Generalization of Soft μ-Closed Sets in Soft Generalized Topological spaces

M. Vigneshwaran [#1], K. Baby [*2]

[1] *Assistant professor, Department of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamilnadu, India.*

[2] *Ph.D Research Scholar, Department of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamilnadu, India.*

Abstract: *In this paper we introduce four new sets namely soft αμ -closed sets, soft gαμ -closed sets, soft *gαμ -closed sets, soft β*gαμ-closed sets in Soft Generalized Topological Spaces and derive their relationship. By using soft β*gαμ -closed sets, we introduce some seperation axioms and investigate their properties with the help of continuity and irresoluteness in Soft Generalized Topological Spaces (SGTS).*

Keywords: *soft αμ -closed sets, soft gαμ-closed sets, soft *gαμ-closed sets, soft β*gαμ -closed sets, soft soft μ*-space, soft soft *gα ½** -space, soft $_c\mu_{\beta^*g\alpha}$-space, soft (μ, η) β*gα -continuous, soft (μ, η) β*gα -open, soft (μ, η) β*gα --pre open, soft (μ, η) β*gα -irresolute, soft generalized β*gαμ -T_0 space, soft generalized β*gαμ-T_1 space, soft generalized β*gα -hausdroff space.*

I. INTRODUCTION

The soft set theory is a rapidly processing field of mathematics. This new set theory has found its applications in Game Theory, Operations Research, Theory of Probability, Riemann Integration, Perron Integration, Smoothness of functions, etc. The main objective behind application of such theory is to derive an effective solution from an uncertain and inadequate data. Molodtsov's **[9]** introduced the concept Soft Set Theory which was originally proposed as general mathematical tool for dealing with uncertainty problems. Maji *et al* **[8]** proposed several operations on soft sets and some basic properties. Shabir and Naz **[11]** introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Aktas and Cagman[**1**] compared soft sets to fuzzy sets and rough sets. Babitha and Sunil[**2**] introduced the soft sets relation and discussed related concepts such as equivalent soft set relation, partition and composition. A.Csaszar [4] introduced the theory of generalized topological spaces. Jyothis and Sunil [**6**] introduced the concept of Soft Generalized Topological Space (SGTS) and studied Soft μ-compactness in SGTSs. The generalized topology is different from general topology by its axioms. According to Csaszar, a collection of subsets of X is a generalized topology on X if and only if it called a Soft Generalized Topological Space (SGTS). contains the empty set and arbitrary union of its elements. But soft generalized topology is based on soft set theory. Jyothis and Sunil [**7**] discussed some separation axioms in soft generalized topological space

II. PRELIMINARIES

Throughout this paper U be an initial universe and E be a set of parameters. Let P (U) denote the power set of U and A be a non-empty subset of E.

Definition: 2.1 [9] Let a soft set F_A over the universe U is defined by the set of ordered pairs

$F_A = \{(e, f_A(e)) / e \in E, f_A(e) \in P(U)\}$, where f_A is a mapping given by $f_A: A \rightarrow P(U)$ such that $f_A(e)=\phi$ if $e \notin A$. Here f_A is called an approximate function of soft set F_A. The set of all soft sets over U is denoted by S(U).

Definition: 2.2 [6] Let $F_A \in S(U)$. A Soft Generalized Topology (SGT) on F_A, denoted by μ or μ_{F_A} is a collection of soft subsets of F_A having the following properties: (i) $F_\emptyset \in \mu$ and (ii) The soft union of any number of soft sets in μ belong to μ. The pair (F_A, μ) is

Definition: 2.3 [6] Let $F_A \in S(U)$ and μ be the collection of all possible soft subsets of F_A, then μ is a SGT on F_A, and is called the discrete SGT on F_A.

Definition: 2.4 [6] Let (F_A, μ) be a SGTS. Then, every element of μ is called a soft μ-open set. Note that $F_\emptyset$ is a soft μ-open set.

Definition: 2.5 [6] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the collection $\mu=\{F_D \cap F_B / F_D \in \mu\}$is called a Subspace Soft Generalized Topology (SSGT) on F_B. The pair (F_A, μ_{F_A}) is called a Soft Generalized Topological Subspace (SGTSS) of F_A.

Definition: 2.6 A soft subset F_B in a generalized topological space is defined to be a Q set iff $i_\mu(c_\mu(F_B)) = c_\mu(i_\mu(F_B))$.

Definition: 2.7 [7] Let (F_A, μ) be a SGTS and $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. If there exists soft μ-open sets F_G and F_H such that $\alpha \in F_G$ and $\beta \notin F_G$ or $\beta \in F_H$ and $\alpha \notin F_H$, then (F_A, μ) is called a soft generalized μ-T_0 space.

Definition: 2.8 [7] Let (F_A, μ) be a SGTS and $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. If there exists soft μ-open sets F_G and F_H such that $\alpha \in F_G$ and $\beta \notin F_G$ and $\beta \in F_H$ and $\alpha \notin F_H$, then (F_A, μ) is called a soft generalized μ-T_1 space.

Definition:2.9 [6] Let (F_A, μ) be a SGTS and $\alpha \in F_A$. If there is a soft μ-open set F_B such that $\alpha \in F_B$, then F_B is called a soft μ-open neighborhood or soft μ-nbd of α. The set of all soft μ-nbds of α, denoted by $\psi(\alpha)$, is called the family of soft μ-nbds of α. i.e, $\psi(\alpha) = \{F_B / F_B \in \mu, \alpha \in F_B\}$.

