sum_{j=N+1}^{P_i} \mu_{ji}^{(k)} + \sum_{j=1}^{N_k} \alpha_j^{(k)} + \iota_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)} - \iota_i \sum_{j=1}^{Q_i} \upsilon_{ji}^{(k)} - \sum_{j=1}^{M_k} \beta_j^{(k)}$$

$$-\iota_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.12}$$

The reals numbers τ_i are positives for $i = 1, \cdots, r$, $\iota_{i^{(k)}}$ are positives for $i^{(k)} = 1 \cdots r^{(k)}$

The contour L_k is in the t_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j^{(k)} - \beta_j^{(k)} t_k)$ with $j = 1$ to M_k are separated from those of $\Gamma(1 - u_j + \sum_{i=1}^{s} \mu_j^{(k)} t_k)$with $j = 1$ to N and $\Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} t_k)$ with $j = 1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

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$$+ \sum_{j=1}^{M_k} \beta_j^{(k)} - \iota_{i^{(k)}} \sum_{j=M_k+1}^{q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1 \cdots, s, i = 1, \cdots, r, i^{(k)} = 1, \cdots, r^{(k)} \tag{1.13}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_s) = 0(|z_1|^{\alpha'_1} \ldots |z_s|^{\alpha'_s}), max(|z_1| \ldots |z_s|) \rightarrow 0$

$\aleph(z_1, \cdots, z_s) = 0(|z_1|^{\beta'_1} \ldots |z_s|^{\beta'_s}), min(|z_1| \ldots |z_s|) \rightarrow \infty$

where, with $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)} / \beta_j^{(k)})], j = 1, \cdots, M_k$ and

$\beta'_k = max[Re((a_j^{(k)} - 1) / \alpha_j^{(k)})], j = 1, \cdots, N_k$

We will use these following notations in this paper

$$U = P_i, Q_i, \iota_i; r'; V = M_1, N_1; \cdots; M_s, N_s \tag{1.15}$$

$$\mathrm{W} = P_{i^{(1)}}, Q_{i^{(1)}}, \iota_{i(1)}; r^{(1)}, \cdots, P_{i^{(r)}}, Q_{i^{(r)}}, \iota_{i(s)}; r^{(s)} \tag{1.16}$$

$$A = \{(u_j; \mu_j^{(1)}, \cdots, \mu_j^{(s)})_{1,N}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(s)})_{N+1,P_i}\} \tag{1.17}$$

$$B = \{\iota_i(v_{ji}; v_{ji}^{(1)}, \cdots, v_{ji}^{(s)})_{M+1,Q_i}\} \tag{1.18}$$

$$C = (a_j^{(1)}; \alpha_j^{(1)})_{1,N_1}, \iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_{i^{(1)}}}, \cdots, (a_j^{(s)}; \alpha_j^{(s)})_{1,N_s}, \iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_{i^{(s)}}} \tag{1.19}$$

$$D = (b_j^{(1)}; \beta_j^{(1)})_{1,M_1}, \iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_{i^{(1)}}}, \cdots, (b_j^{(s)}; \beta_j^{(s)})_{1,M_s}, \iota_{i^{(s)}}(\beta_{ji^{(s)}}^{(s)}; \beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_{i^{(s)}}} \tag{1.20}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots, z_r) = \aleph_{U:W}^{0,\mathrm{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A} : \mathrm{C} \\ \cdot & \cdot\ \cdot\ \cdot \\ \cdot & \\ \mathrm{z}_s & \mathrm{B} : \mathrm{D} \end{array}\right) \tag{1.21}$$

The generalized polynomials defined by Srivastava [6], is given in the following manner :

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[y_1, \cdots, y_u] = \sum_{K_1=0}^{[N_1'/M_1']} \cdots \sum_{K_u=0}^{[N_u'/M_u']} \frac{(-N_1')_{M_1'K_1}}{K_1!} \cdots \frac{(-N_u')_{M_u'K_u}}{K_u!}$$

$$A[N_1', K_1; \cdots; N_u', K_u] y_1^{K_1} \cdots y_u^{K_u} \tag{1.22}$$

Where $M_1', \cdots, M_u'$ are arbitrary positive integers and the coefficients $A[N_1', K_1; \cdots; N_u', K_u]$ are arbitrary constants, real or complex.

