Fuzzy Semi-Super modular Lattice

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Abstract *In this Paper, Fuzzy Semi-Supermodular Lattice – Definition of Fuzzy Semi-Supermodular Lattice - Characterization theorem are given.*

Keywords— *Fuzzy Modular Lattice, Fuzzy Distributive Lattice, Fuzzy Supermodular Lattice, Fuzzy Semi-Supermodular Lattice.*

I. Introduction

The Concept of Fuzzy Lattice was already introduced by Ajmal,N[1], S.Nanda[4] and WilCox,L.R [2] explained modularity in the theory of Lattices, Iqbalunnisa and Vasantha,W.B,[6] explained by Supermodular Lattices, G.Gratzer[3], M.Mullai and B.Chellappa[5] explained Fuzzy L-ideal, V.Vinoba and K.Nithya[8] Explained fuzzy modular pairs in Fuzzy Lattice and Fuzzy Modular Lattice and Iqbalunnisa and Akilandam[7] explained by Semi- Supermodular Lattices, A few of definitions and results are listed that the fuzzy Semi-Supermodular lattice using in this paper we explain fuzzy Semi-Supermodular lattice, Definition of fuzzy Semi-Supermodular lattice, Characterization theorem of Fuzzy Semi-Supermodular Lattice and some examples are given.

Definition: 1.1

A Fuzzy modular lattice L is called a Fuzzy Semi-Supermodular lattice if it satisfies a Fuzzy Semi-Supermodular identity namely.

$(\mu(x)+\mu(a))(\mu(x)+\mu(b))(\mu(x)+\mu(c))$
$[\mu(x)+(\mu(a)+\mu(b))(\mu(b)+\mu(c))$
$(\mu(c)+\mu(a))]$
$=(\mu(x)+\mu(a))(\mu(x)+\mu(b))(\mu(x)+\mu(c))$
$[\mu(x)+(\mu(a)\mu(b)+\mu(c)(\mu(a)+\mu(b))]$
$[\mu(x)+\mu(b)\mu(c)+\mu(a)(\mu(b)+\mu(c))]$
$[\mu(x)+\mu(c)\mu(a)+\mu(b)(\mu(c)+\mu(a))]$,
for all $\mu(x), \mu(a), \mu(b), \mu(c)$ in L .

In the following proposition we denote
$(\mu(x)+\mu(a))(\mu(x)+\mu(b))(\mu(x)+\mu(c))$
$[\mu(x)+(\mu(a)+\mu(b))(\mu(b)+\mu(c))$
$(\mu(c)+\mu(a))]$ as L.H.S of Fuzzy semi-supermodular identity and
$(\mu(x)+\mu(a))(\mu(x)+\mu(b))(\mu(x)+\mu(c))$
$[\mu(x)+(\mu(a)\mu(b)+\mu(c)(\mu(a)+\mu(b))]$
$[\mu(x)+\mu(b)\mu(c)+\mu(a)(\mu(b)+\mu(c))]$
$[\mu(x)+(\mu(c)\mu(a)+\mu(b)(\mu(c)+\mu(a))]$
as R.H.S of the semi-supermodular identity.

Theorem: 1.1

Every Fuzzy distributive lattice is Fuzzy Semi-Supermodular and the converse is not true.

Proof

Assume that L is Fuzzy distributive lattice. To Prove that L is Fuzzy semi-supermodular. We know that every Fuzzy distributive lattice is Fuzzy modular.
Hence it is sufficient to prove that the Fuzzy distributive lattice satisfy the Fuzzy Semi-Supermodular identity.

Now, $(\mu(x)+\mu(a))(\mu(x)+\mu(a))(\mu(x)+\mu(a))$
$[\mu(x)+(\mu(a)+\mu(b))(\mu(b)+\mu(c))$
$(\mu(c)+\mu(a))]$
$=(\mu(x)+\mu(a)\mu(b)\mu(c))[(\mu(x)+$
$(\mu(a)+\mu(b))(\mu(b)+\mu(c))$
$(\mu(c)+\mu(a))]$
$=\mu(x)+[\mu(a)\mu(b)\mu(c)][(\mu(a)+\mu(b))$
$(\mu(b)+\mu(c))(\mu(c)+\mu(a))]$
$=(\mu(x)+[\mu(a)(\mu(a)+\mu(b))][\mu(b)$
$(\mu(b)+\mu(c))][\mu(c)(\mu(c)+\mu(a))]$
$=\mu(x)+[\mu(a)\mu(b)\mu(c)] \longrightarrow (1)$