Definition 2.10 [7] Let (F_A, μ) be a SGTS. If for all $\alpha_1, \alpha_2 \in F_A$ with $\alpha_1 \neq \alpha_2$, there exists $F_G \in \psi(\alpha_1)$ and $F_H \in \psi(\alpha_2)$ such that $F_G \cap F_H = F_\emptyset$, then (F_A, μ) is called a soft generalized μ-T_2 space or soft generalized Hausdorff space (SGHS).

Definition: 2.11 [6] Let (F_A, μ) and (F_B, η) be two SGTS's and $\varphi_\chi:(F_A, \mu) \rightarrow (F_B, \eta)$be a soft function. Then

1. φ_χ is said to be soft (μ, η)-continuous (briefly, soft continuous), if for each soft η-open subset F_G of F_B, the inverse image $\varphi_\chi^{-1}(F_G)$ is a soft μ-open subset of F_A.
2. φ_χ is said to be soft (μ, η)-open, if for each soft μ-open subset F_G of F_A, the image $\varphi_\chi(F_G)$ is a soft η-open subset of F_B.
3. φ_χ is said to be soft (μ, η)-closed, if for each soft μ-closed subset F_G of F_A, the image $\varphi_\chi(F_G)$ is a soft η-closed subset of F_B.

III. Soft α μ-closed sets, Soft gα μ-closed sets, Soft *gα μ-closed sets, Soft β*gα μ-closed sets in Soft Generalized topological spaces.

Throughout this paper (F_A, μ), (F_B, μ) and (F_c, μ) denotes soft generalized topological spaces. $c_\mu(F_D)$, $i_\mu(F_D)$, $c_{\mu_\alpha}(F_D)$ and $c_{\mu_\beta}(F_D)$ denotes soft closure of F_D, soft interior of F_D, soft α-closure of F_D and soft β-closure of F_D in soft generalized topological spaces respectively.

Definition: 3.1 A soft subset $F_D \in S(U)$ is called a soft $\alpha\mu$-closed in soft generalized topological space (F_A, μ) if $c_\mu(i_\mu(c_\mu(F_D))) \subseteq F_D$.

Definition:**3.2** Let (F_A, μ) be a SGTS .A subset F_G of F_A is said to be soft μ-regular if $F_G= i_\mu(c_\mu(F_G))$

Definition:3.3 A soft subset $F_D \in S(U)$ is called a soft pre μ-closed in soft generalized topological space (F_A, μ) if $c_\mu(i_\mu(F_D)) \subseteq F_D$.

Defintion: 3.4 A soft subset $F_D \in S(U)$ is called a soft $\beta\mu$-closed in soft generalized topological space (F_A, μ) if $i_\mu(c_\mu(i_\mu(F_D))) \subseteq F_D$.

Definition: 3.5 A soft subset $F_D \in S(U)$ is called a soft gα μ-closed in soft generalized topological space (F_A, μ) if $c_{\mu_\alpha}(F_D) \subseteq F_E$, whenever $F_D \subseteq F_E$ and F_E is soft α μ-open.

Definition: 3.6 A soft subset $F_D \in S(U)$ is called a soft *gα μ-closed in soft generalized topological space (F_A, μ) if $c_\mu(F_D) \subseteq F_E$, whenever $F_D \subseteq F_E$ and F_E is soft gα μ-open.

Definition: 3.7 A soft subset $F_D \in S(U)$ is called a soft β*gα μ-closed in soft generalized topological space (F_A, μ) if $c_{\mu_\beta}(F_D) \subseteq F_E$, whenever $F_D \subseteq F_E$ and F_E is soft *gα μ-open.

Theorem: 3.8 Every soft μ-closed set is soft gα μ-closed.

Proof: Suppose $F_D \subseteq F_E$ and F_E is soft $\alpha\mu$-open in (F_A, μ). Since F_D is Soft μ -closed and $F_D \subseteq F_E$, $c_\mu (F_D) = F_D \subseteq F_E$. Thus $c_{\mu_\alpha}(F_D) \subseteq c_\mu (F_D) \subseteq F_E$. Hence F_D is soft gα μ-closed.

Converse of the above theorem need not be true by the following example.

Example: 3.9 Let U = {a, b}, E={e_1, e_2, e_3} and A={ e_1, e_2}

The set of soft sets over the universe U with the parameter E is given by

F_ϕ={(e_1, ϕ),(e_2, ϕ)},
F_1={(e_1, ϕ), (e_2,{a})},
F_2= {(e_1, ϕ), (e_2,{b})},
F_3={(e_1, ϕ), (e_2,{a,b})},
F_4= {(e_1, {a}), (e_2, ϕ)},
F_5= {(e_1, {a}), (e_2, {a})},
F_6= {(e_1, {a}), (e_2, {b})},
F_7= {(e_1, {a}), (e_2, {a, b})},
F_8= {(e_1, {b}), (e_2, ϕ)},
F_9= {(e_1, {b}), (e_2, {a})},
F_{10}= {(e_1, {b}), (e_2, {b})},
F_{11}= {(e_1, {b}), (e_2, {a, b})},
F_{12}= {(e_1, {a, b}), (e_2, ϕ)},
F_{13}= {(e_1, {a, b}), (e_2,{a})},
F_{14}= {(e_1, {a, b}), (e_2, {b})},
F_{15}= {(e_1, {a, b}), (e_2, {a, b})}

μ={F_ϕ, F_2, F_4, F_6, F_7, F_{13}, F_{15}} and
μ^c = {F_ϕ, F_2, F_8, F_9, F_{11}, F_{13}, F_{15}}

soft gα μ-closed = { F_ϕ, F_1, F_2, F_3, F_8, F_9, F_{J10}, F_{11}, F_{13}, F_{15} }.

