Bilateral generating functions for systems in several variables and multivariable Aleph-function

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Abstract. In this document, we shall establish three bilateral generating functions for a certain multiples sequences of function of several complex variables involving the multivariable Aleph-function and the generalized Lauricella function. These bilateral generating functions are derivable by using the consequences of Gould's identity ([1],1961) and Lagrange's expansion formula [2, Polya and Szego , 1972, p.348].

1. Introduction and preliminaries.

The object of this document is to establish three bilateral generating formulas from the multivariables aleph-function. These function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [3] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c|} \mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\right.$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\cdots\cdots\cdots\cdots\cdots\ ,[\tau_i(b_{ji};\beta_j^{(1)},\cdots,\beta_j^{(r)})_{m+1,q_i}]:$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^r\alpha_j^{(k)}s_k)}{\sum_{i=1}^R[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^r\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^r\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

$$\text{and }\ \theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

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where $j = 1$ to r and $k = 1$ to r

Suppose , as usual , that the parameters

$a_j, j = 1, \cdots, p; b_j, j = 1, \cdots, q;$

$c_j^{(k)}, j = 1, \cdots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R$, $i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1 \cdots, r, i = 1, \cdots, R , i^{(k)} = 1, \cdots, R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1} \ldots |z_r|^{\alpha_r}) , max(|z_1| \ldots |z_r|) \to 0$

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1} \ldots |z_r|^{\beta_r}) , min(|z_1| \ldots |z_r|) \to \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1) / \gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \ ; \ V = m_1, n_1; \cdots ; m_r, n_r \tag{1.6}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i(1)}; R^{(1)}, \cdots , p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i(r)}; R^{(r)} \tag{1.7}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots , \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots , \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.8}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots , \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots , \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.10}$$

$$D = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots , \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots , z_r) = \aleph_{U:W}^{0,\mathrm{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A : C} \\ \cdot & \\ \cdot & \cdots \\ \cdot & \\ \mathrm{z}_r & \mathrm{B : D} \end{array}\right) \tag{1.12}$$

Let $\mathrm{F}\begin{pmatrix} \mathrm{x}_1 \\ \cdots \\ \mathrm{x}_r \end{pmatrix}$ denote the generalized Lauricella function of several complex variables defined by Srivastava et al [6].

We have : $$\mathrm{F}\begin{pmatrix} \mathrm{x}_1 \\ \cdots \\ \mathrm{x}_r \end{pmatrix} = \sum_{k_1,\cdots,k_r=0}^{\infty} A(k_1, \cdots , k_r) \frac{x_1^{k_1} \cdots x_r^{k_r}}{k_1! \cdots k_r!} \tag{1.13}$$

where : $$A(k_1, \cdots , k_r) = \frac{\prod_{j=1}^{A}(a_j)_{k_1\theta'_j+\cdots+k_r\theta_j^{(r)}} \prod_{j=1}^{B'}(b'_j)_{k_1\phi'_j} \cdots \prod_{j=1}^{B^{(n)}}(b_j^{(r)})_{k_r\phi_j^{(r)}}}{\prod_{j=1}^{C}(c_j)_{k_1\epsilon'_j+\cdots+k_r\epsilon_j^{(r)}} \prod_{j=1}^{D'}(d'_j)_{k_1\delta'_j} \cdots \prod_{j=1}^{D^{(r)}}(d_j^{(r)})_{k_r\delta_j^{(r)}}} \tag{1.14}$$

2. Section 2

In the present paper, we use the following notations.

$$\mathrm{F}^*\begin{pmatrix} \mathrm{x}_1 \\ \cdots \\ \mathrm{x}_r \end{pmatrix} = \sum_{k_1,\cdots,k_r=0}^{\infty} (-n)_{m_1k_1+\cdots+m_rk_r} \ A(m_1, \cdots , m_r) \frac{x_1^{k_1} \cdots x_r^{k_r}}{k_1! \cdots k_r!} \tag{2.1}$$

$$\mathrm{g}\begin{pmatrix} \mathrm{y'}_1 \\ \cdots \\ \mathrm{y'}_r \\ \mathrm{z} \end{pmatrix} = \sum_{n=0}^{\infty} \frac{(\alpha - r)!}{(\alpha - r - n)!} \ \mathrm{F}\begin{pmatrix} \mathrm{y'}_1 \\ \cdots \\ \mathrm{y'}_r \end{pmatrix} z^n \tag{2.2}$$

