Certain finite integrals pertaining to generalized polynomial set, a class of polynomials, Aleph-function and the multivariable Aleph-function

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ABSTRACT

Our aim is to evaluate four integrals pertaining to the products of Aleph-function, a generalized polynomial set $S_n^{\alpha,\beta,0}(x)$, a class of polynomial $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s]$ and the multivariable Aleph-function. On account of the most general nature of the function involved herein a very large number of known and new integrals involving simpler special functions and orthogonal polynomials follows as particular cases of our main integrals.

KEYWORDS : Aleph-function of several variables, Aleph-function, finite integral, special function, general class of polynomials.

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1.Introduction and preliminaries.

The Aleph- function , introduced by Südland [10] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z) = \aleph_{P_i,Q_i,c_i;r}^{M,N}\left(z \;\middle|\; \begin{matrix} (a_j, A_j)_{1,\mathfrak{n}}, [c_i(a_{ji}, A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ (b_j, B_j)_{1,m}, [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix} \right)$$

$$= \frac{1}{2\pi\omega}\int_L \Omega_{P_i,Q_i,c_i;r}^{M,N}(s) z^{-s} \mathrm{d}s \tag{1.1}$$

for all z different to 0 and

$$\Omega_{P_i,Q_i,c_i;r}^{M,N}(s) = \frac{\prod_{j=1}^{M}\Gamma(b_j + B_j s)\prod_{j=1}^{N}\Gamma(1 - a_j - A_j s)}{\sum_{i=1}^{r} c_i \prod_{j=N+1}^{P_i}\Gamma(a_{ji} + A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1 - b_{ji} - B_{ji}s)} \tag{1.2}$$

With :

$$|argz| < \frac{1}{2}\pi\Omega \quad \text{Where } \Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0 \quad \text{with } i = 1,\cdots,r$$

For convergence conditions and other details of Aleph-function , see Südland et al [10].

Serie representation of Aleph-function is given by Chaurasia et al [4].

$$\aleph_{P_i,Q_i,c_i;r}^{M,N}(z) = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(s)}{B_G g!}z^{-s} \tag{1.3}$$

with $s=\eta_{G,g}=\dfrac{b_G+g}{B_G}, P_i<Q_i, |z|<1$ and $\Omega_{P_i,Q_i,c_i;r}^{M,N}(s)$ is given in (1.2) (1.4)

The generalized polynomial set defined by Raizada [5 ,p.64, eq.(2.1.2)] in the following Rodrigues type formula :

$$S_n^{\alpha,\beta,\tau}[x:\gamma,s,q,A,B,m,\zeta,l]=(Ax+B)^{-\alpha}(1-\tau x^{\gamma})^{\beta/\gamma} \tag{1.5}$$

$$\times T_{\zeta,l}^{m+n}[(Ax+B)^{\alpha+qn}(1-\tau x^{\gamma})^{(\beta/\tau)+sn}]$$

with the differential operator $T_{k,l}$ is defined by $T_{k,l}=x^l(k+x\dfrac{d}{dx})$ (1.6)

Moreover it can be expressed in the following serie :

$$S_n^{\alpha,\beta,\tau}[x:\gamma,s,q,A,B,m,\zeta,l]=B^{qn}x^{l(m+n}(1-\tau x^l)^{sn}l^{m+n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\frac{(-)^j(-j)_i(\alpha)_j}{\sigma!\rho!i!j!}$$

$$\frac{(-\sigma)_j(-\alpha-qn)_i}{(1-\alpha-j)_i}\,(-\frac{\beta}{\tau}-sn)_\sigma(\frac{i+\zeta+\gamma\rho}{l})_{m+n}\,(\frac{-\tau x^{\gamma}}{1-\tau x^{\gamma}})^{\sigma}(\frac{Ax}{B})^{i} \tag{1.7}$$

Taking $A=1, B=0$ and $\tau\rightarrow 0$, one arrives at the following polynomial set :

$$S_n^{\alpha,\beta,0}(x)=S_n^{\alpha,\beta,0}[x:\gamma,s,q,1,0,m,\zeta,l]=x^{qn+l(m+n)}l^{m+n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{A_{m,n}}{\sigma!\rho!}(\beta x^{\gamma})^{\sigma} \tag{1.8}$$

Where $A_{m,n}=(-\sigma)_\rho((\alpha+qn+\zeta+\gamma\rho)/l)_{m+n}$ (1.9)

The generalized polynomials defined by Srivastava [8], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s]\ =\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.10}$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex.

