An integral associated with Aleph-functions of several variables

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ABSTRACT
The object of present document is to derive an integral pertaining to a products of two multivariable Aleph-functions, Two general class of polynomials and the M-serie with general arguments of quadratic nature. The result established in this paper are of general nature and hence encompass several particular cases.

Keywords :Multivariable Aleph-function, general class of polynomials, M-serie, general Lauricella function.

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1.Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [2] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|}\mathrm{y}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_r\end{array}\right.$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ ,[\tau_i(b_{ji};\beta_j^{(1)},\cdots,\beta_j^{(r)})_{m+1,q_i}]:$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)y_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^r\alpha_j^{(k)}s_k)}{\sum_{i=1}^R[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^r\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^r\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$a_j, j = 1, \cdots, p; b_j, j = 1, \cdots, q;$

$c_j^{(k)}, j = 1, \cdots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R\ , i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1 \cdots, r, i = 1, \cdots, R\ , i^{(k)} = 1, \cdots, R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\alpha_1} \dots |y_r|^{\alpha_r}) , max(|y_1| \dots |y_r|) \rightarrow 0$

$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\beta_1} \dots |y_r|^{\beta_r}) , min(|y_1| \dots |y_r|) \rightarrow \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$

Serie representation of Aleph-function of several variables is given by

$$\aleph(y_1,\cdots,y_r) = \sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\psi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})$$

$$\times\ \theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r})y_1^{-\eta_{G_1,g_1}}\cdots y_r^{-\eta_{G_r,g_r}} \tag{1.6}$$

Where $\psi(.,\cdots,.), \theta_i(.), i = 1,\cdots,r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1} = \frac{d_{g_1}^{(1)}+G_1}{\delta_{g_1}^{(1)}},\cdots,\ \eta_{G_r,g_r} = \frac{d_{g_r}^{(r)}+G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[d_j^i+p_i] \neq \delta_j^{(i)}[d_{g_i}^i+G_i]$ (1.7)

for $j \neq m_i, m_i = 1,\cdots\eta_{G_i,g_i}; p_i, n_i = 0,1,2,\cdots,; y_i \neq 0, i = 1,\cdots,r$ (1.8)

Consider the Aleph-function of s variables

$$\aleph(z_1,\cdots,z_s) = \aleph^{0,N:M_1,N_1,\cdots,M_s,N_s}_{P_i,Q_i,\iota_i;r:P_{i^{(1)}},Q_{i^{(1)}},\iota_{i^{(1)}};r^{(1)};\cdots;P_{i^{(s)}},Q_{i^{(s)}};\iota_{i^{(s)}};r^{(s)}}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_s \end{array}\middle|\begin{array}{c} [(u_j;\mu_j^{(1)},\cdots,\mu_j^{(r)})_{1,N}] ,[\iota_i(u_{ji};\mu_j^{(1)},\cdots,\mu_j^{(r)})_{N+1,P_i}]: \\ \cdots\cdots\cdots\cdots ,[\iota_i(v_{ji};v_j^{(1)},\cdots,v_j^{(r)})_{M+1,Q_i}]: \end{array}\right.$$

$$\left.\begin{array}{c}[(a_j^{(1)});\alpha_j^{(1)})_{1,N_1}],[\iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)};\alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}];\cdots;[(a_j^{(s)});\alpha_j^{(s)})_{1,N_s}],[\iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)};\alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_i^{(s)}}] \\ [(b_j^{(1)});\beta_j^{(1)})_{1,M_1}],[\iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)};\beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}];\cdots;[(b_j^{(s)});\beta_j^{(s)})_{1,M_s}],[\iota_{i^{(s)}}(b_{ji^{(s)}}^{(s)};\beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_i^{(s)}}]\end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_r}\zeta(t_1,\cdots,t_s)\prod_{k=1}^{s}\phi_k(t_k)z_k^{t_k}\,\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.9}$$

with $\omega = \sqrt{-1}$

$$\zeta(t_1,\cdots,t_s) = \frac{\prod_{j=1}^{N}\Gamma(1-u_j+\sum_{k=1}^{s}\mu_j^{(k)}t_k)}{\sum_{i=1}^{r'}[\iota_i\prod_{j=N+1}^{P_i}\Gamma(u_{ji}-\sum_{k=1}^{s}\mu_{ji}^{(k)}t_k)\prod_{j=1}^{Q_i}\Gamma(1-v_{ji}+\sum_{k=1}^{s}v_{ji}^{(k)}t_k)]} \tag{1.10}$$

and $$\phi_k(t_k) = \frac{\prod_{j=1}^{M_k}\Gamma(b_j^{(k)}-\beta_j^{(k)}t_k)\prod_{j=1}^{N_k}\Gamma(1-a_j^{(k)}+\alpha_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{r^{(k)}}[\iota_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-b_{ji^{(k)}}^{(k)}+\beta_{ji^{(k)}}^{(k)}t_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(a_{ji^{(k)}}^{(k)}-\alpha_{ji^{(k)}}^{(k)}s_k)]} \tag{1.11}$$