Here the soft gα μ-closed sets F_1, F_3, F_{10} are not soft μ -closed.

Theorem: 3.10 Every Soft μ-closed set is Soft *gα μ-closed.

Proof: Suppose $F_D \subseteq F_G$ and F_G is soft gα μ-open in (F_A, μ). Since F_D is soft μ-closed set , $c_\mu (F_D) = F_D \subseteq F_G$ Thus $c_\mu (F_D) \subseteq F_G$. Hence F_D is soft *gα μ-closed.

Converse of the above theorem need not be true by the following example**.**

Example: 3.11 Let U = {a, b}, E={e_1, e_2, e_3} and A={e_1, e_2} with μ = {F_ϕ, F_2, F_4, F_6, F_7, F_{13}, F_{15}} and μ^c = { F_ϕ, F_2, F_8, F_9, F_{11}, F_{13}, F_{15}}.

Soft *g$\alpha\mu$-closed sets of (F_A, μ) = { F_ϕ, F_2, F_8, F_9, F_{10}, F_{11}, F_{13}, F_{14}, F_{15}}

Here the soft *g$\alpha\mu$-closed sets F_{10} and F_{14} are not Soft μ -closed .

Theorem: 3.12 Every Soft α μ-closed set is Soft gα μ-closed.

Proof: Suppose $F_D \subseteq F_G$ and F_G is a soft $\alpha\mu$-open in (F_A, μ). Since F_D is soft $\alpha\mu$-closed, $c_{\mu_\alpha}(F_D) = F_D \subseteq F_G$. Thus $c_{\mu_\alpha}(F_D) \subseteq F_G$. Hence F_D is soft gα μ-closed.

Converse of the above theorem need not be true by the following example.

Example: 3.13 Let U = {a, b}, E = {e_1, e_2, e_3} and A={ e_1, e_2} with μ={F_ϕ, F_6, F_9, F_{15}} and μ^c = { F_ϕ, F_6, F_9, F_{15}}.

Soft α μ -closed sets of (F_A, μ) = {F_ϕ, F_6, F_9, F_{15}} and Soft gα μ-closed sets of (F_A, μ) = S(U).

Here soft gα μ-closed sets F_1, F_2, F_3, F_4, F_5, F_7, F_8, F_{10}, F_{11}, F_{12}, F_{13}, F_{14} are not Soft α μ –closed.

Theorem: 3.14 Every Soft *gα μ -closed set is Soft gα μ-closed.

Proof: Suppose $F_D \subseteq F_G$ and F_G is a soft $\alpha\mu$-open in (F_A, μ). Since every soft $\alpha\mu$-open set is soft g$\alpha\mu$-open, F_G is soft g$\alpha\mu$-open. Since F_D is Soft *g$\alpha\mu$-closed, $c_\mu (F_D) \subseteq F_G$. Thus $c_{\mu_\alpha}(F_D) \subseteq c_\mu (F_B) \subseteq F_G$ which implies $c_{\mu_\alpha}(F_D) \subseteq F_G$. Hence F_D is soft gα μ-closed.

Converse of the above theorem need not be true by the following example.

Example: 3.15 Let U = {a, b}, E={e_1, e_2 , e_3} and A={ e_1, e_2} with μ={F_ϕ, F_6, F_7, F_{12}, F_{14}, F_{15}} and μ^c = { F_ϕ, F_1, F_3, F_8, F_9, F_{15}}.

Soft gαμ-closed sets of (F_A, μ) = { F_ϕ, F_1, F_2, F_3, F_4, F_5, F_8, F_9, F_{10}, F_{11}, F_{15}} and
Soft *gαμ-closed sets of (F_A, μ) = { F_ϕ, F_1, F_3, F_8, F_9, F_{15}}.
Here the Soft gαμ-closed sets F_2, F_4, F_5, F_{10}, F_{11} are not Soft *gα μ-closed.

Theorem: 3.16 Every soft μ-closed set is soft β*gα μ-closed.

Proof: Suppose $F_D \subseteq F_E$ and F_E is soft *gαμ-open in (F_A, μ). Since F_D is Soft μ -closed and $F_D \subseteq F_E$,
$c_\mu (F_D) = F_D \subseteq F_E$. Thus $c_{\mu_\beta}(F_D) \subseteq c_\mu (F_D) \subseteq F_E$. Hence F_D is soft β*gα μ-closed.

Converse of the above theorem need not be true by the following exampl**e.**

Example: 3.17 Let U = {a, b}, E={e_1, e_2, e_3} and
A={ e_1, e_2} with μ={F_ϕ, F_2, F_4, F_6, F_7, F_{13}, F_{15}} and
μ^c = { F_ϕ, F_2, F_8, F_9, F_{11}, F_{13},F_{15}}.
Soft β*gαμ-closed sets of (F_A, μ) = { F_ϕ, F_1, F_2 , F_8, F_9, F_{10}, F_{11}, F_{13},F_{15}}.
Here Soft β*gαμ-closed sets F_1 and F_{10} are not soft μ -closed.