The M-serie is defined, see Sharma [4].

$${}_{p'}M_{q'}^{\alpha}(y) = \sum_{s'=0}^{\infty} \frac{[(a_{p'})]_{s'}}{[(b_{q'})]_{s'}} \frac{y^{s'}}{\Gamma(\alpha s' + 1)} \tag{1.23}$$

Here $\alpha \in \mathbb{C}, Re(\alpha) > 0$. $[(a_{p'})]_{s'} = (a_1)_{s'} \cdots (a_{p'})_{s'}$; $[(b_{q'})]_{s'} = (b_1)_{s'} \cdots (b_{q'})_{s'}$.
The serie (1.23) converge if $p' \leqslant q' and\ |y| < 1$.

$F_n(\rho, \omega; t)$ is Fujiwara's polynomial [1], we have

$$F_n(\rho, \omega; t) = \frac{(-k)^n (b-t)^n (1+\beta)_n}{n!} F\left[-n, n-\alpha; 1+\beta; \frac{a-t}{b-t}\right] \tag{1.24}$$

2. Main results

In the document , we note :

$$G(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}) = \phi(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r})\theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r})$$

$$A_1=\frac{(-N_1)_{M_1'K_1}}{K_1!}\cdots\frac{(-N_u)_{M_u'K_u}}{K_u!}A[N_1',K_1;\cdots;N_u',K_u]$$

$$B_1=\frac{(-)^n\lambda^n(b-a)^{\rho+\sigma+1}\Gamma(\rho+\sigma+1)}{n!}$$ and $U_{22}=P_i+2,Q_i+2,\iota_i;r'$

Formula 1

$$\int_a^b(t-a)^{\rho}(b-t)^{\sigma}\,F_n(\rho,\omega;t)\aleph(z_1(b-t)^{h_1},\cdots,z_s(b-t)^{h_s})\mathrm{d}t$$

$$=\aleph_{U_{22}:W}^{0,N+2:V}\left(\begin{array}{c|c} \mathrm{z}_1(b-a)^{h_1} & (\text{-}\sigma;h_1,\cdots,k_s),\qquad(\omega-\sigma;h_1,\cdots,h_s),A:C\\ \cdot & \cdots\qquad\qquad\cdots\\ \cdot & \cdots\qquad\qquad\cdots\\ \mathrm{z}_s(b-a)^{h_s} & (\text{-1-n-}\sigma-\rho;h_1,\cdots,h_s),(\omega+n-\sigma;h_1,\cdots,h_s),B:D\end{array}\right)$$

(2.1)

Provided :

$$Re(\rho)>-1,Re[\sigma+\sum_{i=1}^{s}h_i\min_{1\leqslant j\leqslant M_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>-1,h_i>0,i=1,\cdots,s$$

Proof of (2.1)

To establish the finite integral (2.1), express the generalized Fujiwara's polynomials occuring on the L.H.S in the series by (1.24) and the multivariable Aleph-function involving there in terms of Mellin-Barnes contour integral by (1.9). We interchange the order of summation and integration (which is permissible under the conditions stated). Now evaluating the t-integral, after simplifications and on reinterpreting the Mellin-Barnes contour integral, we get the desired result.

Formula 2

$$\int_a^b(t-a)^{\rho}(b-t)^{\sigma}F_n(\rho,\omega;t)S_{N_1',\cdots,N_u'}^{M_1',\cdots,M_u'}[x_1(b-t)^{k_1},\cdots,x_u(b-t)^{k_u}]_{p'}M_{q'}^{\alpha}(y(b-t)^{k'})$$

$$\aleph(z_1(b-t)^{h_1'},\cdots,z_r(b-t)^{h_r'})\,\aleph(y_1(b-t)^{h_1},\cdots,y_s(b-t)^{h_s})\mathrm{d}t$$