Suppose , as usual , that the parameters

$u_j, j = 1,\cdots,P; v_j, j = 1,\cdots,Q;$

$a_j^{(k)}, j=1,\cdots,N_k; a_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,P_{i^{(k)}};$

$b_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,Q_{i^{(k)}}; b_j^{(k)}, j=1,\cdots,M_k;$

with $k=1\cdots,s, i=1,\cdots,r'\ , i^{(k)}=1,\cdots,r^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}+\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j^{(k)}+\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}-\iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)}-\sum_{j=1}^{M_k}\beta_j^{(k)}$$

$$-\iota_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}\leqslant 0 \tag{1.12}$$

The reals numbers τ_i are positives for $i=1,\cdots,r$, $\iota_{i^{(k)}}$ are positives for $i^{(k)}=1\cdots r^{(k)}$

The contour L_k is in the t_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j^{(k)}-\beta_j^{(k)}t_k)$ with $j=1$ to M_k are separated from those of $\Gamma(1-u_j+\sum_{i=1}^{s}\mu_j^{(k)}t_k)$with $j=1$ to N and $\Gamma(1-a_j^{(k)}+\alpha_j^{(k)}t_k)$ with $j=1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}B_i^{(k)}\pi$, where

$$B_i^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}-\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}-\iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j^{(k)}-\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{M_k}\beta_j^{(k)}-\iota_{i^{(k)}}\sum_{j=M_k+1}^{q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}>0,\quad \text{with } k=1\cdots,s, i=1,\cdots,r\ , i^{(k)}=1,\cdots,r^{(k)} \tag{1.13}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_s)=0(\,|z_1|^{\alpha_1'}\dots|z_s|^{\alpha_s'}\,), max(\,|z_1|\dots|z_s|\,)\rightarrow 0$

$\aleph(z_1,\cdots,z_s)\ =0(\,|z_1|^{\beta_1'}\dots|z_s|^{\beta_s'}\,), min(\,|z_1|\dots|z_s|\,)\rightarrow\infty$

where, with $k=1,\cdots,z:\alpha_k'=min[Re(b_j^{(k)}/\beta_j^{(k)})], j=1,\cdots,M_k$ and

$\beta_k'=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})], j=1,\cdots,N_k$

We will use these following notations in this paper

$$U = P_i, Q_i, \iota_i; r' ; \; V = M_1, N_1; \cdots ; M_s, N_s \tag{1.15}$$

$$\mathrm{W} = P_{i^{(1)}}, Q_{i^{(1)}}, \iota_{i(1)}; r^{(1)}, \cdots , P_{i^{(r)}}, Q_{i^{(r)}}, \iota_{i(s)}; r^{(s)} \tag{1.16}$$

$$A = \{(u_j; \mu_j^{(1)}, \cdots , \mu_j^{(s)})_{1,N}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots , \mu_{ji}^{(s)})_{N+1,P_i}\} \tag{1.17}$$

$$B = \{\iota_i(v_{ji}; v_{ji}^{(1)}, \cdots , v_{ji}^{(s)})_{M+1,Q_i}\} \tag{1.18}$$

$$C = (a_j^{(1)}; \alpha_j^{(1)})_{1,N_1} , \iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_{i^{(1)}}}, \cdots , (a_j^{(s)}; \alpha_j^{(s)})_{1,N_s} , \iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_{i^{(s)}}} \tag{1.19}$$

$$D = (b_j^{(1)}; \beta_j^{(1)})_{1,M_1}, \iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_{i^{(1)}}}, \cdots , (b_j^{(s)}; \beta_j^{(s)})_{1,M_s}, \iota_{i^{(s)}}(\beta_{ji^{(s)}}^{(s)}; \beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_{i^{(s)}}} \tag{1.20}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots , z_s) = \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A : C} \\ \cdot & \cdot\;\cdot\;\cdot \\ \cdot & \\ \mathrm{z}_s & \mathrm{B : D} \end{array}\right) \tag{1.21}$$

The generalized polynomials of multivariables defined by Srivastava [6], is given in the following manner :

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1, \cdots , y_u] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_u)_{\mathfrak{M}_u K_u}}{K_u!}$$

$$A[N_1, K_1; \cdots ; N_u, K_u] y_1^{K_1} \cdots y_u^{K_u} \tag{1.22}$$

Where $\mathfrak{M}_1, \cdots , \mathfrak{M}_u$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots ; N_u, K_u]$ are arbitrary constants, real or complex.