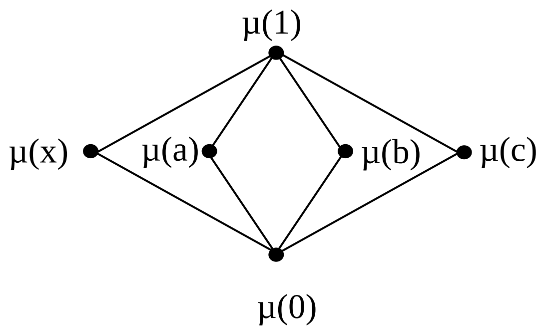
Also

$(\mu(x)+\mu(a))(\mu(x)+\mu(b))(\mu(x)+\mu(c))$
$[\mu(x)+\mu(a)\mu(b)+\mu(c)(\mu(a)+\mu(b)]$
$[(\mu(x)+\mu(b)\mu(c)+\mu(a)(\mu(b)+\mu(c))]$
$[\mu(x)+\mu(c)\mu(a)+\mu(b)(\mu(c)+\mu(a))]$
$=(\mu(x)+\mu(a)\mu(b)\mu(c))$
$[(\mu(x)+\mu(a)\mu(b)+\mu(c)(\mu(a)+\mu(b))$
$[\mu(x)+\mu(b)\mu(c)+\mu(a)(\mu(b)+\mu(c))]$
$[\mu(x)+\mu(c)\mu(a)+\mu(b)(\mu(c)+\mu(a))]$
$=\mu(x)+\mu(a)\mu(b)\mu(c)$
$[\mu(a)\mu(b)+\mu(c)(\mu(a)+\mu(b))]$
$[\mu(b)\mu(c)+\mu(a)(\mu(b)+\mu(c))]$
$[\mu(c)\mu(a)+\mu(b)(\mu(c)+\mu(a))]$
$=\mu(x)+\{\mu(a)\mu(b)\mu(c)$
$[(\mu(a)\mu(b)+\mu(c)(\mu(a)+\mu(b))]\}$
$[\mu(b)\mu(c)+\mu(a)(\mu(b)+\mu(c))]$
$[(\mu(c)\mu(a)+\mu(b)(\mu(c)+\mu(a))]$
$=\mu(x)+\mu(a)\mu(b)\mu(c)$
$[(\mu(b)\mu(c)+\mu(a)(\mu(b)+\mu(c))]$
$[\mu(c)\mu(a)+\mu(b)(\mu(c)+\mu(a))]$
Since $\mu(a)\mu(b)\mu(c) \le \mu(a)\mu(b)+\mu(c)$
$(\mu(a)+\mu(b))$

$=\mu(x)+\{\mu(a)\mu(b)\mu(c)((\mu(b)\mu(c)+\mu(a)$
$(\mu(b)+\mu(c))\}$

$[(\mu (c) \mu (a) + \mu(b) (\mu(c)+ \mu(a))]$
$= \mu (x) + \mu(a) \mu(b) \mu(c)$
$[(\mu (c) \mu (a) + \mu (b) (\mu (c) + \mu (a))]$
Since $\mu(a) \mu(b) \mu(c) \leq \mu(b) \mu(c) + \mu(a) (\mu(b)+\mu(c))$
$= \mu (x) + \mu(a) \mu(b) \mu(c) \longrightarrow (2)$

