Exponentiel Fourier series for the multivariable Aleph-function

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ABSTRACT
In this document, we established three general integrals and employ exponential Fourier series involving the multivariable Aleph-function and the generalized Lauricella function.

KEYWORDS : Aleph-function of several variables , generalized Lauricella function , expansion of exponentialFourier serie

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1. Introduction and preliminaries.

The object of this document is to evaluate three integrals involving the generalized Lauricella function defined by Srivastava and Daoust [7] and the multivariable Aleph-function and use them to establish three exponential form of Fourier series. These results yelds a number of new and known results including the results of Bajpai [1]. These function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [4] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array}\right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{\mathfrak{n}+1,p_i}]:$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ ,[\tau_i(b_{ji};\beta_j^{(1)},\cdots,\beta_j^{(r)})_{m+1,q_i}]:$

$[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]$

$[(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\Bigg)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\phi_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\phi_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$a_j, j = 1, \cdots, p; b_j, j = 1, \cdots, q;$

$c_j^{(k)}, j = 1, \cdots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R$, $i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1 \cdots, r, i = 1, \cdots, R \text{ , } i^{(k)} = 1, \cdots, R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1} \ldots |z_r|^{\alpha_r}), max(|z_1| \ldots |z_r|) \rightarrow 0$

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1} \ldots |z_r|^{\beta_r}), min(|z_1| \ldots |z_r|) \rightarrow \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \; ; \; V = m_1, n_1; \cdots ; m_r, n_r \tag{1.6}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i(1)}; R^{(1)}, \cdots , p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i(r)}; R^{(r)} \tag{1.7}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots , \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots , \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.8}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots , \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots , \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.10}$$

$$D = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots, \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots , z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A} : \mathrm{C} \\ \cdot & \cdot\ \cdot\ \cdot \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B} : \mathrm{D} \end{array}\right) \quad 1.12)$$

In the present paper, we will use the following results :

(i) The result given by MacRobert, T.M. [3,p.340 (95)]

$$\int_{-\pi/2}^{\pi/2} (cos\theta)^{M+N-2} e^{i\omega(M-N)\theta}\, \mathrm{d}\theta = \frac{\pi\Gamma(M+N-1)}{2^{M+N-2}\Gamma(M)\Gamma(N)}, Re(M+N) > 1 \tag{1.13}$$

(ii)The orthogonal property of exponential function [2, p.62]

$$\int_a^b exp(i\frac{2M\pi\omega x}{a-b})exp(i\frac{2N\pi\omega x}{a-b})\mathrm{d}x = b-a \text{ if } M = -N, 0 \text{ else} \tag{1.14}$$

We shall also use the short-hand notations as follows ;

Let $\mathrm{F}\left(\begin{array}{c} \mathrm{x}_1 \\ \cdot\ \cdot\ \cdot \\ \mathrm{x}_r \end{array}\right)$ denote the generalized Lauricella function of several complex variables defined by Srivastava et al [7].

We have. $$\mathrm{F}\left(\begin{array}{c} \mathrm{x}_1 \\ \cdot\ \cdot\ \cdot \\ \mathrm{x}_r \end{array}\right) = \sum_{m_1,\cdots,m_r=0}^{\infty} A(m_1, \cdots , m_r)\, \frac{x_1^{m_1} \cdots x_r^{m_r}}{m_1! \cdots m_r!} \tag{1.15}$$

Where : $$A(m_1, \cdots , m_r) = \frac{\prod_{j=1}^{A}(a_j)_{m_1\theta_j'+\cdots+m_r\theta_j^{(r)}} \prod_{j=1}^{B'}(b_j')_{m_1\phi_j'} \cdots \prod_{j=1}^{B^{(n)}}(b_j^{(r)})_{m_r\phi_j^{(r)}}}{\prod_{j=1}^{C}(c_j)_{m_1\epsilon_j'+\cdots+m_r\epsilon_j^{(r)}} \prod_{j=1}^{D'}(d_j')_{m_1\delta_j'} \cdots \prod_{j=1}^{D^{(r)}}(d_j^{(r)})_{m_r\delta_j^{(r)}}} \tag{1.16}$$

2. Integrals

The integrals to be established are :