Where $$y'_j = y_j(-\zeta)^{m_j}, j = 1 \cdots r \tag{2.3}$$

$$\zeta = t(1 + \zeta)^{\beta+1}, \zeta(0) = 0. \tag{2.4}$$

$$z = -\frac{\zeta}{1 + \zeta} \tag{2.5}$$

$$M = m_1k_1 + \cdots + m_rk_r \tag{2.6}$$

$$\alpha' - \alpha = \gamma' - \gamma = (\beta + 1)M \tag{2.7}$$

and $A(k_1,\cdots,k_r)$ is defined by (1.14)

We have the following results.

Formula 1

$$\sum_{n=0}^{\infty} \mathrm{F}^*\begin{pmatrix} \mathrm{y}_1 \\ \cdots \\ \mathrm{y}_r \end{pmatrix} \aleph_{U_{11}:W}^{0,\mathrm{n}+1:V}\left(\begin{array}{c|c} \mathrm{z}_1 & (-\alpha-(\beta+1)n;\sigma_1,\cdots,\sigma_r), A:C \\ \vdots & \cdots \\ \mathrm{z}_r & (-\alpha-\beta n-M;\sigma_1,\cdots,\sigma_r), B:D \end{array}\right)\frac{t^n}{n!}$$

$$= t(1+\zeta)^{\alpha+1}(1-\beta\zeta)^{-1}\ \mathbf{F}\begin{pmatrix} \mathrm{y'}_1 \\ \cdots \\ \mathrm{y'}_r \end{pmatrix} \aleph_{U:W}^{0,\mathrm{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1(1+\zeta)^{\sigma_1} & \mathrm{A}:\mathrm{C} \\ \vdots & \cdots \\ \mathrm{z}_r(1+\zeta)^{\sigma_r} & \mathrm{B}:\mathrm{D} \end{array}\right) \tag{2.8}$$

Where $U_{11}=p_i+1,q_i+1,\tau_i;R$

Formula 2

$$\sum_{n=0}^{\infty} \mathrm{F}^*\begin{pmatrix} \mathrm{y}_1 \\ \cdots \\ \mathrm{y}_r \end{pmatrix} \aleph_{U_{11}:W}^{0,\mathrm{n}+1:V}\left(\begin{array}{c|c} \mathrm{z}_1 & (-\alpha-(\beta+1)n;\sigma_1,\cdots,\sigma_r), A:C \\ \vdots & \cdots \\ \mathrm{z}_r & (-\alpha-\beta n-M;\sigma_1,\cdots,\sigma_r), B:D \end{array}\right)\frac{t^n}{n!}$$

$$= (1+\zeta)^{\alpha}\ \mathbf{F}\begin{pmatrix} \mathrm{y'}_1 \\ \cdots \\ \mathrm{y'}_r \end{pmatrix} \aleph_{U_{11}:W}^{0,\mathrm{n}+1:V}\left(\begin{array}{c|c} \mathrm{z}_1(1+\zeta)^{\sigma_1} & (1-\alpha';\sigma_1,\cdots,\sigma_r), A:C \\ \vdots & \cdots \\ \mathrm{z}_r(1+\zeta)^{\sigma_r} & (-\alpha';\sigma_1,\cdots,\sigma_r), B:D \end{array}\right) \tag{2.8}$$