In this paper, we note : $A_1=\dfrac{(-N_1)_{M_1K_1}}{K_1!}\cdots\dfrac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s]$ (1.11)

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [6] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}\left(\begin{array}{c|}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\right.$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{\mathfrak{n}+1,p_i}]:$

$\cdots\cdots\cdots\cdots\cdots\cdots\ ,[\tau_i(b_{ji};\beta_j^{(1)},\cdots,\beta_j^{(r)})_{m+1,q_i}]:$

$$\left.\begin{matrix}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{matrix}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.11}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.12}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.13}$$

where $j=1$ to r and $k=1$ to r

Suppose , as usual , that the parameters

$a_j, j=1,\cdots,p; b_j, j=1,\cdots,q;$

$c_j^{(k)}, j=1,\cdots,n_k; c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,p_{i^{(k)}};$

$d_j^{(k)}, j=1,\cdots,m_k; d_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,q_{i^{(k)}};$

with $k=1\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's,\beta's,\gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}+\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}+\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}-\sum_{j=1}^{m_k}\delta_j^{(k)}$$

$$-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}\leqslant 0 \tag{1.14}$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)$ with $j=1$ to m_k are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)}s_k)$ with $j=1$ to n and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)$ with $j=1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0 , \text{ with } k = 1 \cdots, r, i = 1, \cdots, R , i^{(k)} = 1, \cdots, R^{(k)} \tag{1.15}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1} \ldots |z_r|^{\alpha_r}) , max(|z_1| \ldots |z_r|) \rightarrow 0$

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1} \ldots |z_r|^{\beta_r}) , min(|z_1| \ldots |z_r|) \rightarrow \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \; ; \; V = m_1, n_1; \cdots; m_r, n_r \tag{1.16}$$

$$\text{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i(1)}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i(r)}; R^{(r)} \tag{1.17}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.18}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.19}$$

$$C = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots, \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.20}$$

$$D = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots, \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.21}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots, z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V} \left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array} \middle| \begin{array}{c} \text{A : C} \\ \cdot \; \cdot \; \cdot \\ \text{B : D} \end{array} \right) \tag{1.22}$$

2. The main integrals

In this section the following integrals have been derived :

The first integral :

Let $g(x) = \frac{x-s}{x-w}; h(x) = \frac{t-s}{x-w}$

$$\int_s^t (x-s)^{u-1}(t-x)^{v-1}(x-w)^{-u-v} \, S_n^{\alpha,\beta,0}(z(g(x))^h(h(x))^k)\aleph_{P_i,Q_i,c_i;r}^{M,N}(y(g(x))^{h'}(h(x))^{k'})$$

$$S_{Q_1,\cdots,Q_R}^{P_1,\cdots,P_R}[y_1(g(x))^{H_1}(h(x))^{K_1},\cdots,y_R(g(x))^{H_R}(h(x))^{K_R}] \ \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(g(x))^{h_1}(h(x))^{k_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r(g(x))^{h_r}(h(x))^{k_r} \end{array}\right)\mathrm{d}x$$

$$= z^{qn+l(m+n)+\gamma\sigma} l^{m+n} \sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{A_{m,n}}{\sigma!\rho!}(\beta)^\sigma \sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!} y^{\eta_{G,g}} \sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}$$

$$A_1 y_1^{\alpha_1}\cdots y_R^{\alpha_R}(t-w)^{-u-hqn-hl(m+n)-h\gamma\sigma-h'\eta_{G,g}-H_1\alpha_1-\cdots-H_R\alpha_R}$$

$$(s-w)^{-v-kqn-kl(m+n)-k\gamma\sigma-k'\eta_{G,g}-K_1\alpha_1-\cdots-K_R\alpha_R}$$