Srivastava and Garg introduced and defined a general class of multivariable polynomials [8] as follows

$$S_E^{F_1,\cdots,F_v}[z_1, \cdots , z_v] = \sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v \leqslant E} (-E)_{F_1L_1+\cdots+F_vL_v} B(E; L_1, \cdots , L_v) \frac{z_1^{L_1} \cdots z_v^{L_v}}{L_1! \cdots L_v!} \tag{1.23}$$

The M-serie is defined, see Sharma [4].

$${}_{p'}M_{q'}^{\alpha}(y) = \sum_{s'=0}^{\infty} \frac{[(a_{p'})]_{s'}}{[(b_{q'})]_{s'}} \frac{y^{s'}}{\Gamma(\alpha s' + 1)} \tag{1.24}$$

Here $\alpha \in \mathbb{C}, Re(\alpha) > 0$. $[(a_{p'})]_{s'} = (a_1)_{s'} \cdots (a_{p'})_{s'}$; $[(b_{q'})]_{s'} = (b_1)_{s'} \cdots (b_{q'})_{s'}$.
The serie (1.23) converge if $p' \leqslant q' and\, |y| < 1$.

In the document , we note :

$$G(\eta_{G_1,g_1}, \cdots , \eta_{G_r,g_r}) = \phi(\eta_{G_1,g_1}, \cdots , \eta_{G_r,g_r})\theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) \tag{1.25}$$

$$a = \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_u)_{\mathfrak{M}_u K_u}}{K_u!} A[N_1, K_1; \cdots ; N_u, K_u] \tag{1.26}$$

$$b = \frac{(-E)_{F_1L_1+\cdots+F_vL_v}B(E;L_1,\cdots,L_v)}{L_1!\cdots L_v!} \tag{1.27}$$

$$g(x) = \frac{x}{p+qx+sx^2} \tag{1.28}$$

2. Main result

We shall establish the following result :

$$\int_0^\infty x^{1-\beta}(p+qx+sx^2)^{\beta-3/2}\, S_E^{F_1,\cdots,F_v}[z_1g(x)^{n'_1},\cdots,z_vg(x)^{n'_v}]_{p'}M_{q'}^{\alpha}(\tau g(x)^l)$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1g(x)^{n_1},\cdots,y_ug(x)^{n_u}]\,\aleph(x'_1g(x)^{\sigma'_1},\cdots,x'_rg(x)^{\sigma'_r})\aleph(x_1g(x)^{\sigma_1},\cdots,x_sg(x)^{\sigma_s})\mathrm{d}x$$

$$= \sqrt{\frac{\pi}{s}}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\sum_{L=0}^{\infty} ab\, G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\,\frac{[(a_{p'})]_L}{[(b_{q'})]_L}\,\frac{\tau^L}{\Gamma(\alpha L+1)}\,y_1^{K_1}\cdots y_u^{K_u}\,z_1^{L_1}\cdots z_v^{L_v}\,{x'_1}^{\eta_{G_1,g_1}}\cdots {x'_r}^{\eta_{G_r,g_r}}$$

$$(q+2\sqrt{sp})^{\beta-\sum_{i=1}^r\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^u n_iK_i-\sum_{i=1}^v n'_iL_i-Ll-1}$$

$$\aleph_{U_{11}:W}^{0,N+1:V}\left(\begin{array}{c}\mathrm{x}_1\\ \cdot\\ \cdot\\ \mathrm{x}_s\end{array}\middle|\begin{array}{c}(\beta-\sum_{i=1}^r\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^u K_in_i-\sum_{i=1}^v L_in'_i-Ll;\sigma_1,\cdots,\sigma_s),A:C\\ \cdots\\ (\beta-\sum_{i=1}^r\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^u K_in_i-\sum_{i=1}^v L_in'_i-Ll-1/2;\sigma_1,\cdots,\sigma_s),B:D\end{array}\right) \tag{2.1}$$

$$U_{11} = P_i+1, Q_i+1, \iota_i; r'$$

Provided :

a) $\sigma'_i > 0, i=1,\cdots,r\,;\ \sigma_i > 0, i=1,\cdots,s\ ; p' \leqslant q'\, and\, |\tau| < 1, Re(p) > 0, Re(q) > 0, s > 0$

b) $Re[\sum_{i=1}^r \sigma'_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^s \sigma_i \min_{1\leqslant j\leqslant M_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}] > \beta-2$

c) $|arg x_k| < \frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is given in (1.13), $k = 1\cdots,s$

Proof of (2.1)

To prove (2.1), first we express the Aleph-function of r variables, two general class of polynomials of several variables, the M-function in form of serie and the Aleph-function of s-variables in terms of Mellin-Barnes contour integrals. Now interchanging the order of summations and integration wich is possible under the stated conditions, we obtain.