(i) $$\int_{-\pi/2}^{\pi/2}(cos\theta)^{M+N-2}e^{i\omega(M-N)\theta}\,\mathrm{F}\begin{pmatrix}\mathrm{x}_1(e^{i\omega}cos\theta)^{k_1}\\ \cdots\\ \mathrm{x}_r(e^{i\omega}cos\theta)^{k_r}\end{pmatrix}\aleph^{0,\mathfrak{n}:V}_{U:W}\begin{pmatrix}\mathrm{z}_1(e^{i\omega}cos\theta)^{h_1}\\ \cdots\\ \mathrm{z}_r(e^{i\omega}cos\theta)^{h_r}\end{pmatrix}\mathrm{d}\theta$$

$$=\sum_{m_1,\cdots,m_r=0}^{\infty}\frac{\pi A(m_1,\cdots,m_r)}{2^{(M+N+\sum_{i=1}^r k_im_i-2)}\Gamma(N)}\aleph^{0,\mathfrak{n}+1:V}_{U_{11}:W}\left(\begin{matrix}\mathrm{z}_1 2^{-h_1}\\ \cdots\\ \mathrm{z}_r 2^{-h_r}\end{matrix}\ \middle|\ \begin{matrix}(2-M-N-\sum_{i=1}^r k_im_i;h_1,\cdots,h_r),A:C\\ \cdots\\ ((1-M-\sum_{i=1}^r k_im_i;h_1,\cdots,h_r),B:D\end{matrix}\right)\times\frac{x_1^{m_1}\cdots x_m^{m_r}}{m_1!\cdots m_r!} \tag{2.1}$$

Where : $U_{11}=p_i+1,q_i+1,\tau_i;R$

Provided that :

a) $Re[M+N+\sum_{i=1}^r(k_im_i+h_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>1\ ,\ j=1,\cdots,m_i$ (2.2)

b) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

(ii) $$\int_{-\pi/2}^{\pi/2}(cos\theta)^{M+N-2}e^{i\omega(M-N)\theta}\,\mathrm{F}\begin{pmatrix}\mathrm{x}_1(e^{-i\omega}cos\theta)^{k_1}\\ \cdots\\ \mathrm{x}_r(e^{-i\omega}cos\theta)^{k_r}\end{pmatrix}\aleph^{0,\mathfrak{n}:V}_{U:W}\begin{pmatrix}\mathrm{z}_1(e^{-i\omega}cos\theta)^{h_1}\\ \cdots\\ \mathrm{z}_r(e^{-i\omega}cos\theta)^{h_r}\end{pmatrix}\mathrm{d}\theta$$

$$=\sum_{m_1,\cdots,m_r=0}^{\infty}\frac{\pi A(m_1,\cdots,m_r)}{2^{(M+N+\sum_{i=1}^r k_im_i-2)}\Gamma(N)}\aleph^{0,\mathfrak{n}+1:V}_{U_{11}:W}\left(\begin{matrix}\mathrm{z}_1 2^{-h_1}\\ \cdots\\ \mathrm{z}_r 2^{-h_r}\end{matrix}\ \middle|\ \begin{matrix}(2-M-N-\sum_{i=1}^r k_im_i;h_1,\cdots,h_r),A:C\\ \cdots\\ ((1-N-\sum_{i=1}^r k_im_i;h_1,\cdots,h_r),B:D\end{matrix}\right)\times\frac{x_1^{m_1}\cdots x_m^{m_r}}{m_1!\cdots m_r!} \tag{2.3}$$

Where : $U_{11}=p_i+1,q_i+1,\tau_i;R$

Provided that :

a) $Re[M+N+\sum_{i=1}^r(k_im_i+h_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>1\ ,\ j=1,\cdots,m_i$ (2.4)

b) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

|iii) $$\int_{-\pi/2}^{\pi/2}(cos\theta)^{M+N-2}e^{i\omega(M-N)\theta}\,\mathrm{F}\begin{pmatrix}\mathrm{x}_1e^{2i\omega k_1\theta}\\ \cdots\\ \mathrm{x}_re^{2i\omega k_r\theta}\end{pmatrix}\aleph^{0,\mathfrak{n}:V}_{U:W}\begin{pmatrix}\mathrm{z}_1e^{2i\omega h_1\theta}\\ \cdots\\ \mathrm{z}_re^{2i\omega h_r\theta}\end{pmatrix}\mathrm{d}\theta$$

$$= \sum_{m_1,\cdots,m_r=0}^{\infty} \frac{\pi A(m_1,\cdots,m_r)\Gamma(M+N-1)}{2^{(M+N-2)}} \aleph_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\cdot\cdot \\ \mathrm{z}_r \end{array}\middle|\begin{array}{c} (\mathrm{N} - \sum_{i=1}^{r} k_i m_i; h_1,\cdots,h_r), A : C \\ \cdot\cdot\cdot \\ ((1 - \mathrm{M} - \sum_{i=1}^{r} k_i m_i; h_1,\cdots,h_r), B : D \end{array}\right) \times \frac{x_1^{m_1}\cdots x_m^{m_r}}{m_1!\cdots m_r!} \quad (2.5)$$