$$(t-s)^{u+v+(h+k)(qn+\gamma\sigma+l(m+n)-1+(h'+k')\eta_{G,g}+(H_1+K_1)\alpha_1+\cdots+(H_R+K_R)\alpha_R}$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} \mathrm{z}_1(g'(t))^{h_1}(h'(t))^{k_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r(g'(t))^{h_r}(h(t))^{k_r} \end{array}\middle| \begin{array}{c} (1\text{ -u-hqn-hl(m+n)-h}\gamma\sigma - h'\eta_{G,g} - \sum_{i=1}^R H_i\alpha_i; h_1,\cdots,h_r), \\ \cdots \\ \cdots \\ \cdots\cdots\cdots \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{ -v-hqn-kl(m+n)-k}\gamma\sigma - k'\eta_{G,g} - \sum_{i=1}^R K_i\alpha_i; k_1,\cdots,k_r), A : C \\ \cdots \\ \cdots \\ (1\text{-u-v-(hl+kl)(m+n)-(h+k)(qn+}\gamma\sigma) - (h'+k')\eta_{G,g} - \sum_{i=1}^R (H_i+K_i)\alpha_i; h_1+k_1,\cdots,h_r+k_r), B; D \end{array}\right) \quad (2.1)$$

Where :$U_{21} = p_i+2, q_i+1, \tau_i; R$ and $g'(x) = \frac{t-s}{t-w}; h'(x) = \frac{t-s}{s-w}$

Provided that :

a) $h, k, h', k', h_i, k_i > 0, i = 1,\cdots,r$, k is an integer

b) $Re[u + h' \min_{1\leqslant j\leqslant M}\frac{b_L}{B_L} + \sum_{i=1}^r h_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

c) $Re[v + k' \min_{1\leqslant j\leqslant M}\frac{b_L}{B_L} + \sum_{i=1}^r h_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

d)$|argz| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^M \beta_j + \sum_{j=1}^N \alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$

e) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

The second integral :

$$\int_0^t x^u\,(t-x)^{v-1} S_n^{\alpha,\beta,0}[zx^h(t-x)^k] S_{Q_1,\cdots,Q_R}^{P_1,\cdots,P_R}[y_1(t-x)^{K_1}x^{H_1},\cdots,y_R(t-x)^{K_R}x^{H_R}]$$

$$\aleph_{P_i,Q_i,c_i;r}^{M,N}(yx^{h'}(t-x)^{k'})\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 x^{h_1}(t-x)^{k_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{h_r}(t-x)^{k_r} \end{array}\right)\mathrm{d}x$$

$$= z^{qn+l(m+n)+\gamma\sigma} l^{m+n} \sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{A_{m,n}}{\sigma!\rho!}(\beta)^\sigma \sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!} y^{\eta_{G,g}} \sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}$$

$$A_1 y_1^{\alpha_1}\cdots y_R^{\alpha_R}\ \ (t-s)^{u+v+(h+k)(qn+\gamma\sigma)-1+(h'+k')\eta_{G,g}+(H_1+K_1)\alpha_1+\cdots+(H_R+K_R)\alpha_R}$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} \mathrm{z}_1 t^{h_1+k_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r t^{h_r+k_r} \end{array}\right| \begin{array}{c} (1\text{ -u-hqn-hl(m+n)-h}\gamma\sigma - h'\eta_{G,g}-\sum_{i=1}^R H_i\alpha_i; h_1,\cdots,h_r), \\ \cdots \\ \cdots \\ \cdots\cdots\cdots \end{array}$$

$$\left.\begin{array}{c} (1\text{ -v-hqn-kl(m+n)-k}\gamma\sigma - k'\eta_{G,g}-\sum_{i=1}^R K_i\alpha_i; k_1,\cdots,k_r), A : C \\ \cdots \\ \cdots \\ (\text{1-u-v-(hl+kl)(m+n)-(h+k)(qn+}\gamma\sigma) - (h'+k')\eta_{G,g} - \sum_{i=1}^R (H_i+K_i)\alpha_i; h_1+k_1,\cdots,h_r+k_r), B; D \end{array}\right) \quad (2.2)$$