Since $\mu(a) \mu(b) \mu(c) \leq \mu(c) + \mu(b) (\mu(c) + \mu(a))$
From (1) & (2)
$(\mu(x) + \mu(a)) (\mu(x) + \mu(b)) (\mu(x) + \mu(c))$
$[\mu(x) + (\mu(a) + \mu(b)) (\mu (b) + \mu(c)) (\mu(c) + \mu(a))]$
$= (\mu (x) + \mu(a))(\mu (x) + \mu(b))(\mu (x) + \mu(c))$
$[\mu (x) + \mu(a) \mu (b) + \mu(c) (\mu (a) + \mu(b))]$
$[\mu (x) + \mu(b) \mu (c) + \mu(a)(\mu (b) + \mu(c))]$
$[\mu (x) + \mu(c) \mu (a) + \mu(b)(\mu (c) + \mu(b)],$
for all $\mu (x) , \mu(a), \mu (b) , \mu(c)$ in L.
Hence L is a Fuzzy semi-super modular lattice.
The Converse need not be true.
(i.e) Every Fuzzy semi-supermodular lattice is not necessarily Fuzzy distributive.
We shall verify it by the following example.
Consider the Fuzzy lattice M_4 of figure

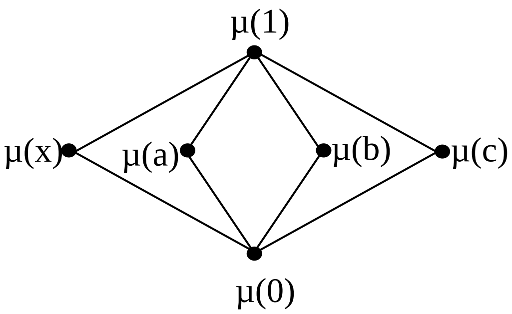


This Fuzzy lattice is Fuzzy semi- supermodular but not Fuzzy distributive because
$\mu (x) + \mu(a) \mu (b) \neq (\mu(x) + \mu(a)) (\mu (x) + \mu(b))$
Since $\mu (x) + \mu (a) \mu (b) = \mu (x) + \mu(0) = \mu (x)$ and $(\mu (x) + \mu (a))(\mu (x) + \mu (b)) = \mu (1) \mu (1)$
$= \mu (1)$

Theorem 7.2
Every fuzzy supermodular lattice is Fuzzy semi-supermodular and converse is not true.
Proof
Assume that L is a Fuzzy supermodular lattice. To prove that L is Fuzzy semi-supermodular. We know that every Fuzzy supermodular lattice is Fuzzy modular, hence it is sufficient to prove that L is satisfy the Fuzzy Semi-supermodular identity.

$(\mu(x) + \mu(a)) (\mu(x) + \mu(b))(\mu(x) + \mu(c))$
$[\mu (x) + (\mu(a) + \mu(b))(\mu(b) + \mu(c)) (\mu(c) + \mu(a))]$
$=[(\mu(x)+\mu(a))(\mu(x)+\mu(b))(\mu(x)+\mu(c))]$
$[\mu(x)+(\mu(a)\mu(b)+\mu(c)(\mu(a)+\mu(b))]$
$[\mu(x)+(\mu(b)\mu(c)+\mu(a) (\mu(b)+\mu(c))]$
$[\mu(x)+(\mu(c)\mu(a)+\mu(b)(\mu(c)+\mu(a))],$
for all $\mu(x),\mu(a),\mu(b),\mu(c)$ in L.