$$\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\sum_{L=0}^{\infty}ab\,G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}$$

$$\frac{[(a_{p'})]_L}{[(b_{q'})]_L}\frac{\tau^L}{\Gamma(\alpha L+1)}y_1^{K_1}\cdots y_u^{K_u}\,z_1^{L_1}\cdots z_v^{L_v}\,{x'_1}^{\eta_{G_1,g_1}}\cdots{x'_r}^{\eta_{G_r,g_r}}\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_r}\zeta(t_1,\cdots,t_s)$$

$$\prod_{k=1}^{s}\phi_k(t_k)x_k^{t_k}\left[\int_0^{\infty}x^{1-(\beta-\sum_{i=1}^{r}\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^{u}n_iK_i-\sum_{i=1}^{v}n'_iL_i-\sum_{i=1}^{s}\sigma_it_i-Ll)}\right.$$

$$\left.(p+qx+sx^2)^{\beta-\sum_{i=1}^{r}\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^{u}n_iK_i-\sum_{i=1}^{v}n'_iL_i-\sum_{i=1}^{s}\sigma_it_i-Ll-3/2}\,\mathrm{d}x\right]\mathrm{d}t_1\cdots\mathrm{d}t_s$$

On solving above x-integral with the help of known theorem, see Saxena [1] and reinterpreting the result obtained in terms of Aleph-function of s-variables, we get the desired result.

3. Particular cases

a) If $p_i = q_i = n = 0$ and $P_i = Q_i = N = 0$ then the Aleph-function of r variables degenere to product of r Aleph-functions of one variable and the Aleph-function of s variables degenere to product of s Aleph-functions of one variable.

$$\int_0^{\infty}x^{1-\beta}(p+qx+sx^2)^{\beta-3/2}\,S_E^{F_1,\cdots,F_v}[z_1g(x)^{n'_1},\cdots,z_vg(x)^{n'_v}]_{p'}M_{q'}^{\alpha}(\tau g(x)^l)$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1g(x)^{n_1},\cdots,y_ug(x)^{n_u}]\prod_{a=1}^{r}\aleph_{p_{i(a)},q_{i(a)},\tau_{i(a)};R^{(a)}}^{m_a,n_a}(x'_ag(x)^{\sigma'_a})$$

$$\prod_{b=1}^{s}\aleph_{P_{i(b)},Q_{i(b)},\iota_{i(b)};r^{(b)}}^{M_b,N_b}(x_ag(x)^{\sigma_a})\mathrm{d}t$$

$$=\sqrt{\frac{\pi}{s}}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\sum_{L=0}^{\infty}ab\,G'(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\frac{[(a_{p'})]_L}{[(b_{q'})]_L}\frac{\tau^L}{\Gamma(\alpha L+1)}y_1^{K_1}\cdots y_u^{K_u}\,z_1^{L_1}\cdots z_v^{L_v}\,{x'_1}^{\eta_{G_1,g_1}}\cdots{x'_r}^{\eta_{G_r,g_r}}$$

$$(q+2\sqrt{sp})^{\beta-\sum_{i=1}^{r}\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^{u}n_iK_i-\sum_{i=1}^{v}n'_iL_i-Ll-1}$$

$$\aleph_{1,1:W}^{0,1:V}\left(\begin{array}{c}\mathrm{x}_1\\ \cdot\\ \cdot\\ \mathrm{x}_s\end{array}\middle|\begin{array}{c}(\beta-\sum_{i=1}^{r}\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^{u}K_in_i-\sum_{i=1}^{v}L_in'_i-Ll;\sigma_1,\cdots,\sigma_s):C\\ \cdots\\ (\beta-\sum_{i=1}^{r}\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^{u}K_in_i-\sum_{i=1}^{v}L_in'_i-Ll-1/2;\sigma_1,\cdots,\sigma_s):D\end{array}\right)\quad(3.1)$$

where $G'(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})=\theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r}),\theta_i(.),i=1,\cdots,r$ are given in (1.2)

b) If $\iota_i=\iota_{i^{(1)}}=\cdots=\iota_{i^{(s)}}=1$ and $r=r^{(1)}=\cdots=r^{(s)}=1$, then the multivariable Aleph-function degenere to the multivariable H-function defined by Srivastava et al [9]. And we have the following result.