Where :$U_{21} = p_i+2, q_i+1, \tau_i; R$

Provided that :

a) $h, k, h', k', h_i, k_i > 0, i = 1,\cdots,r$, k is an integer

b) $Re[u + h' \min\limits_{1\leqslant j\leqslant M}\frac{b_L}{B_L} + \sum\limits_{i=1}^r h_i \min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

c) $Re[v + k' \min\limits_{1\leqslant j\leqslant M}\frac{b_L}{B_L} + \sum\limits_{i=1}^r h_i \min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

d)$|argz| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum\limits_{j=1}^M \beta_j + \sum\limits_{j=1}^N \alpha_j - c_i(\sum\limits_{j=M+1}^{Q_i}\beta_{ji} + \sum\limits_{j=N+1}^{P_i}\alpha_{ji}) > 0$

e) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.15)

The third integral :

$$\int_0^1 x^u\,(1-x)^{v-1}{}_2F_1[\alpha',\beta';u;x]S_n^{\alpha,\beta,0}[z(1-x)^h]S_{Q_1,\cdots,Q_R}^{P_1,\cdots,P_R}[y_1(1-x)^{H_1},\cdots,y_R(1-x)^{H_R}]$$

$$\aleph_{P_i,Q_i,c_i;r}^{M,N}(y(1-x)^{h'})\ \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1(1-x)^{h_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r(1-x)^{h_r}\end{array}\right)\mathrm{d}x$$

$$= z^{qn+l(m+n)+\gamma\sigma}l^{m+n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{A_{m,n}}{\sigma!\rho!}(\beta)^{\sigma}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}y^{\eta_{G,g}}$$

$$\sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}\quad A_1\,y_1^{\alpha_1}\cdots y_R^{\alpha_R}$$

$$\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\right|\begin{array}{c}(1\text{ -v-hqn-hl(m+n)-h}\gamma\sigma-h'\eta_{G,g}-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r),\\ \cdots\\ \cdots\\ (1\text{ -u-v-hqn-hl(m+n)-h}\gamma\sigma-h'\eta_{G,g}+\alpha'-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r),\end{array}$$

$$\left.\begin{array}{c}(1\text{ -u -v-kqn-kl(m+n)-k}\gamma\sigma+\alpha'+\beta'-k'\eta_{G,g}-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r),A:C\\ \cdots\\ \cdots\\ (1\text{-u-v-hqn-hl(m+n)- h}\gamma\sigma)-h'\eta_{G,g}+\beta'-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r),B:D\end{array}\right) \tag{2.3}$$

Where :$U_{22}=p_i+2,q_i+2,\tau_i;R$

Provided that :

a) $h,k,h',k',h_i,k_i>0,i=1,\cdots,r$, k is an integer

b) $Re[u+h'\min\limits_{1\leqslant j\leqslant M}\frac{b_L}{B_L}+\sum_{i=1}^r h_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c) $Re[v+k'\min\limits_{1\leqslant j\leqslant M}\frac{b_L}{B_L}+\sum_{i=1}^r h_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

d)$|argz|<\frac{1}{2}\pi\Omega$ Where $\Omega=\sum_{j=1}^M\beta_j+\sum_{j=1}^N\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0$

e) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.15)

f) $Re(u+v+\alpha'+\beta')>0,Re(u)>0$

The fourth integral :

$$\int_{-1}^{1}(1+x)^{u-1}(1-x)^{v-1}P_w^{(\alpha',\beta')}(1-st(1-x)/2)S_n^{\alpha,\beta,0}[z(1+x)^h(1-x)^k]$$

$$S_{Q_1,\cdots,Q_R}^{P_1,\cdots,P_R}[y_1(1+x)^{H_1}(1-x)^{K_1},\cdots,y_R(1+x)^{H_R}(1-x)^{K_R}]\aleph_{P_i,Q_i,c_i;r}^{M,N}(y(1+x)^{h'}(1-x)^{k'})$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(1+x)^{h_1}(1-x^{k_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r(1-x)^{h_r}(1-x)^{k_r} \end{array}\right)\mathrm{d}x$$