Theorem: 3.18 Every Soft *gα μ -closed set is Soft β*gα μ-closed but not conversely.

Proof: Suppose $F_D \subseteq F_G$ and F_G is a soft *gαμ-open in (F_A, μ). Since every soft *gαμ-open set is soft gαμ-open, F_G is soft gα μ-open. Since F_D is Soft *gαμ-closed, $c_\mu (F_D) \subseteq F_G$. Thus $c_{\mu_\beta}(F_D) \subseteq c_\mu (F_B) \subseteq F_G$ which implies $c_{\mu_\beta}(F_D) \subseteq F_G$. Hence F_D is soft β*gα μ-closed.

Example: 3.19 Let U = {a, b}, E={e_1, e_2, e_3} and
A={ e_1, e_2} with μ={F_ϕ, F_2, F_4, F_6, F_7, F_{13}, F_{15}} and
μ^c = { F_ϕ, F_2, F_8, F_9, F_{11}, F_{13},F_{15}}.
Soft *gαμ-closed sets of (F_A, μ) = { F_ϕ, F_2, F_8, F_9, F_{10}, F_{11}, F_{13},F_{15}}.
Soft β*gαμ-closed sets of (F_A, μ) = { F_ϕ, F_1, F_2 , F_8, F_9, F_{10}, F_{11}, F_{13},F_{15}}.
Here Soft β*gαμ-closed sets F_1 is not soft *gαμ -closed.

Definition: 3.20 A space (F_A, μ) is said to be

(i)soft μ*-space if every soft gα μ-closed set is soft αμ-closed,
(ii) soft *gα ½** -space if every soft *gα μ-closed set is soft μ-closed,
(iii) soft $_c\mu_{\beta^*g\alpha}$ -space if every soft β*gα μ-closed set is soft μ-closed.

Theorem: 3.21 If (F_A, μ) is a soft μ*-space, then every singleton of (F_A, μ) is soft μ-closed or soft α μ-open.

Proof: Assume that for $\alpha \in F_A$, {α} is not soft μ-closed. Then $F_A \setminus \{\alpha\}$ is not soft μ- open. Thus the only soft μ -open set containing $F_A \setminus \{\alpha\}$ is F_A itself and hence $F_A \setminus \{ \alpha \}$is trivially soft gα μ-closed. Since (F_A, μ) is a soft μ*-space, $F_A \setminus \{\alpha\}$ soft α μ-closed which implies {α} is soft α μ-open.

1V. SOFT GENERALİZED SEPARATION AXIOMS IN SGTSs

Definition: 4.1 Let (F_A, μ) be a SGTS and α, β ∈ F_A such that α ≠ β. If there exists soft β*gαμ-open sets F_G and F_H such that $\alpha \in F_G$ and $\beta \notin F_G$ or $\beta \in F_H$ and $\alpha \notin F_H$, then (F_A, μ) is called a soft generalized β*gα μ-T_0 space.

Theorem: 4.2 Let (F_A, μ) be a SGTS and α, β ∈F_A such that α ≠ β. If there exists soft β*gαμ-open sets F_G and F_H such that $\alpha \in F_G$ and $\beta \in (F_G)^c$ or $\beta \in F_H$ and $\alpha \in (F_H)^c$, then (F_A, μ) is a soft generalized β*gαμ-T_0 space.

Proof: Let α, β ∈ F_A such that α ≠ β and
F_G, F_H ∈ β*gαμ such that $\alpha \in F_G$ and $\beta \in (F_G)^c$ or
$\beta \in F_H$ and $\alpha \in (F_H)^c$. If $\alpha \in (F_H)^c$ then $\alpha \notin ((F_H)^c)^c = F_H$. Similarly if $\beta \in (F_G)^c$ then $\beta \notin ((F_G)^c)^c = F_G$. Hence there exists soft β*gαμ-open sets F_G, F_H such that $\alpha \in F_G$ and $\beta \notin F_G$ or $\beta \in F_H$ and $\alpha \notin F_H$. Hence (F_A, μ) is a soft generalized β*gαμ -T_0 space.

Remark: 4.3 Every soft generalized μ-T_0 space is soft generalized β*gαμ -T_0 space.

Proof: Since every soft μ-open set is soft β*gαμ-open, proof is obvious.

Definition: 4.4 Let (F_A, μ) and (F_B, η) be two SGTS's and φ_χ: $(F_A, \mu) \rightarrow (F_B, \eta)$ be a soft function. Then

1. φ_χ is said to be soft (μ, η) $\beta^*g\alpha$ -continuous, if for each soft η-open subset F_G of F_B, the inverse image $\varphi_\chi^{-1}(F_G)$ is a soft $\beta^*g\alpha\mu$-open subset of F_A.

2. φ_χ is said to be soft (μ, η) $\beta^*g\alpha$ -open, if for each soft μ-open subset F_E of F_A, the image $\varphi_\chi(F_F)$ is a soft $\beta^*g\alpha\eta$-open subset of F_B.

3. φ_χ is said to be soft (μ, η) $\beta^*g\alpha$ -closed, if for each soft μ-closed subset F_H of F_A, the image $\varphi_\chi(F_H)$ is a soft $\beta^*g\alpha\eta$-closed subset of F_B.