$$=B_1\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_u=0}^{[N_u'/M_u']}\sum_{l=0}^{\infty}A_1\,G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})\frac{[(a_{p'})]_l}{[(b_{q'})]_l}\frac{y^l}{\Gamma(\alpha l+1)}$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}x_1^{K_1}\cdots x_u^{K_u}\,z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}(b-a)^{h_1'\eta_{G_1,g_1}+\cdots+h_r'\eta_{G_r,g_r}+K_1k_1+\cdots+K_uk_u+k'l}$$

$$\aleph_{U_{22}:W}^{0,N+2:V}\left(\begin{array}{c|c} \mathrm{y}_1(b-a)^{h_1} & (\omega-\sigma-\sum_{i=1}^{r}h_i'\eta_{G_i,g_i}-\sum_{i=1}^{u}K_ik_i-k'l;h_1,\cdots,h_s),\\ \cdot & \cdots\\ \cdot & \\ \mathrm{y}_s(b-a)^{h_s} & (\omega+n-\sigma-\sum_{i=1}^{r}h_i'\eta_{G_i,g_i}-\sum_{i=1}^{u}K_ik_i-k'l;h_1,\cdots,h_s),\end{array}\right.$$

$$\left.\begin{array}{c} (\text{-}\sigma-\sum_{i=1}^{r}h_i'\eta_{G_i,g_i}-\sum_{i=1}^{u}K_ik_i-k'l;h_1,\cdots,h_s),A:C\\ \cdots\\ (\text{-n-}\rho-\sigma-1-\sum_{i=1}^{r}h_i'\eta_{G_i,g_i}-\sum_{i=1}^{u}K_ik_i-k'l;h_1,\cdots,h_s),B:D\end{array}\right) \qquad (2.2)$$

Provided :

a) $h_i > 0, i = 1,\cdots,s; h'_i > 0, i = 1,\cdots,r; Re(\rho) > -1; p' \leqslant q' and\, |y| < 1$

b) $Re[\sigma + \sum_{i=1}^{r} h'_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^{s} h_i \min_{1\leqslant j\leqslant M_i} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > -1$

c) $|arg y_k| < \frac{1}{2} B_i^{(k)} \pi$, where $B_i^{(k)}$ is given in (1.13)

Proof of (2.2)

To derive (2.2), we express the general class of polynomials, M-series, the Aleph-function of r variables in series form with the help of (1.22), (1.23) and (1.6) and then changing the order of integration and summation which is valid with the conditions stated and evaluating the remaining integral with the help of the formula (2.1), we arrive at the desired result.

3. Particular cases

a) If $\iota_i = \iota_{i^{(1)}} = \cdots = \iota_{i^{(s)}} = 1$ and $r = r^{(1)} = \cdots = r^{(s)} = 1$, then the multivariable Aleph-function degenere to the multivariable H-function defined by Srivastava et al [7]. And we have the following result.

$$\int_a^b (t-a)^{\rho}(b-t)^{\sigma} F_n(\rho,\omega;t) S_{N'_1,\cdots,N'_u}^{M'_1,\cdots,M'_u}[x_1(b-t)^{k_1},\cdots,x_u(b-t)^{k_u}]_{p'}M_{q'}^{\alpha}(y(b-t)^{k'})$$

$$\aleph(z_1(b-t)^{h'_1},\cdots,z_r(b-t)^{h'_r})\, H(y_1(b-t)^{h_1},\cdots,y_s(b-t)^{h_s})\mathrm{d}t$$

$$= B_1 \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N'_1/M'_1]} \cdots \sum_{K_u=0}^{[N'_u/M'_u]} \sum_{l=0}^{\infty} A_1\, G(\eta_{G_1,g_1},\cdots \eta_{G_r,g_r}) \frac{[(a_{p'})]_l}{[(b_{q'})]_l} \frac{y^l}{\Gamma(\alpha l+1)}$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} x_1^{K_1} \cdots x_u^{K_u}\, z_1^{\eta_{G_1,g_1}} \cdots z_r^{\eta_{G_r,g_r}} (b-a)^{h'_1\eta_{G_1,g_1}+\cdots+h'_r\eta_{G_r,g_r}+K_1k_1+\cdots+K_uk_u+k'l}$$