$$\int_0^\infty x^{1-\beta}(p+qx+sx^2)^{\beta-3/2}\, S_E^{F_1,\cdots,F_v}[z_1g(x)^{n'_1},\cdots,z_vg(x)^{n'_v}]_{p'}M_{q'}^{\alpha}(\tau g(x)^l)$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1g(x)^{n_1},\cdots,y_ug(x)^{n_u}]\,\aleph(x'_1g(x)^{\sigma'_1},\cdots,x'_rg(x)^{\sigma'_r})H(x_1g(x)^{\sigma_1},\cdots,x_sg(x)^{\sigma_s})\mathrm{d}x$$

$$=\sqrt{\frac{\pi}{s}}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\sum_{L=0}^{\infty} ab\,G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\frac{[(a_{p'})]_L}{[(b_{q'})]_L}\frac{\tau^L}{\Gamma(\alpha L+1)}y_1^{K_1}\cdots y_u^{K_u}\,x_1^{L_1}\cdots x_v^{L_v}\,{x'_1}^{\eta_{G_1,g_1}}\cdots {x'_r}^{\eta_{G_r,g_r}}$$

$$(q+2\sqrt{sp})^{\beta-\sum_{i=1}^r\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^u n_iK_i-\sum_{i=1}^v n'_iL_i-Ll-1}$$

$$H_{P+1,Q+1:W}^{0,N+1:V}\left(\begin{array}{c}\mathrm{x}_1\\ \cdot\\ \cdot\\ \mathrm{x}_s\end{array}\middle|\begin{array}{c}(\beta-\sum_{i=1}^r\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^u K_in_i-\sum_{i=1}^v L_in'_i-Ll;\sigma_1,\cdots,\sigma_s),A':C'\\ \cdots\\ (\beta-\sum_{i=1}^r\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^u K_in_i-\sum_{i=1}^v L_in'_i-Ll-1/2;\sigma_1,\cdots,\sigma_s),B':D'\end{array}\right)\quad(3.2)$$

Provided :

a) $\sigma'_i>0, i=1,\cdots,r\ ;\ \sigma_i>0, i=1,\cdots,s\ ; p'\leqslant q' and\, |\tau|<1, Re(p)>0, Re(q)>0, s>0$

b) $Re[\sum_{i=1}^r\sigma'_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^s\sigma_i\min_{1\leqslant j\leqslant M_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>\beta-2$

c) $|argx_k|<\frac{1}{2}B_i\pi\ ,\ k=1,\cdots,s$

where $B_i=\sum_{j=1}^N\mu_j^{(i)}-\sum_{j=N+1}^P\mu_j^{(i)}-\sum_{j=1}^Q\upsilon_j^{(i)}+\sum_{j=1}^{N_i}\alpha_j^{(i)}-\sum_{j=N_i+1}^{P_i}\alpha_j^{(i)}+\sum_{j=1}^{M_i}\beta_j^{(i)}-\sum_{j=M_i+1}^{Q_i}\beta_j^{(i)}>0$

c) If $r=s=2$, we obtain two Aleph-functions of two variables defined by K. Sharma [3]

$$\int_0^\infty x^{1-\beta}(p+qx+sx^2)^{\beta-3/2}\, S_E^{F_1,\cdots,F_v}[z_1g(x)^{n'_1},\cdots,z_vg(x)^{n'_v}]_{p'}M_{q'}^{\alpha}(\tau g(x)^l)$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1g(x)^{n_1},\cdots,y_ug(x)^{n_u}]\ \aleph(x'_1g(x)^{\sigma'_1},x'_2g(x)^{\sigma'_2})\aleph(x_1g(x)^{\sigma_1},x_2g(x)^{\sigma_2})\mathrm{d}x$$

$$=\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{m_1}\sum_{g_2=0}^{m_2}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\,G(\eta_{G_1,g_1},\eta_{G_2,g_2})\frac{[(a_{p'})]_L}{[(b_{q'})]_L}\frac{\tau^L}{\Gamma(\alpha L+1)}$$

$$y_1^{K_1}\cdots y_u^{K_u}z_1^{L_1}\cdots z_v^{L_v}{x'_1}^{\eta_{G_1,g_1}}{x'_2}^{\eta_{G_2,g_2}}\frac{(-)^{G_1+G_2}}{\delta_{g_1}G_1!\delta_{g_2}G_2!}$$

$$(q+2\sqrt{sp})^{\beta-\sum_{i=1}^r\sigma'_i\eta_{G_i,g_i}-\sum_{i=1}^u n_iK_i-\sum_{i=1}^v n'_iL_i-Ll-1}$$

$$\aleph_{U_{11}:W}^{0,N+1:V}\left(\begin{array}{c|c} \mathrm{x}_1 & (\beta-\sum_{i=1}^{2}\sigma_i'\eta_{G_i,g_i}-\sum_{i=1}^{u}K_in_i-\sum_{i=1}^{v}L_in_i'-Ll;\sigma_1,\sigma_2),A:C \\ \cdot & \cdots \\ \mathrm{x}_2 & (\beta-\sum_{i=1}^{2}\sigma_i'\eta_{G_i,g_i}-\sum_{i=1}^{u}K_in_i-\sum_{i=1}^{v}L_in_i'-Ll-1/2;\sigma_1,\sigma_2),B:D \end{array}\right) \quad (3.3)$$