$$= z^{qn+l(m+n)+\gamma\sigma}l^{m+n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{A_{m,n}}{\sigma!\rho!}(\beta)^\sigma\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}y^{\eta_{G,g}}\sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}A_1$$

$$y_1^{\alpha_1}\cdots y_R^{\alpha_R}\frac{2^{u+v+(n+k)qn+(h+k)(m+n)+(h+k)\gamma\sigma+(h'+k')\eta_{G,g}+(H_1+k_1)\alpha_1+\cdots+(H_R+K_R)\alpha_R-1}(\alpha+1;w)}{w!}$$

$$\sum_{R=0}^{w}\frac{(-w:R)(1+\alpha'+\beta'+w;R)}{R!(\alpha'+1;R)}(st/2)^R$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} \mathrm{z}_1 2^{h_1+k_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r 2^{h_r+k_r} \end{array}\right|\begin{array}{c} (1\text{ -u-hqn-hl(m+n)-h}\gamma\sigma-h'\eta_{G,g}-\sum_{i=1}^{R}H_i\alpha_i;h_1,\cdots,h_r), \\ \cdots \\ \cdots \\ \cdots\cdots \end{array}$$

$$\left.\begin{array}{c} (1\text{ -v-hqn-kl(m+n)-k}\gamma\sigma-k'\eta_{G,g}-\sum_{i=1}^{R}K_i\alpha_i;k_1,\cdots,k_r),A:C \\ \cdots \\ \cdots \\ (1\text{-u-v-(hl+kl)(m+n)-(h+k)(qn+}\gamma\sigma)-(h'+k')\eta_{G,g}-\sum_{i=1}^{R}(H_i+K_i)\alpha_i;h_1+k_1,\cdots,h_r+k_r),B;D \end{array}\right) \quad (2.4)$$

Where : $U_{21}=p_i+2,q_i+1,\tau_i;R$

Provided that :

a) $h,k,h',k',h_i,k_i>0,i=1,\cdots,r$, k is an integer

b) $Re[u+h'\min_{1\leqslant j\leqslant M}\frac{b_L}{B_L}+\sum_{i=1}^{r}h_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c) $Re[v+k'\min_{1\leqslant j\leqslant M}\frac{b_L}{B_L}+\sum_{i=1}^{r}h_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

d) $|argz| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$

e) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.15)

Proof :

To establish the finite integral (2.1), express the generalized class of polynomials $S_n^{\alpha,\beta,0}(z(g(x))^h(h(x))^k)$ and $S_{Q_1,\cdots,Q_R}^{P_1,\cdots,P_R}[y_1(g(x))^{H_1}(h(x))^{K_1},\cdots,y_R(g(x))^{H_R}(h(x))^{K_R}]$ occuring on the L.H.S in the series form given by (1.8) and (1.10), the Aleph-function in serie form given by (1.3) and the multivariable Aleph-function involving there in terms of Mellin-Barnes contour integral by (1.11). Now interchange the order of summation and integration (which is permissible under the conditions stated), so that the L.H.S of (2.1) say I assume the following from after a little simplification :

$$I = z^{qn+l(m+n)+\gamma\sigma}l^{m+n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{A_{m,n}}{\sigma!\rho!}(\beta)^{\sigma}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}y^{\eta_{G,g}}\sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}$$

$$A_1 y_1^{\alpha_1}\cdots y_R^{\alpha_R}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)\,z_k^{s_k}\Big[\int_s^t$$

$$(x-s)^{u+hqn+hl(m+n)+h\gamma\sigma+h'\eta_{G,g}+H_1\alpha_1+\cdots+H_R\alpha_R+h_1s_1+\cdots+h_rs_r-1}$$

$$(t-x)^{v+kqn+kl(m+n)+k\gamma\sigma+k'\eta_{G,g}+K_1\alpha_1+\cdots+K_R\alpha_R+k_1s_1+\cdots+k_rs_r-1}$$

$$(x-w)^{-u-v-(h+k)qn-(h+k)l(m+n)-(h+k)\gamma\sigma-(h'+k')\eta_{G,g}-(H_1+K_1)\alpha_1-\cdots-(H_R+K_R)\alpha_R-(h_1+k_1)s_1-\cdots-(h_r+k_r)s_r-1}$$