$$H_{P+2:Q+2:W}^{0,N+2:V}\left(\begin{array}{c} \mathrm{y}_1(b-a)^{h_1} \\ \cdot \\ \cdot \\ \mathrm{y}_s(b-a)^{h_s} \end{array} \middle| \begin{array}{l} (\omega-\sigma-\sum_{i=1}^{r} h'_i\eta_{G_i,g_i} - \sum_{i=1}^{u} K_ik_i - k'l; h_1,\cdots,h_s), \\ \cdots \\ (\omega+n-\sigma-\sum_{i=1}^{r} h'_i\eta_{G_i,g_i} - \sum_{i=1}^{u} K_ik_i - k'l; h_1,\cdots,h_s), \end{array}\right.$$

$$\left.\begin{array}{l} (\text{-}\sigma-\sum_{i=1}^{r} h'_i\eta_{G_i,g_i} - \sum_{i=1}^{u} K_ik_i - k'l; h_1,\cdots,h_s), A' : C' \\ \cdots \\ (\text{-n-}\rho-\sigma-1-\sum_{i=1}^{r} h'_i\eta_{G_i,g_i} - \sum_{i=1}^{u} K_ik_i - k'l; h_1,\cdots,h_s), B' : D' \end{array}\right) \tag{3.1}$$

Provided :

a) $h_i > 0, i = 1,\cdots,s; h'_i > 0, i = 1,\cdots,r; Re(\rho) > -1; p' \leqslant q' and\, |y| < 1$

b) $Re[\sigma + \sum_{i=1}^{r} h'_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^{s} h_i \min_{1\leqslant j\leqslant M_i} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > -1$

c) $|arg y_k| < \frac{1}{2} B_i \pi$, $k = 1,\cdots,s$

where $B_i = \sum_{j=1}^{N} \mu_j^{(i)} - \sum_{j=N+1}^{P} \mu_j^{(i)} - \sum_{j=1}^{Q} \upsilon_j^{(i)} + \sum_{j=1}^{N_i} \alpha_j^{(i)} - \sum_{j=N_i+1}^{P_i} \alpha_j^{(i)} + \sum_{j=1}^{M_i} \beta_j^{(i)} - \sum_{j=M_i+1}^{Q_i} \beta_j^{(i)} > 0$

b) If $p_i = q_i = n = 0$ and $P_i = Q_i = N = 0$ then the Aleph-function of r variables degenere to product of r Aleph-functions of one variable and the Aleph-function of s variables degenere to product of s Aleph-functions of one variable.

$$\int_a^b (t-a)^{\rho}(b-t)^{\sigma} F_n(\rho,\omega;t) S_{N'_1,\cdots,N'_u}^{M'_1,\cdots,M'_u}[x_1(b-t)^{k_1},\cdots,x_u(b-t)^{k_u}]_{p'} M_{q'}^{\alpha}(y(b-t)^{k'})$$

$$\prod_{u=1}^{r} \aleph_{p_{i(u)},q_{i(u)},\tau_{i(u)};R^{(u)}}^{m_u,n_u}\left(z_u(b-t)^{h'_u}\right) \prod_{v=1}^{s} \aleph_{P_{i(v)},Q_{i(v)},\iota_{i(v)};r^{(v)}}^{M_v,N_v}\left(y_v(b-t)^{h_v}\right) \mathrm{d}t$$

$$= B_1 \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N'_1/M'_1]} \cdots \sum_{K_u=0}^{[N'_u/M'_u]} \sum_{l=0}^{\infty} A_1\, G'(\eta_{G_1,g_1},\cdots \eta_{G_r,g_r}) \frac{[(a_{p'})]_l}{[(b_{q'})]_l} \frac{y^l}{\Gamma(\alpha l+1)}$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} x_1^{K_1} \cdots x_u^{K_u}\, z_1^{\eta_{G_1,g_1}} \cdots z_r^{\eta_{G_r,g_r}} (b-a)^{h'_1\eta_{G_1,g_1}+\cdots+h'_r\eta_{G_r,g_r}+K_1k_1+\cdots+K_uk_u+k'l}$$