Where : $U_{11} = p_i + 1, q_i + 1, \tau_i; R$

Provided that :

a) $Re[M + N + \sum_{i=1}^{r}(k_i m_i + h_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 1\ ,\ j = 1,\cdots,m_i$ (2.6)

b) $|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)}$ is given in (1.5)

c) $Re(M + N) > 1$

Proof:

Expressing the multivariable Aleph-function and the Lauricella function involving in the left side of (2.1) in terms of its Mellin-Barnes integral(1.1) and by the definition of generalized lauricella function [7], interchanging the order of integration and summation. Finally evaluating the inner integral with the help of the result (1.13), the result (2.1) is obtained. Results (2.2) and (2.3) can be similarily established on applying the same procedure as above with the help of (1.13).

3. Exponential form of Fourier series

(i) $$(cos\theta)^{2(M+N-1)}\, \mathrm{F}\left(\begin{array}{c} \mathrm{x}_1(e^{i\omega}cos\theta)^{k_1} \\ \cdot\cdot\cdot \\ \mathrm{x}_r(e^{i\omega}cos\theta)^{k_r} \end{array}\right) \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(e^{i\omega}cos\theta)^{h_1} \\ \cdot\cdot\cdot \\ \mathrm{z}_r(e^{i\omega}cos\theta)^{h_r} \end{array}\right)$$

$$= \sum_{L=-\infty}^{\infty} \sum_{m_1,\cdots,m_r=0}^{\infty} \frac{\pi A(m_1,\cdots,m_r)}{2^{(L+N-1)+\sum_{i=1}^{r} k_i m_i}\Gamma(2N)} \aleph_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} \mathrm{z}_1 2^{-h_1} \\ \cdot\cdot\cdot \\ \mathrm{z}_r 2^{-h_r} \end{array}\middle|\begin{array}{c} (2 - 2\mathrm{L} - 2\mathrm{N} - \sum_{i=1}^{r} k_i m_i; h_1,\cdots,h_r), A : C \\ \cdot\cdot\cdot \\ ((1 - 2\mathrm{L} - \sum_{i=1}^{r} k_i m_i; h_1,\cdots,h_r), B : D \end{array}\right) e^{-2i\omega(L-N)\theta} \times \frac{x_1^{m_1}\cdots x_m^{m_r}}{m_1!\cdots m_r!} \quad \textbf{(3.1)}$$

Where : $U_{11} = p_i + 1, q_i + 1, \tau_i; R$

(ii) $$(cos\theta)^{2(M+N-1)}\, \mathrm{F}\left(\begin{array}{c} \mathrm{x}_1(e^{-i\omega}cos\theta)^{k_1} \\ \cdot\cdot\cdot \\ \mathrm{x}_r(e^{-i\omega}cos\theta)^{k_r} \end{array}\right) \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(e^{-i\omega}cos\theta)^{h_1} \\ \cdot\cdot\cdot \\ \mathrm{z}_r(e^{-i\omega}cos\theta)^{h_r} \end{array}\right)$$

$$= \sum_{L=-\infty}^{\infty} \sum_{m_1,\cdots,m_r=0}^{\infty} \frac{\pi A(m_1,\cdots,m_r)}{2^{(L+N-1)+\sum_{i=1}^{r} k_i m_i}\Gamma(2L)} \aleph_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} \mathrm{z}_1 2^{-h_1} \\ \cdot\cdot\cdot \\ \mathrm{z}_r 2^{-h_r} \end{array}\middle|\right.$$

$$\left.\begin{matrix}(2\text{ - }2L\text{ - }2N\text{ - }\sum_{i=1}^{r}k_i m_i; h_1,\cdots,h_r), A: C\\ \cdots\\ ((1\text{ - }2N\text{ - }\sum_{i=1}^{r}k_i m_i; h_1,\cdots,h_r), B: D\end{matrix}\right) e^{-2i\omega(L-N)\theta} \times \frac{x_1^{m_1}\cdots x_m^{m_r}}{m_1!\cdots m_r!} \quad \textbf{(3.2)}$$