Where $G(\eta_{G_1,g_1},\eta_{G_2,g_2})=\phi(\eta_{G_1,g_1},\eta_{G_2,g_2})\theta_1(\eta_{G_1,g_1})\,\theta_2(\eta_{G_2,g_2})$and $U_{21}=P_i+2,Q_i+1,\iota_i;r'$

Provided

a) $\sigma_i'>0,i=1,2\,;\;\sigma_i>0,i=1,2\;;p'\leqslant q' and\,|\tau|<1,Re(p)>0,Re(q)>0,s>0$

b) $Re[\sum_{i=1}^{2}\sigma_i'\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^{2}\sigma_i\min_{1\leqslant j\leqslant M_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>\beta-2$

c) $|arg(x_1)|<A_1\frac{\pi}{2}$ and $|arg(x_2)|<A_2\frac{\pi}{2}\;;i=1,2\,;i'=1,2\,;i''=1,2,$ where :

$$A_1=\iota_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(1)}-\iota_i\sum_{j=1}^{Q_i}\beta_{ji}^{(1)}+\sum_{j=1}^{M_1}\beta_j-\iota_{i'}\sum_{j=M_1+1}^{Q_{i'}^{(1)}}\beta_{ji'}+\sum_{j=1}^{N_1}\alpha_j-\iota_{i'}\sum_{j=N_1+1}^{P_{i''}^{(1)}}\alpha_{ji'}>0$$

$$A_2=\iota_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(1)}-\iota_i\sum_{j=1}^{Q_i}\beta_{ji}^{(2)}+\sum_{j=1}^{M_1}\delta_j-\iota_{i''}\sum_{j=M_2+1}^{Q_{i''}^{(2)}}\delta_{ji''}+\sum_{j=1}^{N_2}\gamma_j-\iota_{i''}\sum_{j=N_2+1}^{P_{i''}^{(2)}}\gamma_{ji''}>0$$

d) If $r=s=1$, we obtain two Aleph-functions of one variable defined by Südland [10].

$$\int_0^\infty x^{1-\beta}(p+qx+sx^2)^{\beta-3/2}\,S_E^{F_1,\cdots,F_v}[z_1g(x)^{n_1'},\cdots,z_vg(x)^{n_v'}]_{p'}M_{q'}^{\alpha}(\tau g(x)^l)$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1g(x)^{n_1},\cdots,y_ug(x)^{n_u}]\;\aleph(y'g(x)^{\sigma'})\aleph(yg(x)^{\sigma})\mathrm{d}x$$

$$=\sum_{G=1}^{m}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}ab\,G(\eta_{G,g})\frac{[(a_{p'})]_L}{[(b_{q'})]_L}\frac{\tau^L}{\Gamma(\alpha L+1)}y'^{\eta_{G,g}}y_1^{K_1}\cdots y_u^{K_u}$$

$$z_1^{L_1}\cdots z_v^{L_v}\,(q+2\sqrt{sp})^{\beta-\sum_{i=1}^{r}\sigma_i'\eta_{G_i,g_i}-\sum_{i=1}^{u}n_iK_i-\sum_{i=1}^{v}n_i'L_i-Ll-1}$$

$$\aleph_{P_i+1,Q_i+1,c_i;r}^{M,N+1}\left(z\,\middle|\,\begin{array}{c}(\beta-\sigma'\eta_{G,g}-\sum_{i=1}^{u}K_in_i-\sum_{i=1}^{v}L_in_i'-Ll;\sigma),\\ \cdots\\ (\beta-\sigma'\eta_{G,g}-\sum_{i=1}^{u}K_in_i-\sum_{i=1}^{v}L_in_i'-Ll-1/2;\sigma),\end{array}\right.$$

$$\left.\begin{array}{c}(\,\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\ \cdots\\ (\,\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{array}\right) \quad (3.4)$$

Where $G(\eta_{G,g})=\frac{(-)^G\Omega_{P_i,Q_i,c_i,r}^{M,N}(s)}{B_gG!}$, $\Omega_{P_i,Q_i,c_i,r}^{M,N}(s)$ is defined by Südland [10]