$$\aleph_{2,2:W}^{0,2:V}\left(\begin{array}{c} \mathrm{y}_1(b-a)^{h_1} \\ \cdot \\ \cdot \\ \mathrm{y}_s(b-a)^{h_s} \end{array}\middle| \begin{array}{c} (\omega-\sigma-\sum_{i=1}^{r} h'_i\eta_{G_i,g_i} - \sum_{i=1}^{u} K_ik_i - k'l; h_1,\cdots,h_s), \\ \cdots \\ (\omega+n-\sigma-\sum_{i=1}^{r} h'_i\eta_{G_i,g_i} - \sum_{i=1}^{u} K_ik_i - k'l; h_1,\cdots,h_s), \end{array}\right.$$

$$\left.\begin{array}{c} (\text{-}\sigma-\sum_{i=1}^{r} h'_i\eta_{G_i,g_i} - \sum_{i=1}^{u} K_ik_i - k'l; h_1,\cdots,h_s) : C \\ \cdots \\ (\text{-n-}\rho-\sigma-1-\sum_{i=1}^{r} h'_i\eta_{G_i,g_i} - \sum_{i=1}^{u} K_ik_i - k'l; h_1,\cdots,h_s) : D \end{array}\right) \tag{3.2}$$

Where $G'(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r}) = \theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r}), \theta_i(.), i=1,\cdots,r$ is given respectively in (1.2)

Provided :

a) $h_i > 0, i=1,\cdots,s; h'_i > 0, i=1,\cdots,r; Re(\rho) > -1; p' \leqslant q' and\, |y| < 1$

b) $Re[\sigma + \sum_{i=1}^{r} h'_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^{s} h_i \min_{1\leqslant j\leqslant M_i} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > -1$

c)$|argy_k| < \frac{1}{2} B_i^{(k)} \pi$, where

$$B_i^{(k)} = +\sum_{j=1}^{N_k} \alpha_j^{(k)} - \iota_{i^{(k)}} \sum_{j=N_k+1}^{N_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)} + \sum_{j=1}^{M_k} \beta_j^{(k)} - \iota_{i^{(k)}} \sum_{j=M_k+1}^{q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} > 0,$$

with $k = 1\cdots, s$, $i^{(k)} = 1,\cdots, R^{(k)}$

c) If $r = s = 2$, we obtain two Aleph-functions of two variables defined by K. Sharma [3].

$$\int_a^b (t-a)^{\rho}(b-t)^{\sigma}F_n(\rho,\omega;t)S_{N_1',\cdots,N_u'}^{M_1',\cdots,M_u'}[x_1(b-t)^{k_1},\cdots,x_u(b-t)^{k_u}]_{p'}M_{q'}^{\alpha}(y(b-t)^{k'})$$

$$\aleph(z_1(b-t)^{h_1'},z_2(b-t)^{h_2'})\,\aleph(y_1(b-t)^{h_1},y_2(b-t)^{h_2})\mathrm{d}t$$

$$= B_1\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{m_1}\sum_{g_2=0}^{m_2}\sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_u=0}^{[N_u'/M_u']}\sum_{l=0}^{\infty}A_1G(\eta_{G_1,g_1},\eta_{G_2,g_2})\frac{[(a_{p'})]_l}{[(b_{q'})]_l}\frac{y^l}{\Gamma(\alpha l+1)}\frac{(-)^{G_1+G_2}}{\delta_{g_1}G_1!\delta_{g_2}G_2!}$$

$$x_1^{K_1}\cdots x_u^{K_u}\,z_1^{\eta_{G_1,g_1}}\,z_2^{\eta_{G_2,g_2}}(b-a)^{h_1'\eta_{G_1,g_1}+h_2'\eta_{G_2,g_2}+K_1k_1+\cdots+K_uk_u+k'l}$$

$$\aleph_{U_{22}:W}^{0,N+2:V}\left(\begin{array}{c|c} \mathrm{y}_1(b-a)^{h_1} & (\omega-\sigma-h_1'\eta_{G_1,g_1}-h_2'\eta_{G_2,g_2}-\sum_{i=1}^u K_ik_i-k'l;h_1,h_2), \\ \cdot & \cdots \\ \mathrm{y}_2(b-a)^{h_2} & (\omega+n-\sigma-h_1'\eta_{G_1,g_1}-h_2'\eta_{G_2,g_2}-\sum_{i=1}^u K_ik_i-k'l;h_1,h_2), \end{array}\right.$$

$$\left.\begin{array}{c} (\text{-}\sigma-h_1'\eta_{G_1,g_1}-h_2'\eta_{G_2,g_2}-\sum_{i=1}^u K_ik_i-k'l;h_1,h_2),A:C \\ \cdots \\ (\text{-n-}\rho-\sigma-1-h_1'\eta_{G_1,g_1}-h_2'\eta_{G_2,g_2}-\sum_{i=1}^u K_ik_i-k'l;h_1,h_2),B:D \end{array}\right) \tag{3.3}$$