Where : $U_{11} = p_i + 1, q_i + 1, \tau_i; R$

iii) $(cos\theta)^{2(M+N-1)}\,\mathrm{F}\begin{pmatrix}\mathrm{x}_1 e^{2i\omega k_1\theta}\\ \cdots\\ \mathrm{x}_r e^{2i\omega k_r\theta}\end{pmatrix}\aleph_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix}\mathrm{z}_1 e^{2i\omega h_1\theta}\\ \cdots\\ \mathrm{z}_r e^{2i\omega h_r\theta}\end{pmatrix}$

$$= \sum_{L=-\infty}^{\infty}\frac{\pi A(m_1,\cdots,m_r)\Gamma(2L+2N-2)}{2^{2(L+N-2)}}\,\aleph_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\ \middle|\ \begin{matrix}(\text{- }2N\text{ - }\sum_{i=1}^{r}k_i m_i; h_1,\cdots,h_r), A: C\\ \cdots\\ ((1\text{ - }2L\text{ - }\sum_{i=1}^{r}k_i m_i; h_1,\cdots,h_r), B: D\end{matrix}\right) e^{-2i\omega(L-N)\theta} \times \frac{x_1^{m_1}\cdots x_m^{m_r}}{m_1!\cdots m_r!} \quad (3.3)$$

Where : $U_{11} = p_i + 1, q_i + 1, \tau_i; R$

Proof : Taking

$$f(\theta) = (cos\theta)^{2(M+N-1)}\,\mathrm{F}\begin{pmatrix}\mathrm{x}_1(e^{i\omega}cos\theta)^{k_1}\\ \cdots\\ \mathrm{x}_r(e^{i\omega}cos\theta)^{k_r}\end{pmatrix}\aleph_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix}\mathrm{z}_1(e^{i\omega}cos\theta)^{h_1}\\ \cdots\\ \mathrm{z}_r(e^{i\omega}cos\theta)^{h_r}\end{pmatrix}$$

$$= \sum_{L=-\infty}^{\infty} C_L e^{-2i\omega(L-N)\theta}\quad , [-\pi/2 < \theta < \pi/2] \quad (3.4)$$

which is valid due to $f(\theta)$ is continuous and bounded variation in the open interval (-$\pi/2$,$\pi/2$). Now multiplying by $e^{2i\omega(L-N)\theta}$ both sides in (3.4) and integrating it with respect to θ from $\pi/2$ to $\pi/2$ and then making use (1.14) and (2.1), we get :

$$C_T = \frac{\pi A(m_1,\cdots,m_r)}{2^{2(T+N-1)+\sum_{i=1}^{r}k_i m_i}\Gamma(2N)}\,\aleph_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix}\mathrm{z}_1 2^{-h_1}\\ \cdots\\ \mathrm{z}_r 2^{-h_r}\end{matrix}\ \middle|\ \begin{matrix}(2\text{ - }2T\text{ - }2N\text{ - }\sum_{i=1}^{r}k_i m_i; h_1,\cdots,h_r), A: C\\ \cdots\\ ((1\text{ - }2T\text{ - }\sum_{i=1}^{r}k_i m_i; h_1,\cdots,h_r), B: D\end{matrix}\right) \quad (3.5)$$

From (3.4) and (3.5), we obtain the desired result (3.1).

Results (3.2) and (3.3) can be similarily established on applying the same procedure as above with the help (1.14).

Remarks : If $\tau_i = \tau_{i^{(k)}} = 1$, then the Aleph-function of several variables degenere in the I-function of several variables defined by Sharma and Ahmad [4], for more detail see C.K.sharma et al [5].

And if $R = R^{(1)} =, \cdots, R^{(r)} = 1$, the multivariable I-function degenere in the multivariable H-function defined by srivastava et al [6],

4. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as I-function of several variables defined by Sharma and Ahmad [5] , multivariable H-function , see Srivastava et al [9] , the Aleph-function of two variables defined by K.sharma [7], the I-function of two variables defined by Goyal and Agrawal [1,2,3] , sharma and Mishra [6] ,and the h-function of two variables , see Srivastava et a[9].

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