$$\mathrm{d}x\mathrm{d}s_1\cdots\mathrm{d}s_r\Big] \tag{2.5}$$

On evaluating the inner integral occuring on the R.H.S. Of (2.5), we get after simplification :

$$I = (t-s)^{u+v+(h+k)(qn+l(m+n)+\gamma\sigma)-1}(t-w)^{-u-h(qn+l(m+n)+\gamma\sigma)}(s-w)^{-v-k(qn+l(m+n)+\gamma\sigma)}$$

$$z^{qn+l(m+n)+\gamma\sigma}l^{m+n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{A_{m,n}}{\sigma!\rho!}(\beta)^{\sigma}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}y^{\eta_{G,g}}\sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}$$

$$A_1 y_1^{\alpha_1}\cdots y_R^{\alpha_R}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}$$

$$[g(t)]^{(H_1+K_1)\alpha_1+\cdots+(H_R+K_R)\alpha_R+(h_1+k_1)\alpha_1+\cdots+(h_r+k_r)\alpha_r]}$$

$$\Gamma(u+hqn+hl(m+n)+h\gamma\sigma+h'\eta_{G,g}+H_1\alpha_1+\cdots+H_R\alpha_R+h_1s_1+\cdots+h_rs_r)$$

$$\times\Gamma(v+kqn+kl(m+n)+k\gamma\sigma+k'\eta_{G,g}+k_1\alpha_1+\cdots+k_R\alpha_R+k_1s_1+\cdots+k_rs_r)$$

$$\times[\,\Gamma(u+v+(h+k)qn+(h+k)l(m+n)+(h+k)\gamma\sigma+(h'+k')\eta_{G,g}+$$

$$+(H_1+K_1)\alpha_1+\cdots+(H_R+K_R)\alpha_R+(h_1+k_1)s_1+\cdots+(h_r+k_r)s_r)]^{-1}$$

$$\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.6}$$

where $g(t) = \frac{t-s}{t-w}$

Finally, on reinterpreting the Mellin-Barnes contour integral in the R.H.S. Of (2.6) in term of the multivariable Aleph-function given by (1.11), we arrive at the desired result. The proofs of the integrals (2.2), (2.3) and (2.4) can be developed on similar method.

3. Particular cases

Replace $S_n^{\alpha,\beta,0}(x)$ by $H_n^{(\mathfrak{r})}(x,\alpha,\beta)$, we get :

a) $$\int_s^t (x-s)^{u-1}(t-x)^{v-1}(x-w)^{-u-v}\, H^{(\mathfrak{r})}(z(g(x))^h(h(x))^k,\alpha,\beta)\aleph_{P_i,Q_i,c_i;r}^{M,N}(y(g(x))^{h'}(h(x))^{k'})$$

$$S_{Q_1,\cdots,Q_R}^{P_1,\cdots,P_R}[y_1(g(x))^{H_1}(h(x))^{K_1},\cdots,y_R(g(x))^{H_R}(h(x))^{K_R}]\ \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(g(x))^{h_1}(h(x))^{k_1} \\ . \\ . \\ . \\ \mathrm{z}_r(g(x))^{h_r}(h(x))^{k_r} \end{array}\right)\mathrm{d}x$$

$$= z^{\gamma\sigma-n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{B_{m,n}}{\sigma!\rho!}(\beta)^\sigma\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}y^{\eta_{G,g}}\sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}$$

$$A_1\frac{(-Q_1)_{P_1\alpha_1}}{\alpha_1!}\cdots\frac{(-Q_R)_{P_R\alpha_R}}{\alpha_R!}\ (t-s)^{u+v+(h+k)\gamma\sigma+(h'+k')\eta_{G,g}+(H_1+K_1)\alpha_1+\cdots+(H_R+K_R)\alpha_R}$$