Where $G(\eta_{G_1,g_1},\eta_{G_2,g_2})=\phi(\eta_{G_1,g_1},\eta_{G_2,g_2})\theta_1(\eta_{G_1,g_1})\,\theta_2(\eta_{G_2,g_2})$

Provided :

a) $h_1>0; Re(\rho)>-1, h_2>0, h_1'>0, h_2'>0; p'\leqslant q' and\,|\tau|<1$

b) $Re[\sigma+h_1'\min_{1\leqslant j\leqslant m_1}\frac{d_j^{(1)}}{\delta_j^{(1)}}+h_2'\min_{1\leqslant j\leqslant m_2}\frac{d_j^{(2)}}{\delta_j^{(2)}}+h_1\min_{1\leqslant j\leqslant M_1}\frac{b_j^{(1)}}{\beta_j^{(1)}}+h_2\min_{1\leqslant j\leqslant M_2}\frac{b_j^{(2)}}{\beta_j^{(2)}}]>-1$

c) $|arg(y_1)|<A_1\frac{\pi}{2}$ and $|arg(y_2)|<A_2\frac{\pi}{2}$; $i=1,\cdots,r$; $i'=1,\cdots r'$; $i''=1,\cdots,r''$, where :

$$A_1=\iota_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(1)}-\iota_i\sum_{j=1}^{Q_i}\beta_{ji}^{(1)}+\sum_{j=1}^{M_1}\beta_j-\iota_{i'}\sum_{j=M_1+1}^{Q_{i'}^{(1)}}\beta_{ji'}+\sum_{j=1}^{N_1}\alpha_j-\iota_{i'}\sum_{j=N_1+1}^{P_{i''}^{(1)}}\alpha_{ji'}>0$$

$$A_2=\iota_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(1)}-\iota_i\sum_{j=1}^{Q_i}\beta_{ji}^{(2)}+\sum_{j=1}^{M_1}\delta_j-\iota_{i''}\sum_{j=M_2+1}^{Q_{i''}^{(2)}}\delta_{ji''}+\sum_{j=1}^{N_2}\gamma_j-\iota_{i''}\sum_{j=N_2+1}^{P_{i''}^{(2)}}\gamma_{ji''}>0$$

d) If $r = s = 1$, we obtain two Aleph-functions of one variable defined by Südland [8].

$$\int_a^b (t-a)^{\rho}(b-t)^{\sigma}F_n(\rho,\omega;t)S_{N_1',\cdots,N_u'}^{M_1',\cdots,M_u'}[x_1(b-t)^{k_1},\cdots,x_u(b-t)^{k_u}]_{p'}M_{q'}^{\alpha}(y(b-t)^{k'})$$

$$\aleph(z(b-t)^{h'})\,\aleph(y(b-t)^{h})\mathrm{d}t$$

$$= B_1 \sum_{G=0}^{\infty} \sum_{g=0}^{m} \sum_{K_1=0}^{[N_1'/M_1']} \cdots \sum_{K_u=0}^{[N_u'/M_u']} \sum_{l=0}^{\infty} A_1\, G(\eta_{G,g}) \frac{[(a_{p'})]_l}{[(b_{q'})]_l} \frac{y^l}{\Gamma(\alpha l+1)} x_1^{K_1} \cdots x_u^{K_u}\, z^{\eta_{G,g}}$$

$$(b-a)^{h\eta_{G,g}+K_1k_1+\cdots+K_uk_u+k'l}$$

$$\aleph^{M,N+2}_{P_i+2,Q_i+2,c_i;r}\left(y(b-a)^h \,\middle|\, \begin{matrix} (\omega-\sigma-h'\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-k'l;h), \\ \cdots \\ (\omega+n-\sigma-h'\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-k'l;h), \end{matrix}\right.$$

$$\left.\begin{matrix} (\text{-}\sigma-h'\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-k'l;h), & (\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ \cdots & \cdots \\ (\text{-n-}\rho-\sigma-1-h'\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-k'l;h), & (\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix}\right) \quad (3.4)$$