For $(\mu(x) + \mu(a)) (\mu(x) + \mu(b))(\mu(x) + \mu(c))$
$\geq((\mu(x)+\mu(a))(\mu(x)+\mu(b))(\mu(x)+\mu(c))$
$[(\mu(x)+(\mu(a)+(\mu(b))(\mu(b)+\mu(c))(\mu(c)+\mu(a))]$
$\geq((\mu(x)+\mu(a))(\mu(x)+\mu(b))(\mu(x)+\mu(c))$
$[(\mu(x)+\mu(a)\mu(b)+\mu(c)(\mu(a)+\mu(b))]$
Since $\mu(a) \leq \mu(a) + \mu(b), \mu(c) + \mu(a); \mu(b) \leq \mu(b) + \mu(c)$
$\Rightarrow \mu(a) \leq (\mu(a) + \mu(b))(\mu(c) + \mu(a)); \mu(b) \leq \mu(b) + \mu(c)$
$\Rightarrow \mu(a) \mu(b) \geq(\mu(a)+ \mu(b))(\mu(b)+\mu(c)(\mu(c)+\mu(a))$
and $(\mu(b)+\mu(c))(\mu(c)+\mu(a)) \geq \mu(c)$
$\Rightarrow(\mu(a) + \mu(b)) (\mu(b) + \mu(c))(\mu(c) + \mu(a))$
$\geq \mu(c)(\mu(a) + \mu(b))$
$\Rightarrow(\mu(a) + \mu(b)) (\mu(b) + \mu(c))(\mu(c) + \mu(a))$
$\geq \mu(a) \mu(b)+ \mu(c) (\mu(a) + \mu(b))$
Similarly we can prove that
$(\mu(x) + \mu(a)) (\mu(x) + \mu(b))(\mu(x) + \mu(c))$
$\geq (\mu(x) + \mu(a)) (\mu(x) + \mu(b)) (\mu(x) + \mu(c))$
$(\mu(x)+\mu(b)\mu(c)+\mu(a)(\mu(b)+(\mu(c)]$ and
$(\mu(x) + \mu(a)) (\mu(x) + \mu(b))(\mu(x) + \mu(c))$
$\geq (\mu(x) + \mu(a)) (\mu(x) + \mu(b))(\mu(x) + \mu(c))$
$[\mu(x)+(\mu(a)+\mu(b))(\mu(b)+(\mu(c))(\mu(c)+\mu(a))]$
$\geq (\mu(x) + \mu(a)) (\mu(x) + \mu(b))(\mu(x) + \mu(c))$
$[\mu(x)+(\mu(a)\mu(b) + \mu(c)(\mu(a)+\mu(b))]$
$[\mu(x)+(\mu(b)\mu(c) + \mu(a)(\mu(b)+\mu(c))]$
$[\mu(x)+(\mu(c)\mu(a)+\mu(b)(\mu(c)+\mu(a))]$
$\geq (\mu(x), \mu(a) \mu(b)(\mu(x)+\mu(c)), \mu(b)\mu(c) (\mu(x)+\mu(a)), \mu(c)\mu(a)(\mu(x)+ \mu(b))$
$\geq (\mu(x)+ \mu(a) \mu(b)(\mu(x)+\mu(c))+ \mu(b)\mu(c) (\mu(x)+\mu(a)) + \mu(c) \mu(a)(\mu(x)+ \mu(b)) \longrightarrow (1)$
Now,
$(\mu(x) + \mu(a)) (\mu(x) + \mu(b)) (\mu(x) + \mu(c))$
$= \mu(x) + \mu(a) \mu(b) (\mu(x)+\mu(c)) + \mu(b)\mu(c) (\mu(x) + \mu(a))+ \mu(c)\mu(a) (\mu(x)+\mu(b))$
as L is Fuzzy supermodular so that(1) becomes
$(\mu(x) + \mu(a)) (\mu(x) + \mu(b))(\mu(x) + \mu(c))$
$[\mu(x)+(\mu(a)+\mu(b))(\mu(b)+\mu(c))(\mu(c)+\mu(a))]$
$= (\mu(x) + \mu(a)) (\mu(x) + \mu(b))(\mu(x) + \mu(c))$
$[\mu(x) + \mu(a) \mu(b)+ \mu(c) (\mu(a) + \mu(b))]$
$[\mu(x) + \mu(b) \mu(c) + \mu(a) (\mu(b) + \mu(c))]$
$[\mu(x) + \mu(c) \mu(a)+ \mu(b) (\mu(c) + \mu(a))],$
for all $\mu(x),\mu(a),\mu(b),\mu(c)$ in L.
Hence L is Fuzzy semi supermodular.
The converse need not be true.
(i.e) Every Fuzzy semi supermodular lattice is not necessarily super modular.
We shall verify it by the following example
consider Fuzzy lattice M_4 of figure



This Fuzzy lattice is Fuzzy semi-supermodular but not Fuzzy super modular, because

$$(\mu(x) + \mu(a))\ (\mu(x) + \mu(b))\ (\mu(x) + \mu(c)) \neq \mu(x) + \mu(a)\ \mu(b)\ (\mu(x) + \mu(c)) + \mu(b)\ \mu(c)\ (\mu(x) + \mu(a)) + \mu(c)\ \mu(a)\ (\mu(x) + \mu(b))$$

