Line Join Connected Domination on a Graph

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Abstract**:** *In this paper we introduce a new concept line join graph of a set of vertices S of a graph. We construct a new graph, called the line join graph of S and define a new domination parameter called line join connected domination number. We determine this number for some standard graphs and obtain bounds for general graphs. We prove various important results connecting with the domination number.*

Key words: *Connected graph, complete, bipartite, wheel graphs, distance, dominating set*

I. Introduction

The various domination parameters introduced in [6], [7], [8] till now find many applications in covering of entire graph by the different sets with each of which has some specified property. These concepts are helpful to find centrally located sets to cover the entire graph in which they are defined. The concept of line join connected set has practical application in checking the transport and various facilities of the interior villages which are considered as edges and the main cities as vertices in a graph in the case of bye pass ways. In this paper, we define a new concept named line join connected domination and study the structural properties of a graph using this concept.

II. PRELIMINARIES

Definition 2.1[7]: A set $D \subseteq V(G)$ is a **dominating set** of G, if every vertex in $V-D$ is adjacent to some vertex in D. The dominating set D is a minimal dominating set if no proper subset D' of D is a dominating set. The minimal dominating set with minimum cardinality is known as a minimum dominating set. The cardinality of minimum dominating set is known as the **domination number** and is denoted by $\gamma(G)$.

A dominating set $D \subseteq V(G)$ is a **connected dominating set** if the induced subgraph $\langle D\rangle$ is connected. The connected dominating set D is minimal connected dominating set if no proper subset D' of D is a connected dominating set. The minimal connected dominating set with minimum cardinality is known as a minimum connected dominating set. The cardinality of minimum connected dominating set is known as a **connected domination number** and is denoted by $\gamma_c(G)$.

Definition 2.2[4]: Let G be a connected graph and v be a vertex of G. The **eccentricity** $e(v)$ of v is the distance to a vertex farthest from v. Thus G, $e(v) = max\{d(u, v): u, v \in V(G)\}$. The **radius** $rad(G)$ is the minimum eccentricity the vertices, whereas the **diameter** $diam(G)$ is the maximum eccentricity. For any connected graph G, $rad(G) \leq diam(G) \leq 2\, rad(G)$. A vertex v is called a **central vertex** if $e(v)=rad(G)$.The **centre** $C(G)$ is the set of all central vertices. The central subgraph $\langle C(G)\rangle$ of a graph G is the subgraph induced by the centre $C(G)$. The vertex v is a **peripheral vertex** if $e(v)=diam(G)$.The periphery $P(G)$ is the set of all peripheral vertices.

Definition 2.3[4]: The **open neighbourhood** $N(v)$ of a vertex v is the set of all vertices adjacent to v in G. $N(v)\cup\{v\}$ is called the **closed neighbourhood** of v.

Definition 2.4[4]: Let x and z be two distinct vertices in G. A vertex y distinct from x and z is said to **lie between** x and z if $d(x, z) = d(x, y) + d(y, z)$.

Definition 2.5[4]: The **girth** of a graph G is the length of a shortest cycle contained in the graph. If the graph does not contain any cycles its girth is defined to be infinity. For example, a 4-cycle has girth 4.

III. LINE JOIN CONNECTED DOMINATION

In this section we define our new concept of line join graph of the set of vertices, line join connected set and its characterisation.

Definition 3.1: Let $G = (V, E)$ be a graph. Let S be a subset of vertices of G. Let $E_0 = \{v_1, v_2,.., v_s\}$; $s \leq q$ be the set of edges incident with the vertices of S where q is the number of edges of G. Draw a new graph with the members of E_0 as vertices and there is an edge between e_i and e_j $\forall$ $e_i, e_j \in E_0$ if e_i and e_j are adjacent in G. Then the new graph is said to be the **line join graph of the set of vertices *S*** and is denoted by $LJ(S)$.

Definition 3.2: Let G = (V, E) be a graph. A subset S of V is said to be a **line join connected set** if the line join graph $LJ(S)$ is connected.