Where $G(\eta_{G,g}) = \dfrac{(-)^G\Omega^{M,N}_{P_i,Q_i,c_i,r}(s)}{B_g G!}$

Provided

a) $Re(\rho)>0\,;p'\leqslant q' and\,|y|<1\,;h>0,h'>0$

b) $Re[\sigma+h\min\limits_{1\leqslant j\leqslant M}\dfrac{b_j}{B_j}+h'\min\limits_{1\leqslant j\leqslant m}\dfrac{d_j}{\delta_j}]>-1$

c) $|argz|<\dfrac{1}{2}\pi\Omega$ Where $\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0$

e) If $x_2=\cdots=x_u=0$, then the class of polynomials $S^{M_1',\cdots,M_u'}_{N_1',\cdots,N_u'}(x_1,\cdots,x_u)$ defined of (1.14) degenere to the class of polynomials $S^{M'}_{N'}(x)$ defined by Srivastava [5].

$$\int_a^b (t-a)^\rho(b-t)^\sigma F_n(\rho,\omega;t)S^{M'}_{N'}[x(b-t)^k]_{p'}M^{\alpha}_{q'}(y(b-t)^{k'})$$

$$\aleph(z_1(b-t)^{h_1'},\cdots,z_r(b-t)^{h_r'})\,\aleph(y_1(b-t)^{h_1},\cdots,y_s(b-t)^{h_s})\mathrm{d}t$$

$$= B_1 \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K=0}^{[N'/M']} \sum_{l=0}^{\infty} A_1\; G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r}) \frac{[(a_{p'})]_l}{[(b_{q'})]_l} \frac{y^l}{\Gamma(\alpha l+1)}$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!} x_1^{K_1}\cdots x_u^{K_u}\, z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}(b-a)^{h_1'\eta_{G_1,g_1}+\cdots+h_r'\eta_{G_r,g_r}+Kk+k'l}$$

$$\aleph^{0,N+2:V}_{U_{22}:W}\left(\begin{matrix} \mathrm{y}_1(b-a)^{h_1} \\ \cdot \\ \cdot \\ \mathrm{y}_s(b-a)^{h_s} \end{matrix}\,\middle|\, \begin{matrix} (\omega-\sigma-\sum_{i=1}^{r}h_i'\eta_{G_i,g_i}-Kk-k'l;h_1,\cdots,h_s), \\ \cdots \\ (\omega+n-\sigma-\sum_{i=1}^{r}h_i'\eta_{G_i,g_i}-Kk-k'l;h_1,\cdots,h_s), \end{matrix}\right.$$

$$\left.\begin{matrix} (\text{-}\sigma - \sum_{i=1}^{r} h_i'\eta_{G_i,g_i} - Kk - k'l; h_1, \cdots, h_s), A : C \\ \cdots \\ (\text{-n-}\rho - \sigma - 1 - \sum_{i=1}^{r} h_i'\eta_{G_i,g_i} - Kk - k'l; h_1, \cdots, h_s), B : D \end{matrix}\right) \tag{3.5}$$

Where $U_{22} = P_i + 2, Q_i + 2, \iota_i; r$

Provided :

a) $h_i > 0, i = 1, \cdots, s; h_i' > 0, i = 1, \cdots, r; Re(\rho) > -1; p' \leqslant q' and\,|y| < 1$

b) $Re[\sigma + \sum_{i=1}^{r} h_i' \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^{s} h_i \min_{1\leqslant j\leqslant M_i} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > -1$

c) $|argy_k| < \frac{1}{2} B_i^{(k)} \pi$, where $B_i^{(k)}$ is given in (1.13)

4. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as , multivariable H-function , defined by Srivastava et al [8] , the Aleph-function of two variables defined by K.sharma [3].

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