$$(s-w)^{-v-kn-k\gamma\sigma-k'\eta_{G,g}-K_1\alpha_1-\cdots-K_R\alpha_R}(t-w)^{-u-hn-hl(m+n)-h\gamma\sigma-h'\eta_{G,g}-H_1\alpha_1-\cdots-H_R\alpha_R}$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} \mathrm{z}_1(g'(t))^{h_1}(h'(t))^{k_1} \\ . \\ . \\ . \\ \mathrm{z}_r(g'(t))^{h_r}(h(t))^{k_r} \end{array}\left|\begin{array}{c} (1\text{ -u-hn-hl(m+n)-h}\gamma\sigma-h'\eta_{G,g}-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r), \\ \cdots \\ \cdots \\ \cdots\cdots\cdots \end{array}\right.\right.$$

$$\left.\begin{array}{c} (1\text{ -v-hn-k}\gamma\sigma-k'\eta_{G,g}-\sum_{i=1}^R K_i\alpha_i;k_1,\cdots,k_r),A:C \\ \cdots \\ \cdots \\ (1\text{-u-v-(h+k)(m+n)-(h+k)(qn+}\gamma\sigma)-(h'+k')\eta_{G,g}-\sum_{i=1}^R(H_i+K_i)\alpha_i;h_1+k_1,\cdots,h_r+k_r),B;D \end{array}\right) \quad (3.1)$$

Where $g(x)=\dfrac{x-s}{x-w}$ and $B_{m,n}=(-\sigma)_\rho((-\alpha-\gamma\rho)/l)_n$ and Where :$U_{21}=p_i+2,q_i+1,\tau_i;R$

b) $$\int_0^t x^u\,(t-x)^{v-1}H_n^{(\mathfrak{r})}[zx^h(t-x)^k,\alpha,\beta]S_{Q_1,\cdots,Q_R}^{P_1,\cdots,P_R}[y_1(t-x)^{K_1}x^{H_1},\cdots,y_R(t-x)^{K_R}x^{H_R}]$$

$$\aleph_{P_i,Q_i,c_i;r}^{M,N}(yx^{h'}(t-x)^{k'})\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1x^{h_1}(t-x)^{k_1} \\ . \\ . \\ . \\ \mathrm{z}_rx^{h_r}(t-x)^{k_r} \end{array}\right)\mathrm{d}x$$

$$= z^{\gamma\sigma-n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{B_{m,n}}{\sigma!\rho!}(\beta)^\sigma\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}y^{\eta_{G,g}}\sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}$$

$$A_1 y_1^{\alpha_1}\cdots y_R^{\alpha_R}(t-s)^{u+v+(h+k)\gamma\sigma-1+(h'+k')\eta_{G,g}+(H_1+K_1)\alpha_1+\cdots+(H_R+K_R)\alpha_R}$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} z_1 t^{h_1+k_1} & (1\text{ -u+hn-h}\gamma\sigma-h'\eta_{G,g}-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r), \\ . & \cdots \\ . & \cdots \\ . & \\ z_r t^{h_r+k_r} & \cdots\cdots \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{ -v-hn+kn-k}\gamma\sigma-k'\eta_{G,g}-\sum_{i=1}^R K_i\alpha_i;k_1,\cdots,k_r),A:C \\ \cdots \\ \cdots \\ (1\text{-u-v-(h+k)(m+n)-(h+k)}\gamma\sigma-(h'+k')\eta_{G,g}-\sum_{i=1}^R(H_i+K_i)\alpha_i;h_1+k_1,\cdots,h_r+k_r),B;D \end{array}\right) \quad (3.2)$$

Where :$U_{21}=p_i+2,q_i+1,\tau_i;R$

c) $\int_0^1 x^u\,(1-x)^{v-1}{}_2F_1[\alpha',\beta';u;x]S_n^{\alpha,\beta,0}[z(1-x)^h]S_{Q_1,\cdots,Q_R}^{P_1,\cdots,P_R}[y_1(1-x)^{H_1},\cdots,y_R(1-x)^{H_R}]$

$$\aleph_{P_i,Q_i,c_i;r}^{M,N}(y(1-x)^{h'})\ \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} z_1(1-x)^{h_1} \\ . \\ . \\ . \\ z_r(1-x)^{h_r} \end{array}\right)\mathrm{d}x$$