Definition 3.3: A dominating set $D \subseteq V(G)$ of G is said to be a **line join connected dominating set** if the line join graph D in which the edges incident to each vertex in D considered as vertices is connected. The minimum cardinality taken over all line join connected dominating sets is called the **line join connected domination number** of G and is denoted by γ_{ljcd}

.

Theorem 3.4: In any connected graph G, there is a dominating set D such that LJ (D) is connected.

Proof: Let G be a connected graph. Let D be a dominating set of G. If LJ (D) is connected then $|D|=\gamma_{ljcd}(G)$. Otherwise, we have to find a dominating set D for which LJ (D) is connected. We know that the whole vertex set V of G dominates itself and LJ (V) is connected. Hence $\gamma_{ljcd}(G) \leq |V|$. Therefore for any connected graph G, we can find a minimum dominating set for which the line join graph is connected.

Theorem 3.5: A dominating set D of a connected graph G is a line join connected dominating set of G if for any pair of non-adjacent vertices $(x, y) \in V-D$ there exists a pair of vertices $(u, v) \in D$ such that $d(x, y) \geq d(u, v)$.

Proof: Assume that D is a line join connected dominating set of a conneced graph G with n vertices. This means that LJ (D) is connected. This is possible only if the vertices in D are adjacent among themselves or the vertices in $V-D$ lie between the vertices in D. Suppose that the vertices in D are adjacent among themselves and since each vertex in $V-D$ is adjacent to atleast one vertex in D for any pair of non-adjacent vertices $(x, y) \in V-D$ there exists a pair of vertices $(u, v) \in D$ such that $d(x, y) \geq d(u, v)$. Suppose that the vertices in $V-D$ lie between the vertices of D in G. If n is odd let $D = \{v_1, v_2, \ldots, v_{n-1}\}$ and $V-D = \{v_2, v_4, \ldots, v_n\}$. Then obviously for any pair of non-adjacent vertices $(x, y) \in V-D$ there exists a pair of vertices $(u, v) \in D$ such that $d(x, y) \geq d(u, v)$.If n is even let $D = \{v_1, v_3, \ldots, v_n\}$ and $V-D = \{v_2, v_4, \ldots, v_{n-1}\}$. Then obviously for any pair of non-adjacent vertices $(x, y) \in V-D$ there exists a pair of vertices $(u, v) \in D$ such that $d(x, y) \geq d(u, v)$. Hence a dominating set D of G is a line join connected dominating set of G if for any pair of non-adjacent vertices $(x, y) \in V-D$ there exists a pair of vertices $(u, v) \in D$ such that $d(x, y) \geq d(u, v)$.

Theorem 3.6: For any connected graph G, $\gamma(G) \leq \gamma_{ljcd}(G)$.

Proof: : Let D be a minimum dominating set of G. Draw the line join graph of D. If LJ (D) is connected, then D is a line join connected dominating set of G. Since D is a minimum and LJ (D) is connected D is the minimum line join connected dominating set of G. Therefore $\gamma(G) = \gamma_{ljcd}(G)$. Suppose LJ (D) is not connected. Then D is not a line join connected dominating set of G. Consider another dominating set D' of G such that LJ (D') is connected and it is the minimum line join connected dominating set of G. D' is a dominating set of G but it is not a minimum dominating set of G. For, if D' is the minimum dominating set of G it must be a contradiction to our assumption that D is the minimum dominating set of G. Therefore, $\gamma(G) < \gamma_{ljcd}(G)$. In general, for any connected graph G, $\gamma(G) \leq \gamma_{ljcd}(G)$.

Example 3.7:

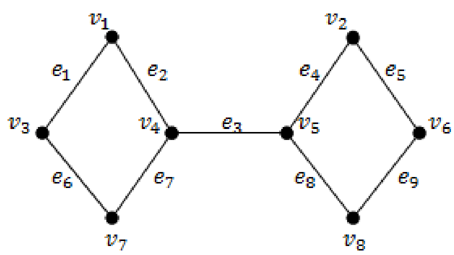

Fig. 1 A graph G

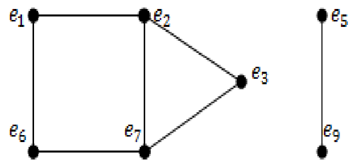

Fig. 2 $LJ(D_1)$

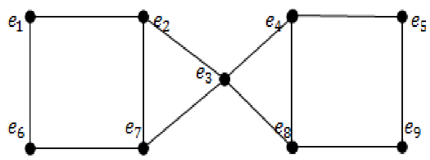

Fig. 3 LJ (D_2)