4. φ_χ is said to be soft (μ, η) $\beta^*g\alpha$ pre-closed, if for each soft $\beta^*g\alpha\mu$-closed subset F_H of F_A, the image $\varphi_\chi(F_H)$ is a soft $\beta^*g\alpha\eta$-closed subset of F_B.

5. φ_χ is said to be soft (μ, η) $\beta^*g\alpha$ pre-open, if for each soft $\beta^*g\alpha\mu$-open subset F_E of F_A, the image $\varphi_\chi(F_E)$ is a soft $\beta^*g\alpha\eta$-open subset of F_B.

6. φ_χ is said to be soft (μ, η) $\beta^*g\alpha$ -irresolute, if for each soft $\beta^*g\alpha\eta$-open subset F_G of F_B, the inverse image $\varphi_\chi^{-1}(F_G)$ is a soft $\beta^*g\alpha\mu$-open subset of F_A.

Theorem: 4.5 If $\varphi_\chi : (F_A, \mu) \rightarrow (F_B, \eta)$ soft (μ, η) $\beta^*g\alpha$-continuous and (F_A, μ) is soft ${}_c\mu_{\ \beta^*g\alpha}$ -space, then φ_χ is soft (μ, η)-continuous.

Proof: Let F_E be soft η-closed in (F_B, η). Since φ_χ: $(F_A, \mu) \rightarrow (F_B, \eta)$ is soft (μ, η) $\beta^*g\alpha$ -continuous, $\varphi_\chi^{\ -1}(F_E)$ is $\beta^*g\alpha\mu$-closed in (F_A, μ). Since (F_A, μ) is soft ${}_c\mu_{\ \beta^*g\alpha}$ -space, $\varphi_\chi^{\ -1}(F_E)$ is μ-closed. Therefore φ_χ is soft (μ, η)-continuous.

Theorem: 4.6 Let $\varphi_\chi : (F_A, \mu) \rightarrow (F_B, \eta)$ and $\Psi_\lambda : (F_B, \eta) \rightarrow (F_C, \gamma)$

(i) Ψ_λ □ $\varphi_\chi : (F_A, \mu) \rightarrow (F_C, \gamma)$ is soft (μ, γ) β $*g\alpha$ -continuous, if Ψ_λ is soft (η, γ)-continuous and φ_χ is soft (μ, η) β $*g\alpha$ -continuous.

(ii) Ψ_λ □ $\varphi_\chi : (F_A, \mu) \rightarrow (F_C, \gamma)$ is soft (μ, γ) β $*g\alpha$ -irresolute, if is Ψ_λ is soft (η, γ) $\beta^*g\alpha$ -irresolute and φ_χ is soft (μ, η) β $*g\alpha$ -irresolute.

(iii) Ψ_λ □ $\varphi_\chi : (F_A, \mu) \rightarrow (F_C, \gamma)$ is soft (μ, γ) β $*g\alpha$-continuous, if Ψ_λ is soft (η, γ) $\beta^*g\alpha$-continuous and φ_χ is soft (μ, η) β $*g\alpha$-irresolute.

(iv) Ψ_λ □ $\varphi_\chi : (F_A, \mu) \rightarrow (F_C, \gamma)$ is soft (μ, γ) β $*g\alpha$-continuous, if φ_χ and Ψ_λ are soft (μ, η) $\beta^*g\alpha$-continuous and soft (η, γ) $*g\alpha$-continuous respectively and (F_B, η) soft ${}_c\mu_{\ \beta^*g\alpha}$ -space.

Proof (i) Let F_E be soft γ -closed in (F_C, γ). Since Ψ_λ is continuous, $\Psi_\lambda^{\ -1}(F_E)$ is soft η -closed. Since φ_χ is soft (μ, η) $\beta^*g\alpha$-continuous, $(\Psi_\lambda$ □ $\varphi_\chi)^{\ -1}(F_E) = \varphi_\chi^{\ -1}(\Psi_\lambda^{\ -1}(F_E))$ is $\beta^*g\alpha$ μ-closed in (F_A, μ). Therefore Ψ_λ □ φ_χ is soft
(μ, γ) β $*g\alpha$-continuous.

(ii) Let F_E be soft $\beta^*g\alpha$ γ -closed in (F_C, γ). Since Ψ_λ is soft (η, γ) $\beta^*g\alpha$ -irresolute, $\Psi_\lambda^{-1}(F_E)$ is soft $\beta^*g\alpha$ η-closed. Since φ_χ is soft (μ, η) $\beta^*g\alpha$-irresolute,
$(\Psi_\lambda$ □ $\varphi_\chi)^{\ -1}(F_E) = \varphi_\chi^{\ -1}(\Psi_\lambda^{\ -1}(F_E))$ is $\beta^*g\alpha$ μ-closed in (F_A, μ). Therefore Ψ_λ □ φ_χ is soft (μ, γ) β $*g\alpha$-irresolute.

(iii) Let F_E be soft γ -closed in (F_C, γ). Since Ψ_λ is soft (η, γ) $\beta^*g\alpha$ -continuous, $\Psi_\lambda^{\ -1}(F_E)$ is soft $\beta^*g\alpha\eta$-closed. Since φ_χ is soft (μ, η) β $*g\alpha$-irresolute,
$(\Psi_\lambda$ □ $\varphi_\chi)^{\ -1}(F_E) = \varphi_\chi^{\ -1}(\Psi_\lambda^{\ -1}(F_E))$ is β $*g\alpha$ μ-closed in (F_A, μ). Therefore Ψ_λ □ φ_χ is soft (μ, γ) β $*g\alpha$-continuous.