Where $U_{11}=p_i+1,q_i+1,\tau_i;R$

Formula 3

$$\sum_{n=0}^{\infty} \mathrm{F}^*\begin{pmatrix} \mathrm{y}_1 \\ \cdots \\ \mathrm{y}_r \end{pmatrix} \aleph_{U_{22}:W}^{0,\mathrm{n}+2:V}\left(\begin{array}{c|c} \mathrm{z}_1 & (1-\gamma-(\beta+1)n;\sigma_1,\cdots,\sigma_r),\ (-\alpha-(\beta+1)n;\sigma_1,\cdots,\sigma_r), A:C \\ \vdots & \cdots \qquad \cdots \\ \mathrm{z}_r & (-\alpha-\beta n-M;\sigma_1,\cdots,\sigma_r),\ (-\gamma-(\beta+1)n;\sigma_1,\cdots,\sigma_r), B:D \end{array}\right)\frac{t^n}{n!}$$

$$= (1+\zeta)^{\alpha}\mathrm{g}\begin{pmatrix} \mathrm{y'}_1 \\ \cdots \\ \mathrm{y'}_r \\ \mathrm{z} \end{pmatrix} \aleph_{U_{22}:W}^{0,\mathrm{n}+2:V}\left(\begin{array}{c|c} \mathrm{z}_1(1+\zeta)^{\sigma_1} & (1-\gamma';\sigma_1,\cdots,\sigma_r),\ (\text{-M }-\gamma";\sigma'_1,\cdots,\sigma'_r), A:C \\ \vdots & \cdots \qquad \cdots \\ \mathrm{z}_r(1+\zeta)^{\sigma_r} & (-\gamma';\sigma_1,\cdots,\sigma_r),\ (\text{-n-M }-\gamma";\sigma'_1,\cdots,\sigma'_r), B:D \end{array}\right) \tag{2.9}$$

Where $U_{22}=p_i+2,q_i+2,\tau_i;R\,;\gamma"=\gamma/(\beta+1);\sigma'_i=\sigma_i/(\beta+1),i=1\cdots r$

The conditions (1.4) and (1.5) are satisfied.

Proof :
To prove (2.7), first applying the definitions (1.1) and (1.13) on left-hand side of (2.7) and changing the order of summations and integrations. Now evaluating the inner summation by using the Lagrange's expansion formula [2 ,Polya and Szego (1972), p.349, problem 216]

$$\sum_{n=0}^{\infty}\binom{\alpha+(\beta+1)n}{n}t^n=\frac{(1+\zeta)^{\alpha+1}}{(1-\beta\zeta)} \tag{2.10}$$

Similarly, we can (2.8) and (2.9) by using he Lagrange's expansion formula [2 ,Polya and Szego (1972), p.348, problem 212]

$$\sum_{n=0}^{\infty}\frac{\alpha}{\alpha+(\beta+1)n}\binom{\alpha+(\beta+1)n}{n}t^n=(1+\zeta)^{\alpha} \tag{2.11}$$

and Gould's identity [1, (1961), p.196, (6.1)]

$$\sum_{n=0}^{\infty}\frac{\gamma}{\gamma+(\beta+1)n}\binom{\alpha+(\beta+1)n}{n}t^n=(1+\zeta)^{\alpha}\sum_{n=0}^{\infty}\binom{\alpha-\gamma}{n}\binom{n+\gamma"}{n}^{-1}z^n \tag{2.12}$$

respectively.

with $\quad z=-\dfrac{\zeta}{1+\zeta}$ and $\gamma"=\gamma/(\beta+1)$.

3. Section 3

In this section, particular cases will be treated,

a) If $\tau_i=\tau_{i^{(k)}}=1$, then the Aleph-function of several variables degenere in the I-function of several variables defined by Sharma and Ahmad [3], for more detail see C.K.sharma et al [4].

b) If $\tau_i=\tau_{i^{(k)}}=1$ and $R=R^{(1)}=,\cdots,R^{(r)}=1$, then the multivariable Aleph-function degenere in the multivariable H-function defined by Srivastava et al [5]. And we have the following results.