Here $D_1 = \{ v_3, v_4, v_6\}$ is the minimum dominating set of G and $D_1 = \{ v_3, v_4, v_5, v_6\}$ is the dominating set of G. But D_1 is not line join connected because LJ (D_1) is not a connected graph. But LJ (D_2) is connected.Therefore D_2 is the minimum line join connected dominating set of G and $\gamma_{ljcd}(G) = 4$.

Theorem 3.8: For any path P_n , where $n \geq 2$

$$\gamma_{ljcd}(P_n) = \begin{cases} \frac{n}{2}\ ; \text{if n is even} \\ \left\lfloor \frac{n}{2} \right\rfloor + 1; \text{if n is odd} \end{cases}$$

Proof: Let $v_1, v_3, \ldots, v_n$ represent a path P_n.
Case (i): If n is even. Let $D = \{v_1, v_3, \ldots, v_{n-1}\}$ be a minimum dominating set of P_n. Since each vertex in $V-D$ is adjacent to atmost two vertices in D, the edges incident to the vertices in D constitute all the

edges of the path. Therefore the line join graph of $LJ(D)$ is connected and also it is a minimum dominating set. Hence D is a minimum line join connected dominating set of P_n and $\gamma_{ljcd}(P_n) = \frac{n}{2}$.

Case (ii): If n is odd. Let $D = \{v_1, v_2, \ldots, v_n\}$ be a minimum dominating set of P_n. Since each vertex in $V-D$ is adjacent to exactly two vertices in D, the edges incident to the vertices in D constitute all the edges of the path. Therefore the line join graph of $LJ(D)$ is connected and also it is a minimum dominating set. Hence D is a minimum line join connected dominating set of P_n and $\gamma_{ljcd}(P_n) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Theorem 3.9: For any cycle C_n, where $n \geq 3$

$$\gamma_{ljcd}(C_n) = \begin{cases} \frac{n}{2} & ; \text{if n is even} \\ \left\lfloor \frac{n}{2} \right\rfloor & ; \text{if n is odd} \end{cases}$$

Proof: Let $v_1 v_3 \ldots v_n v_1$ represent a cycle C_n.

Case (i): If n is even. Let $D = \{v_1, v_3, \ldots, v_{n-1}\}$ be a minimum dominating set of C_n. Since each vertex in $V-D$ is adjacent to exactly two vertices in D, the edges incident to the vertices in D constitute all the edges of the cycle. Therefore the line join graph $LJ(D)$ is connected which is a path and also it is a minimum dominating set. Hence D is a minimum line join connected dominating set of C_n and $\gamma_{ljcd}(C_n) = \frac{n}{2}$.

Case (ii): If n is odd. Let $D = \{v_1, v_2, \ldots, v_{n-2}\}$ be a minimum dominating set of C_n. Since each vertex in $V-D$ is adjacent to atmost two vertices in D, the edges incident to the vertices in D constitute the edges of the cycle except the edge which connects the vertices adjacent to exactly one vertex in the dominating set. Therefore the line join graph $LJ(D)$ is connected and also it is a minimum dominating set. Hence D is a minimum line join connected dominating set of C_n and $\gamma_{ljcd}(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$.

Theorem 3.10: For a complete graph K_n, where $n \geq 2$, $\gamma_{ljcd}(K_n) = 1$.

Proof: Let $D = \{v_i\}$ be a dominating set of K_n and also it is minimum. Since K_n is a complete graph v_i is adjacent to the remaining $n-1$ vertices. That is $n-1$ edges are incident to the vertex v_i and each edge incident to the vertex v_i is adjacent to the remaining $n-2$ edges. Then the line join graph $LJ(D)$ is a complete graph K_{n-1} Therefore D is a minimum line join connected dominating set of K_n and $\gamma_{ljcd}(K_n) = 1$.