(iv) F_E be soft γ -closed in (F_C, γ). Since Ψ_λ is soft (η, γ) $\beta^*g\alpha$-continuous, $\Psi_\lambda^{\ -1}(F_E)$ is soft $\beta^*g\alpha$ η-closed. Since (F_B, η) is is soft ${}_c\mu_{\ \beta^*g\alpha}$ -space, $\Psi_\lambda^{\ -1}(F_E)$ is soft η-closed. Since φ_χ is soft (μ, η) β $*g\alpha$-continuous,
$(\Psi_\lambda$ □ $\varphi_\chi)^{\ -1}(F_E) = \varphi_\chi^{\ -1}(\Psi_\lambda^{\ -1}(F_E))$ is $\beta^*g\alpha\mu$-closed in (F_A, μ). Therefore Ψ_λ □ φ_χ is soft (μ, γ) β $*g\alpha$-continuous.

Theorem: 4.7 A surjective function
$\varphi_\chi : (F_A, \mu) \rightarrow (F_B, \eta)$ is soft (μ, η) $\beta^*g\alpha$-closed if and only if for each subset F_E of F_B and each μ-open set F_D of F_A containing $\varphi_\chi^{-1}(F_E)$, there exists a soft $\beta^*g\alpha$ η-open set F_F of F_B such that $F_E \subseteq F_F$ and $\varphi_\chi^{-1}(F_F) \subseteq F_D$.

Proof: Suppose that φ_χ is soft (μ, η) $\beta^*g\alpha$ -closed. Let F_E be any soft subset of F_B and F_D be μ-open set of F_A containing $\varphi_\chi^{-1}(F_E)$. Put $F_F = F_B - \varphi_\chi (F_A - F_D)$. Then, the complement of F_F is $F_B - F_F = \varphi_\chi (F_A - F_D)$. Since $F_A - F_D$ is μ-closed in (F_A, μ) and φ_χ is soft (μ, η) $\beta^*g\alpha$ -closed, $F_B - F_F = \varphi_\chi (F_A - F_D)$ is soft β $*g\alpha\eta$ -closed in (F_B, η) that is F_F is soft β $*g\alpha\eta$ -open in

(F_B, η). Hence $F_E \subseteq F_F$ and $\varphi_\chi^{-1}(F_F) \subseteq F_D$.

Conversely, let F_G be a soft μ -closed set of F_A. Put $F_E = F_B - \varphi_\chi(F_G)$, then we have $\varphi_\chi^{-1}(F_E) = F_A - F_G$ and $F_A - F_G$ is soft μ -open in (F_A, μ). Then by the assumption, there exists a soft $\beta^*g\alpha\eta$ -open set F_F of (F_B, η) such that $F_E = F_B - \varphi_\chi(F_G) \subseteq F_F$ and $\varphi_\chi^{-1}(F_F) \subseteq F_A - F_G$. Now $\varphi_\chi^{-1}(F_F) \subseteq F_A - F_G$ implies $F_F \subseteq F_B - \varphi_\chi(F_G) = F_E$. Also $F_E \subseteq F_F$ and so $F_E = F_F$. Therefore, we obtain $\varphi_\chi(F_G) = F_B - F_F$ and hence $\varphi_\chi(F_G)$ is soft $\beta^*g\alpha\eta$ -closed. This shows that the soft mapping φ_χ is soft (μ, η) $\beta^*g\alpha$ -closed.

Theorem: 4.8 A surjective soft function $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ is soft (μ, η) $\beta^*g\alpha$ -irresolute and soft (μ, η)-closed function. Then $\varphi_\chi(F_G)$ is soft $\beta^*g\alpha$ η-closed in (F_B, η), for every soft $\beta^*g\alpha$ μ -closed set F_G of (F_A, μ).

Proof: Let F_G be any soft $\beta^*g\alpha$ μ -closed set of (F_A, μ) and F_K be soft $\beta^*g\alpha$ η-open set of (F_B, η) such that $\varphi_\chi(F_G) \subseteq F_K$. Since φ_χ is surjective soft function and soft (μ, η) $\beta^*g\alpha$ -irresolute, $\varphi_\chi^{-1}(F_K)$ is soft $\beta^*g\alpha\mu$-open in (F_A, μ). Since $F_G \subseteq \varphi_\chi^{-1}(F_K)$ and F_G is soft $\beta^*g\alpha$ μ -closed set of (F_A, μ), $c_\mu(F_G) \subseteq \varphi_\chi^{-1}(F_K)$. Then $\varphi_\chi(c_\mu(F_G)) \subseteq \varphi_\chi(\varphi_\chi^{-1}(F_K)) = F_K$. Since φ_χ is soft (μ, η)-closed, $\varphi_\chi(c_\mu(F_G)) = c_\mu(\varphi_\chi(c_\mu(F_G)))$. This implies

$c_\mu(\varphi_\chi(F_G)) \subseteq c_\mu(\varphi_\chi(c_{\mu_\beta}(F_G))) = \varphi_\chi(c_\mu(F_G)) \subseteq F_K$.

Therefore $\varphi_\chi(F_G)$ is soft $\beta^*g\alpha$ η-closed in (F_B, η).