Since $(\mu(x) + \mu(a))\ (\mu(x) + \mu(b))(\ \mu(x) + \mu(c))$

$= \mu(1).\ \mu(1).\ \mu(1)$

$= \mu(1)$

and $\mu(x) + \mu(a)\ \mu(b)(\ \mu(x) + \mu(c))+ \mu(b)\ \mu(c)\ (\mu(x) + \mu(a))+ \mu(c)\ \mu(a)(\ \mu(x) + \mu(b))$

$= \mu(x)+ \mu(0)$

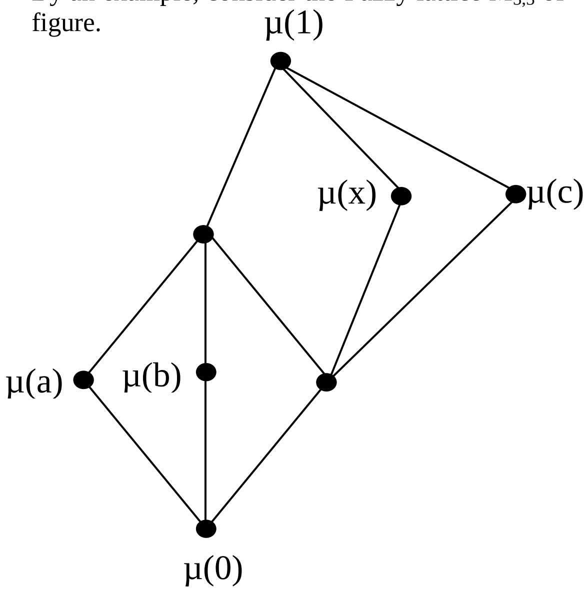
$= \mu(x)$

Theorem 1.3

Every Fuzzy modular lattice is not necessarily a fuzzy semi-supermodular lattice.

Proof

By an example, consider the Fuzzy lattice $M_{3,3}$ of figure.



$M_{3,3}$

This fuzzy lattice $M_{3,3}$ of figure in Fuzzy modular but not Fuzzy semi-supermodular.

For consider the set $\mu(x),\mu(a),\mu(b),\mu(c)$ of elements of $M_{3,3}$ as in the figure.

Then $(\mu(x) + \mu(a))\ (\mu(x) + \mu(b))(\ \mu(x) + \mu(c))$

$[\mu(x)+(\mu(a)+\mu(b))(\mu(b)+\mu(c))(\mu(c)+\mu(a))]$

$= \mu(1).\ \mu(1).\ \mu(1).\ \mu(1)$

$= \mu(1)$,

for all $\mu(x)$, $\mu(a)$, $\mu(b)$, $\mu(c)$ in L.

and

$(\mu(x) + \mu(a))\ (\mu(x) +\mu(b))\ (\mu(x) + \mu(c))$

$[\mu(x)+\mu(a)\mu(b)+\mu(c)(\mu(a)+\mu(b))]$

$[\mu(x) + \mu(b)\ \mu(c) + \mu(a)(\ \mu(b) + \mu(c))]$

$[\mu(x) + \mu(c)\ \mu(a) + \mu(b)(\ \mu(c) + \mu(a))]$

$= \mu(1).\ \mu(1).\ \mu(1)\ (\ \mu(x) + \mu(0) + \mu(c))$

$(\mu(x) + \mu(0) + \mu(a))\ (\mu(x) + \mu(0) + \mu(b))$

$= \mu(1).\ \mu(x).\ \mu(1)\ .\ \mu(1)$

$= \mu(x)$

Theorem: 1.4

The class of Fuzzy semi-supermodular lattice is a distinct equation class of Fuzzy lattice. Lying between the equation class of Fuzzy modular lattices, and the equation class of supermodular lattices.

Proof

Follows from Theorem 1.2 and Theorem 1.3

Conclusion:

This paper is proved that Every Fuzzy distributive lattice is Fuzzy Semi-Supermodular and the converse is not true, Every fuzzy supermodular lattice is Fuzzy semi-supermodular and converse is not true, Every Fuzzy modular lattice is not necessarily a fuzzy semi-supermodular lattice and The class of Fuzzy semi-supermodular lattice is a distinct equation class of Fuzzy lattice. Lying between the equation class of Fuzzy modular lattices, and the equation class of supermodular lattices.

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