Theorem 3.11: For a complete bipartite graph $K_{m,n}$ where $m, n \geq 2$ $\gamma_{ljcd}(K_{m,n}) = 2$.

Proof: Let $D = \{v_i, v_j\}$ where $v_i \in V_1$ and $v_j \in V_2$ be a minimum dominating set of a complete bipartite graph $K_{m,n}$. Since $v_i \in V_1$ and $v_j \in V_2$ and $K_{m,n}$ is a complete bipartite graph n edges are incident to v_i and m edges are incident to v_j. Since $K_{m,n}$ is complete is complete one edge among the edges incident with the vertices v_i, v_j is common. Therefore, $m + n - 1$ edges are incident with the vertices v_i, v_j and these edges are adjacent among themselves. Also the line join graph $LJ(D)$ is connected. Therefore D is a minimum line join connected dominating set of $K_{m,n}$ and $\gamma_{ljcd}(K_{m,n}) = 2$.

Theorem 3.12: For a wheel W_n, $\gamma_{ljcd}(W_n) = 1$.

Proof: Let $D = \{v_i\}$ be a dominating set of W_n and also it is minimum. Since v_i is the unique central vertex of maximum degree $n-1$. That is $n-1$ edges are incident to the vertex v_i and these edges are adjacent among themselves. Then the line join graph $LJ(D)$ is connected. Therefore D is a minimum line join connected dominating set of W_n and $\gamma_{ljcd}(W_n) = 1$.

Theorem 3.13: If T is a tree with l leaves and n vertices, then $\gamma_{ljcd}(T) \geq \frac{n-l+2}{3}$.

Proof: E.Delavina[3] proved that $\gamma(T) \geq \frac{n-l+2}{3}$.

From theorem 3.6, we know that for any connected graph G, $\gamma(G) \leq \gamma_{ljcd}(G)$. Therefore, for a tree T with l leaves and n vertices $\gamma_{ljcd}(T) \geq \frac{n-l+2}{3}$

Theorem 3.14: For any connected graph G with x cut vertices, then $\gamma_{ljcd}(G) \geq \frac{x+2}{3}$.

Proof: E.Delavina[3] proved that $\gamma(G) \geq \frac{x+2}{3}$.

From theorem 3.6, we know that for any connected graph G, $\gamma(G) \leq \gamma_{ljcd}(G)$. Therefore, for any connected graph G with x cut vertices $\gamma_{ljcd}(G) \geq \frac{x+2}{3}$

IV. BOUNDS BASED ON DIAMETRE, RADIUS, GIRTH AND Δ(G)

Theorem 4.1: Let G be a connected graph with $n > 1$ and diametre $diam(G)$, then $\frac{diam(G)+1}{3} \leq \gamma_{ljcd}(G)$.

Proof: Let D be a minimum line join connected dominating set of G. That is the line join graph of D $LJ(D)$is connected. Consider any arbitrary path of length D. Then this diametral path includes atmost two edges from the induced subgraph $< N[v] >$ for each $v \in D$. Furtermore, since D is a minimum line join connected set of G, the diametral path includes atmost $\gamma_{ljcd}(G) - 1$ edges joining the neighborhoods of the vertices of D. Hence

$$diam(G) \leq 2\,\gamma_{ljcd}(G) + \gamma_{ljcd}(G) - 1$$

$$= 3\,\gamma_{ljcd}(G) - 1$$

$$diam(G) + 1 \leq 3\,\gamma_{ljcd}(G)$$

$$\frac{diam(G)+1}{3} \leq \gamma_{ljcd}(G)$$

Hence for any connected graph G with $n > 1$ and diametre $diam(G)$, $\frac{diam(G)+1}{3} \leq \gamma_{ljcd}(G)$.

Theorem 4.2: Let G be a connected graph with and $n > 1$ radius $rad(G)$, then $rad(G) \leq \frac{3}{2}\gamma_{ljcd}(G)$.