Formula 1

$$\sum_{n=0}^{\infty}F^*\begin{pmatrix}y_1\\\cdots\\y_r\end{pmatrix}H^{0,n+1:V}_{p+1;q+1:W}\begin{pmatrix}(\text{-}\alpha-(\beta+1)n;\sigma_1,\cdots,\sigma_r),A':C'\\\cdots\\(\text{-}\alpha-\beta n-M;\sigma_1,\cdots,\sigma_r),B':D'\end{pmatrix}\frac{t^n}{n!}$$

$$=t(1+\zeta)^{\alpha+1}(1-\beta\zeta)^{-1}\ \mathbf{F}\begin{pmatrix}y'_1\\\cdots\\y'_r\end{pmatrix}H^{0,n+1:V}_{p+1;q+1:W}\left(\begin{matrix}z_1(1+\zeta)^{\sigma_1}\\\cdot\\\cdot\\z_r(1+\zeta)^{\sigma_r}\end{matrix}\ \middle|\ \begin{matrix}A':C'\\\cdots\\B':D'\end{matrix}\right) \tag{3.1}$$

Formula 2

$$\sum_{n=0}^{\infty}F^*\begin{pmatrix}y_1\\\cdots\\y_r\end{pmatrix}H^{0,n+1:V}_{p+1;q+1:W}\left(\begin{matrix}z_1\\\cdot\\\cdot\\\cdot\\z_r\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\alpha-(\beta+1)n;\sigma_1,\cdots,\sigma_r),A':C'\\\cdots\\(\text{-}\alpha-\beta n-M;\sigma_1,\cdots,\sigma_r),B':D'\end{matrix}\right)\frac{t^n}{n!}$$

$$=(1+\zeta)^{\alpha}\ \mathbf{F}\begin{pmatrix}y'_1\\\cdots\\y'_r\end{pmatrix}H^{0,n+1:V}_{p+1;q+1:W}\left(\begin{matrix}z_1(1+\zeta)^{\sigma_1}\\\cdot\\\cdot\\z_r(1+\zeta)^{\sigma_r}\end{matrix}\ \middle|\ \begin{matrix}(1\text{-}\alpha';\sigma_1,\cdots,\sigma_r),A':C'\\\cdots\\(\text{-}\alpha';\sigma_1,\cdots,\sigma_r),B':D'\end{matrix}\right) \tag{3.2}$$

$$\sum_{n=0}^{\infty} \mathrm{F}^*\begin{pmatrix} \mathrm{y}_1 \\ \cdots \\ \mathrm{y}_r \end{pmatrix} H^{0,\mathrm{n}+2:V}_{p+2;q+2:W}\left(\begin{matrix} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{matrix}\ \middle|\ \begin{matrix} (1\text{-}\gamma-(\beta+1)n;\sigma_1,\cdots,\sigma_r),\ (\text{-}\alpha-(\beta+1)n;\sigma_1,\cdots,\sigma_r),A:C \\ \cdots \qquad\qquad \cdots \\ (\text{-}\alpha-\beta n-M;\sigma_1,\cdots,\sigma_r),\ (\text{-}\gamma-(\beta+1)n;\sigma_1,\cdots,\sigma_r),B:D \end{matrix}\right)\frac{t^n}{n!}$$

$$= (1+\zeta)^\alpha \mathrm{g}\begin{pmatrix} \mathrm{y'}_1 \\ \cdots \\ \mathrm{y'}_r \\ \mathrm{z} \end{pmatrix} H^{0,\mathrm{n}+2:V}_{p+2;q+2:W}\left(\begin{matrix} \mathrm{z}_1(1+\zeta)^{\sigma_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r(1+\zeta)^{\sigma_r} \end{matrix}\ \middle|\ \begin{matrix} (1\text{-}\gamma';\sigma_1,\cdots,\sigma_r),\ (\text{-M -}\gamma";\sigma'_1,\cdots,\sigma'_r),A:C \\ \cdots \qquad\qquad \cdots \\ (\text{-}\gamma';\sigma_1,\cdots,\sigma_r),\ (\text{-n-M -}\gamma";\sigma'_1,\cdots,\sigma'_r),B:D \end{matrix}\right) \quad (3.3)$$

Where $\gamma" = \gamma/(\beta+1); \sigma'_i = \sigma_i/(\beta+1), i = 1\cdots r$

4. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as I-function of several variables defined by Sharma and Ahmad [3] , multivariable H-function , see Srivastava et al [5]

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