$$=z^{\gamma\sigma-n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{B_{m,n}}{\sigma!\rho!}(\beta)^\sigma\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}y^{\eta_{G,g}}\sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}$$

$$A_1 y_1^{\alpha_1}\cdots y_R^{\alpha_R}\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} z_1 & (1\text{ -v-hn-h}\gamma\sigma-h'\eta_{G,g}-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r), \\ . & \cdots \\ . & \cdots \\ . & \\ z_r & (1\text{ -u-v+hn-h}\gamma\sigma-h'\eta_{G,g}+\alpha'-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r), \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-u-v-hn- h}\gamma\sigma-h'\eta_{G,g}+\alpha'+\beta'-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r),A:C \\ \cdots \\ \cdots \\ (1\text{-u-v-hn- h}\gamma\sigma-h'\eta_{G,g}+\beta'-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r),B;D \end{array}\right) \quad (3.3)$$

Where :$U_{22}=p_i+2,q_i+2,\tau_i;R$

d) $\int_{-1}^1 (1+x)^{u-1}(1-x)^{v-1}P_w^{(\alpha',\beta')}(1-st(1-x)/2)H_n^{(\mathfrak{r})}[z(1+x)^h(1-x)^k,\alpha,\beta]$

$$S_{Q_1,\cdots,Q_R}^{P_1,\cdots,P_R}[y_1(1+x)^{H_1}(1-x)^{K_1},\cdots,y_R(1+x)^{H_R}(1-x)^{K_R}]\aleph_{P_i,Q_i,c_i;r}^{M,N}(y(1+x)^{h'}(1-x)^{k'})$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(1+x)^{h_1}(1-x)^{k_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r(1-x)^{h_r}(1-x)^{k_r} \end{array}\right)\mathrm{d}x$$

$$= z^{\gamma\sigma-n}\sum_{\sigma=0}^{m+n}\sum_{\rho=0}^{\sigma}\frac{B_{m,n}}{\sigma!\rho!}(\beta)^{\sigma}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}y^{\eta_{G,g}}\sum_{\alpha_1=0}^{[Q_1/P_1]}\cdots\sum_{\alpha_R=0}^{[Q_R/P_R]}$$

$$A_1 y_1^{\alpha_1}\cdots y_R^{\alpha_R}\ \frac{2^{u+v+(n+k)n+(h+k)\gamma\sigma+(h'+k')\eta_{G,g}+(H_1+k_1)\alpha_1+\cdots+(H_R+K_R)\alpha_R-1}(\alpha+1;w)}{w!}$$

$$\sum_{R=0}^{w}\frac{(-w:R)(1+\alpha'+\beta'+w;R)}{R!(\alpha'+1;R)}(st/2)^R$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} \mathrm{z}_1 2^{h_1+k_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r 2^{h_r+k_r} \end{array}\left|\begin{array}{c} (1\text{ -u+hn-h}\gamma\sigma-h'\eta_{G,g}-\sum_{i=1}^R H_i\alpha_i;h_1,\cdots,h_r), \\ \cdots \\ \cdots \\ \cdots\cdots \end{array}\right.\right.$$

$$\left.\begin{array}{c} (1\text{-v+kn-k}\gamma\sigma-k'\eta_{G,g}-\sum_{i=1}^R K_i\alpha_i;k_1,\cdots,k_r),A:C \\ \cdots \\ \cdots \\ (1\text{-u-v+(h+k)n- h}\gamma\sigma-h'\eta_{G,g}+\alpha'-\sum_{i=1}^R(H_i+K_i)\alpha_i;h_1+k_1,\cdots,h_r+k_r),B;D \end{array}\right) \quad (3.4)$$

Where : $U_{21}=p_i+2,q_i+1,\tau_i;R$

4. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as I-function of several variables defined by Sharma and Ahmad [6] , multivariable H-function , see Srivastava et al [9] , the Aleph-function of two variables defined by K.sharma [7], the I-function of two variables defined by Goyal and Agrawal [1,2,3] , ,and the h-function of two variables , see Srivastava et al [9].

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