Proof: Let D be a minimum line join connected dominating set of G. Form a spanning tree T of G such that D is also a minimum line join connected dominating set of T and therefore $rad(G) \leq rad(T)$. Since $2rad(T) - 1 \leq diam(T)$ and $\gamma_{ljcd}(T) = \gamma_{ljcd}(G)$. We have $2rad(G) - 1 \leq 2rad(T) - 1 \leq diam(T)$. By applying the previous theorem we have

$$2rad(G) - 1 \leq diam(G) \leq 3\,\gamma_{ljcd}(G) - 1$$

$$2rad(G) - 1 \leq 3\,\gamma_{ljcd}(G) - 1 = 2\,rad(G) \leq 3\,\gamma_{ljcd}(G)$$

$$rad(G) \leq \frac{3}{2}\gamma_{ljcd}(G)$$

Hence for any connected graph G with $n > 1$ and radius $rad(G)$, then $rad(G) \leq \frac{3}{2}\gamma_{ljcd}(G)$.

Theorem 4.3: Let G be a connected graph with $n > 1$. Then $\gamma_{ljcd}(G) \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$

Proof: Let G be a tree with n vertices. Let D be the minimum dominating set and $\gamma_{ljcd}(G)$.be the line join connected domination number of G. We prove this theorem by induction on n. The result is obviosly true for n=1. Assume that the result is true for a connected graph G with $n - 1$ vertices.That is, for a connected graph G with $n - 1$vertices $\gamma_{ljcd}(G) \leq (n-1) - \left\lfloor \frac{diam(G)}{3} \right\rfloor$. Now we prove the result for the case of n vertices. By induction hypothesis the result is true for n−1. We now add one vertex to the graph G of n−1vertices and we call the new graph with n vertices as G'. Then we have to consider the following two cases:

Case (i): If the newly added vertex is adjacent two one vertex in G.

Subcase (i): If the newly added vertex is adjacent to a vertex in the set D of G. Then $\gamma_{ljcd}(G) = \gamma_{ljcd}(G')$. But $diam(G') > diam(G)$. In this case $\gamma_{ljcd}(G') \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$.

Subcase (ii): If the newly added vertex is adjacent to a vertex in $V-D$ of G, then $\gamma_{ljcd}(G) < \gamma_{ljcd}(G')$. But $diam(G') > diam(G)$. In this case $\gamma_{ljcd}(G') \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$.

Case (ii): If the newly added vertex is adjacent two one vertex in G.

Subcase (i): If the newly added vertex is adjacent to two vertices of G, where one vertex is in the set D and the other vertex belongs to the set $V-D$ of G. Then $\gamma_{ljcd}(G) = \gamma_{ljcd}(G')$. But $diam(G') > diam(G)$. In this case $\gamma_{ljcd}(G') \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$.

Subcase (ii): If the newly added vertex is adjacent to any two vertices in $V-D$ of G. Then $\gamma_{ljcd}(G')$ increases by 1 also the $diam(G')$ increases by 1. In this case $\gamma_{ljcd}(G') \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$.

Subcase (iii): If the newly added vertex is adjacent to any two vertices in the set D of G. Then $\gamma_{ljcd}(G) = \gamma_{ljcd}(G')$. But $diam(G') > diam(G)$. In this case $\gamma_{ljcd}(G') \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$.

Case (iii): If the newly added vertex is adjacent to all the vertices in of G. Then $\gamma_{ljcd}(G')$ equal to 1 and $diam(G')$ also equal to 1. In this case $\gamma_{ljcd}(G') \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$

In all the cases $\gamma_{ljcd}(G) \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$ is true for all n. Hence for any connected graph G with $n > 1$, $\gamma_{ljcd}(G) \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$.

Note 4.4: From Theorem 4.1 and Theorem 4.3 we have the bound that $\frac{diam(G)+1}{3} \leq \gamma_{ljcd}(G) \leq n - \left\lfloor \frac{diam(G)}{3} \right\rfloor$.

Theorem 4.5: For any graph G, $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq \gamma_{ljcd}(G) \leq n - \Delta(G)$.