Provided :

a) $\sigma' > 0, ;\ \sigma > 0,\ ; p' \leqslant q' and |\tau| < 1, Re(p) > 0, Re(q) > 0, s > 0$

b) $Re[\sigma' \min_{1 \leqslant j \leqslant m} \frac{d_j}{\delta_j} + \sigma_i \min_{1 \leqslant j \leqslant M} \frac{b_j}{\beta_j}] > \beta - 2$

c) $|argx| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^{M} \beta_j + \sum_{j=1}^{N} \alpha_j - c_i(\sum_{j=M+1}^{Q_i} \beta_{ji} + \sum_{j=N+1}^{P_i} \alpha_{ji}) > 0$

e) If $B(E; L_1, \cdots, L_v) = \frac{\prod_{j=1}^{A}(a_j)_{L_1\theta'_j+\cdots+L_v\theta_j^{(v)}} \prod_{j=1}^{B'}(b'_j)_{L_1\phi'_j} \cdots \prod_{j=1}^{B^{(v)}}(b_j^{(v)})_{L_v\phi_j^{(v)}}}{\prod_{j=1}^{C}(c_j)_{m_1\psi'_j+\cdots+m_v\psi_j^{(v)}} \prod_{j=1}^{D'}(d'_j)_{L_1\delta'_j} \cdots \prod_{j=1}^{D^{(v)}}(d_j^{(v)})_{L_r\delta_j^{(v)}}}$ (3.5)

then the general class of multivariable polynomial $S_E^{F_1,\cdots,F_v}[z_1, \cdots, z_v]$ reduces to generalized Lauricella function defined by Srivastava et al [7].

$$F_{C:D';\cdots;D^{(v)}}^{1+A:B';\cdots;B^{(v)}}\left(\begin{array}{c} z_1 \\ \cdots \\ z_v \end{array}\middle| \begin{array}{c} [(\text{-E})\text{: } F_1, \cdots, F_v], [(a) : \theta', \cdots, \theta^{(v)}]; [(b') : \phi']; \cdots ; [(b^{(v)}) : \phi^{(v)}] \\ [(\text{c})\text{: } \psi', \cdots, \psi^{(v)}]; [(d') : \delta']; \cdots ; [(b)^{(v)} : \delta^{(v)}] \end{array}\right) \quad (3.6)$$

We have the following result.

$$\int_0^\infty x^{1-\beta}(p+qx+sx^2)^{\beta-3/2}\ {}_{p'}M_{q'}^{\alpha}(\tau g(x)^l) S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1 g(x)^{n_1}, \cdots, y_u g(x)^{n_u}]$$

$$F_{C:D';\cdots;D^{(v)}}^{1+A:B';\cdots;B^{(v)}}\left(\begin{array}{c} z_1 g(x)^{n'_1} \\ \cdots \\ z_v g(x)^{n'_v} \end{array}\middle| \begin{array}{c} [(\text{-E})\text{: } F_1, \cdots, F_v], [(a) : \theta', \cdots, \theta^{(v)}]; [(b') : \phi']; \cdots ; [(b^{(v)}) : \phi^{(v)}] \\ [(\text{c})\text{: } \psi', \cdots, \psi^{(v)}]; [(d') : \delta']; \cdots ; [(b)^{(v)} : \delta^{(v)}] \end{array}\right)$$

$$\aleph(x'_1 g(x)^{\sigma'_1}, \cdots, x'_r g(x)^{\sigma'_r}) \aleph(x_1 g(x)^{\sigma_1}, \cdots, x_s g(x)^{\sigma_s}) \mathrm{d}x$$

$$= \sqrt{\frac{\pi}{s}} \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v \leqslant E} \sum_{L=0}^{\infty} a\, G(\eta_{G_1,g_1}, \cdots \eta_{G_r,g_r})$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!} \frac{[(a_{p'})]_L}{[(b_{q'})]_L} \frac{\tau^L}{\Gamma(\alpha L+1)} (-E)_{F_1L_l+\cdots+F_vL_v} B(E; L_1, \cdots, L_v)\, y_1^{K_1} \cdots y_u^{K_u} \frac{z_1^{L_1} \cdots z_v^{L_v}}{L_1! \cdots L_v!}$$

$$x_1'^{\eta_{G_1,g_1}} \cdots x_r'^{\eta_{G_r,g_r}} \quad (q+2\sqrt{sp})^{\beta-\sum_{i=1}^r \sigma'_i\eta_{G_i,g_i} - \sum_{i=1}^u n_iK_i - \sum_{i=1}^v n'_iL_i - Ll - 1}$$

$$\aleph_{U_{11}:W}^{0,N+1:V}\left(\begin{array}{c} \mathrm{x}_1 \\ \cdot \\ \cdot \\ \mathrm{x}_s \end{array}\middle| \begin{array}{c} (\beta - \sum_{i=1}^r \sigma'_i\eta_{G_i,g_i} - \sum_{i=1}^u K_in_i - \sum_{i=1}^v L_in'_i - Ll; \sigma_1, \cdots, \sigma_s), A : C \\ \cdots \\ (\beta - \sum_{i=1}^r \sigma'_i\eta_{G_i,g_i} - \sum_{i=1}^u K_in_i - \sum_{i=1}^v L_in'_i - Ll - 1/2; \sigma_1, \cdots, \sigma_s), B : D \end{array}\right) \quad (3.7)$$