Theorem: 4.9 A surjective soft function $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ is soft (μ, η) $\beta^*g\alpha$ -irresolute and soft (μ, η)-closed. If (F_A, μ) is soft ${}_c\mu_{\beta^*g\alpha}$ -space, then (F_B, η) is also soft ${}_c\mu_{\beta^*g\alpha}$ -space.

Proof: Let F_G be soft $\beta^*g\alpha\eta$-closed in (F_B, η). Since φ_χ is soft (μ, η) $\beta^*g\alpha$ -irresolute, $\varphi_\chi^{-1}(F_G)$ is soft $\beta^*g\alpha$ μ-open in (F_A, μ). Since (F_A, μ) is soft ${}_c\mu_{\beta^*g\alpha}$ -space, $\varphi_\chi^{-1}(F_G)$ is closed in (F_A, μ). Since φ_χ is soft (μ, η)-closed function, $\varphi_\chi(\varphi_\chi^{-1}(F_G))$ is soft μ- closed in (F_B, η). Since φ_χ is surjective soft function, $\varphi_\chi(\varphi_\chi^{-1}(F_G)) = F_G$ and hence F_G is soft η - closed in (F_B, η). Therefore (F_B, η) is soft ${}_c\mu_{\beta^*g\alpha}$ -space.

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Theorem: 4.10 Let $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft (μ, η) $\beta^*g\alpha$-irresolute, soft bijective function. If (F_B, η) is a soft generalized $\beta^*g\alpha$ η-T_0 space, then (F_A, μ) is also a soft generalized $\beta^*g\alpha$ μ-T_0 space.

Proof: Let (F_B, η) be a soft generalized $\beta^*g\alpha$ η-T_0 space. Suppose $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. Since φ_χ is soft injective, there exist $\gamma, \delta \in F_B$ such that $\gamma = \varphi_\chi(\alpha)$, $\delta = \varphi_\chi(\beta)$ and $\gamma \neq \delta$. Since (F_B, η) is a soft generalized $\beta^*g\alpha$ η-T_0 space, there exist soft $\beta^*g\alpha$ η-open sets F_G, F_H such that $\gamma \in F_G$ and $\delta \notin F_G$ or $\delta \in F_H$ and $\gamma \notin F_H$. This implies that $\varphi_\chi(\alpha) \in F_G$ and $\varphi_\chi(\beta) \notin F_G$ or $\varphi_\chi(\beta) \in F_H$ and $\varphi_\chi(\alpha) \notin F_H \Rightarrow \alpha \in \varphi_\chi^{-1}(F_G)$ and $\beta \notin \varphi_\chi^{-1}(F_G)$ or $\beta \in \varphi_\chi^{-1}(F_H)$ and $\alpha \notin \varphi_\chi^{-1}(F_H)$. Since φ_χ is a soft (μ, η) $\beta^*g\alpha$ -irresolute function, $\varphi_\chi^{-1}(F_G)$ and $\varphi_\chi^{-1}(F_H)$ are soft $\beta^*g\alpha$ μ-open sets. Hence (F_A, μ) is a soft generalized $\beta^*g\alpha$ μ-T_0 space.

Theorem: 4.11 Let $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft (μ, η) $\beta^*g\alpha$ pre-open soft bijective function. If (F_A, μ) is a soft generalized $\beta^*g\alpha$ μ-T_0 space, then (F_B, η) is a soft generalized $\beta^*g\alpha$ η-T_0 space.

Proof: Suppose that (F_A, μ) is a soft generalized $\beta^*g\alpha$ μ-T_0 space. Let $\alpha, \beta \in F_B$ such that $\alpha \neq \beta$. Since φ_χ is a soft bijective function there exist $\gamma, \delta \in F_A$ such that $\alpha = \varphi_\chi(\gamma)$, $\beta = \varphi_\chi(\delta)$ and $\gamma \neq \delta$. Since (F_A, μ) is a soft generalized $\beta^*g\alpha$ μ-T_0 space, there exist soft $\beta^*g\alpha\mu$-open sets F_G, F_H such that $\gamma \in F_G$ and $\delta \notin F_G$ or $\delta \in F_H$ and $\gamma \notin F_H$. This implies that $\varphi_\chi(\gamma) \in \varphi_\chi(F_G)$ and $\varphi_\chi(\delta) \notin \varphi_\chi(F_G)$ or $\varphi_\chi(\delta) \in \varphi_\chi(F_H)$ and $\varphi_\chi(\gamma) \notin \varphi_\chi(F_H) \Rightarrow \alpha \in \varphi_\chi(F_G)$ and $\beta \notin \varphi_\chi(F_G)$ or $\beta \in \varphi_\chi(F_H)$ and $\alpha \notin \varphi_\chi(F_H)$. Since φ_χ is a soft (μ, η) $\beta^*g\alpha$ pre-open function, both $\varphi_\chi(F_G)$ and $\varphi_\chi(F_H)$ are soft $\beta^*g\alpha$ η-open sets. Hence (F_B, η) is a soft generalized $\beta^*g\alpha$ η -T_0 space.

Definition: 4.12 Let (F_A, μ) be a SGTS and $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. If there exists soft $\beta^*g\alpha$ μ-open sets F_G and F_H such that $\alpha \in F_G$ and $\beta \notin F_G$ and $\beta \in F_H$ and $\alpha \notin F_H$, then (F_A, μ) is called a soft generalized $\beta^*g\alpha$ μ-T_1 space.