Proof: Let D be a minimium line join connected dominating set of G that is the line join graph of D $LJ(D)$ is connected. Let $\gamma_{ljcd}(G)$ be the line join connected domination number of G. First we consider the lower bound. Since each vertex can dominate atmost itself and $\Delta(G)$ other vertices and also the dominating set's line join graph will be a connected graph with $\Delta(G)$ vertices and satisfies the line join connected condition and so $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq \gamma_{ljcd}(G)$. For the upper bound let v be a vertex of maximum degree $\Delta(G)$ then v dominates $N[v]$ and the vertices in $V-N[v]$ dominates themselves. Hence $V-N[v]$ is a dominating set of cardinality $n - \Delta(G)$ and set. Therefore, $\gamma_{ljcd}(G) \leq n - \Delta(G)$. Therefore for any graph G, $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq \gamma_{ljcd}(G) \leq n - \Delta(G)$.

Corollary 4.6: For any tree T with $n \geq 2$, $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq \gamma_{ljcd}(T) \leq 2m - n + 1$.

Proof: Let D be a γ_{ljcd} -set of T. The lower bound follows from the Theorem 4.4. Now we consider the upper bound. For any tree T, $\gamma_{ljcd}(T) \leq n-1 = 2(n-1) - n + 1 = 2m - n + 1$ and so $\gamma_{ljcd}(T) \leq 2m - n + 1$. Hence for any tree T with $n \geq 2$, $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq \gamma_{ljcd}(T) \leq 2m - n + 1$.

Theorem 4.7: If a graph has $\delta(G) \geq 2$ and $g(G) \geq 5$, then $\gamma_{ljcd}(G) \leq \left\lceil \frac{n-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil + 1$ if $g(G)$ is even and $\gamma_{ljcd}(G) \leq \left\lceil \frac{n-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil$ if $g(G)$ is odd.

Proof: Let G be a graph with $\delta(G) \geq 2$ and $g(G) \geq 5$ and let D be a γ_{ljcd} - set of G that is the line join graph of $LJ(D)$ is a connected graph with girth 3. Remove a g-cycle from G to form G'. Suppose a vertex $v \in V(G')$ has two neighbors, say x and y on the g-cycle which was removed from G. If $d(x, y) \leq 2$, then v, x, y are on either a C_3 or C_4 in G, contradicting the hypothesis that $g(G) \geq 5$. If $d(x, y) \geq 3$, then replacing the path from x to y on the g-cycle with the x, v, y reduce the girth of G to 3. This is a contradiction to the fact that $g(G) \geq 5$. Hence no vertex in G' has two or more neighbors on the g-cycle. Since $\delta(G) \geq 2$, the graph G' has minimum degree atleast $\delta(G) - 1 \geq 1$. Since we know that if a graph G has no isolated vertices then the domination number must be less than $\frac{n}{2}$, $\gamma_{ljcd}(G) \leq \left\lceil \frac{n-g(G)}{2} \right\rceil$ and obviously G' will be a line join connected grpah . In general, a cycle with length g can be dominated by $\left\lceil \frac{g-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil$ vertices if g is odd and a cycle with length g can be dominated by $\left\lceil \frac{g-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil + 1$ vertices if g is even. Hence , if $g(G)$ is odd

$$\gamma_{ljcd}(G) \leq \left\lceil \frac{n-g(G)}{2} \right\rceil + \left\lceil \frac{g-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil = \left\lceil \frac{n-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil.$$

and if $g(G)$ is even

$$\gamma_{ljcd}(G) \leq \left\lceil \frac{n-g(G)}{2} \right\rceil + \left\lceil \frac{g-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil + 1 = \left\lceil \frac{n-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil + 1.$$

Hence, if a graph G has $\delta(G) \geq 2$ and $g(G) \geq 5$, then $\gamma_{ljcd}(G) \leq \left\lceil \frac{n-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil + 1$ if $g(G)$ is even and $\gamma_{ljcd}(G) \leq \left\lceil \frac{n-\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil$ if $g(G)$ is odd.

V. CONCLUSION

Many researchers are concentrating various dominating concepts in graphs. In this paper we have taken a set of vertices and then constructed a new graph by using line graph concepts. Then we have defined a parameter the line join connected domination in graphs which is not the general connected domination. We have investigated the line join connected domination number for different types of graphs. Many results have been found and compared for some graphs.

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