$U_{11} = P_i + 1, Q_i + 1, \iota_i; r', B(E; L_1, \cdots, L_v)$ is defined by (3.5)

Provided :

a) $\sigma'_i > 0, i = 1, \cdots, r\, ;\ \sigma_i > 0, i = 1, \cdots, s\ ; p' \leqslant q' and\, |\tau| < 1, Re(p) > 0, Re(q) > 0, s > 0$

b) $Re[\sum_{i=1}^{r} \sigma'_i \min_{1 \leqslant j \leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^{s} \sigma_i \min_{1 \leqslant j \leqslant M_i} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > \beta - 2$

c) $|arg x_k| < \frac{1}{2} B_i^{(k)} \pi$, where $B_i^{(k)}$ is given in (1.13), $k = 1 \cdots, s$

f) If $y_2 = \cdots = y_u = 0$, then the class of polynomials $S_{N_1, \cdots, N_u}^{M_1, \cdots, M_u}(y_1, \cdots, y_u)$ defined of (1.14) degenere to the class of polynomials $S_N^M(y)$ defined by Srivastava [5] and we have.

$$\int_0^\infty x^{1-\beta}(p + qx + sx^2)^{\beta - 3/2}\, S_E^{F_1, \cdots, F_v}[z_1 g(x)^{n'_1}, \cdots, z_v g(x)^{n'_v}]\, S_N^M[yg(x)^n]\, {}_{p'}M_{q'}^{\alpha}(\tau g(x)^l)$$

$$\aleph(x'_1 g(x)^{\sigma'_1}, \cdots, x'_r g(x)^{\sigma'_r}) \aleph(x_1 g(x)^{\sigma_1}, \cdots, x_s g(x)^{\sigma_s}) \mathrm{d}x$$

$$= \sqrt{\frac{\pi}{s}} \sum_{G_1, \cdots, G_r = 0}^{\infty} \sum_{g_1 = 0}^{m_1} \cdots \sum_{g_r = 0}^{m_r} \sum_{K=0}^{[N/M]} \sum_{L_1, \cdots, L_v = 0}^{F_1 L_1 + \cdots F_v L_v \leqslant E} \sum_{L=0}^{\infty} a' b\, G(\eta_{G_1, g_1}, \cdots \eta_{G_r, g_r})$$

$$\frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} \frac{[(a_{p'})]_L}{[(b_{q'})]_L} \frac{\tau^L}{\Gamma(\alpha L + 1)} y^K\, z_1^{L_1} \cdots z_v^{L_v}\, x_1'^{\,\eta_{G_1, g_1}} \cdots x_r'^{\,\eta_{G_r, g_r}}$$

$$(q + 2\sqrt{sp})^{\beta - \sum_{i=1}^{r} \sigma'_i \eta_{G_i, g_i} - \sum_{i=1}^{u} n_i K_i - \sum_{i=1}^{v} n'_i L_i - Ll - 1}$$

$$\aleph_{U_{11}:W}^{0, N+1:V} \left(\begin{array}{c} \mathrm{x}_1 \\ \cdot \\ \cdot \\ \mathrm{x}_s \end{array} \middle| \begin{array}{c} (\beta - \sum_{i=1}^{r} \sigma'_i \eta_{G_i, g_i} - Kn - \sum_{i=1}^{v} L_i n'_i - Ll; \sigma_1, \cdots, \sigma_s), A : C \\ \cdots \\ (\beta - \sum_{i=1}^{r} \sigma'_i \eta_{G_i, g_i} - Kn - \sum_{i=1}^{v} L_i n'_i - Ll - 1/2; \sigma_1, \cdots, \sigma_s), B : D \end{array} \right) \quad (3.8)$$

$U_{11} = P_i + 1, Q_i + 1, \iota_i; r'$

Provided :

a) $\sigma'_i > 0, i = 1, \cdots, r\, ;\ \sigma_i > 0, i = 1, \cdots, s\ ; p' \leqslant q' and\, |\tau| < 1, Re(p) > 0, Re(q) > 0, s > 0$

b) $Re[\sum_{i=1}^{r} \sigma'_i \min_{1 \leqslant j \leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^{s} \sigma_i \min_{1 \leqslant j \leqslant M_i} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > \beta - 2$

c) $|arg x_k| < \frac{1}{2} B_i^{(k)} \pi$, where $B_i^{(k)}$ is given in (1.13), $k = 1 \cdots, s$

4. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as , multivariable H-function , defined by Srivastava et al [9] , the Aleph-function of two variables defined by K.sharma [3].

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