Theorem: 4.13 Every soft generalized $\beta^*g\alpha\mu$-T_1 space is a soft generalized $\beta^*g\alpha$ μ-T_0 space.

Proof: Let (F_A, μ) be a soft generalized $\beta^*g\alpha$ μ-T_1 space and $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. So there exists soft $\beta^*g\alpha$ μ-open sets F_G and F_H such that $\alpha \in F_G$ and $\beta \notin F_G$ and $\beta \in F_H$ and $\alpha \notin F_H$. Obviously then we have $\alpha \in F_G$ and $\beta \notin F_G$ or $\beta \in F_H$ and $\alpha \notin F_H$. Hence (F_A, μ) is a soft generalized $\beta^*g\alpha\mu$-T_0 space.

Theorem: 4.14 Let (F_A, μ) be a soft generalized $\beta^*g\alpha$ μ-T_1 space and $\alpha \in F_A$.Then for each soft $\beta^*g\alpha$ μ-open set F_G with $\alpha \in F_G$, $\{\alpha\} \subseteq (\cap F_G)$.

Proof: Since $\alpha \in F_G$ for each soft $\beta^*g\alpha\mu$-open set F_G, $\alpha \in (\cap F_G)$. Then it is obvious that $\{\alpha\} \subseteq \cap F_G$.

Theorem: 4.15 Let $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft (μ, η) $\beta^*g\alpha$-irresolute soft bijective function. If (F_B, η) is a soft generalized $\beta^*g\alpha$ η-T_1 space.

Proof: it is similar to the proof of 4.9

Definition: 4.16 Let (F_A, μ) be a SGTS and $\alpha \in F_A$. If there is a soft $\beta^*g\alpha$ μ-open set F_B such that $\alpha \in F_B$, then F_B is called a soft $\beta^*g\alpha$ μ-open neighborhood or soft $\beta^*g\alpha$ μ-nbd of α. The set of all soft $\beta^*g\alpha\mu$-nbds of α, denoted by $N(\alpha)$.

Definition: 4.17 Let (F_A, μ) be a SGTS. If for all $\alpha_1, \alpha_2 \in F_A$ with $\alpha_1 \neq \alpha_2$, there exists $F_G \in N(\alpha_1)$ and $F_H \in N(\alpha_2)$such that $F_G \cap F_H = F_\emptyset$, then (F_A, μ) is called a soft generalized $\beta^*g\alpha$ μ-T_2 space or soft $\beta^*g\alpha$ generalized Hausdorff space.

Theorem 4.18 Every soft $\beta^*g\alpha$ generalized Hausdorff space is soft generalized $\beta^*g\alpha$ μ-T_1 space.

Proof: Let (F_A, μ) be a soft $\beta^*g\alpha$ generalized Hausdorff space and $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. Then there exist soft $\beta^*g\alpha$ μ-open sets F_G, F_H such that $\alpha \in F_G$, $\beta \in F_H$ and $F_G \cap F_H = F_\emptyset$. Since $F_G \cap F_H = F_\emptyset$, $\alpha \notin F_H$, and $\beta \notin F_G$. Thus F_G, F_H are $\beta^*g\alpha$ μ-open sets such that $\alpha \in F_G$, $\beta \notin F_G$ and $\beta \in F_H$ and $\alpha \notin F_H$. Hence (F_A, μ) is a soft generalized $\beta^*g\alpha$ μ-T_1 space.

Theorem: 4.19 Let $\varphi_\chi : (F_A,\mu) \to (F_B, \eta)$ be a soft bijective soft (μ, η) $\beta^*g\alpha$ pre-open function. If (F_A, μ) is soft generalized $\beta^*g\alpha$ Hausdorff space then (F_B, η) is soft generalized $\beta^*g\alpha$ Hausdorff space.

Proof: Let $\alpha, \beta \in F_B$ such that $\alpha \neq \beta$. Since φ_χ is a soft bijective function, there exist $\gamma, \delta \in F_A$ such that $\alpha = \varphi_\chi(\gamma)$, $\beta = \varphi_\chi(\delta)$ and $\gamma \neq \delta$. Since (F_A, μ) is soft generalized $\beta^*g\alpha$ Hausdorff space, there exist soft $\beta^*g\alpha$ μ-open set F_G, F_H such that $\gamma \in F_G$, $\delta \in F_H$ and $F_G \cap F_H = F_\emptyset$. This implies that $\varphi_\chi(\gamma) \in \varphi_\chi(F_G)$, $\varphi_\chi(\delta) \in \varphi_\chi(F_H) \Rightarrow \alpha \in \varphi_\chi(F_G)$ and $\beta \in \varphi_\chi(F_H)$. Since φ_χ is a soft (μ, η) $\beta^*g\alpha$ pre-open function, $\varphi_\chi(F_G)$ and $\varphi_\chi(F_H)$ are soft $\beta^*g\alpha$ η-open sets. Again since φ_χ is a soft bijective, $\varphi_\chi(F_G) \cap \varphi_\chi(F_H) = \varphi_\chi(F_G \cap F_H) = \varphi_\chi(F_\emptyset) = F_\emptyset$.

Theorem: 4.20 Let $\varphi_\chi : (F_A, \mu) \to (F_B,)$ be a soft bijective soft (μ, η) $\beta^*g\alpha$-irresolute function. If (F_B, η) is soft generalized $\beta^*g\alpha$ Hausdorff space, then (F_A, μ) is also soft generalized $\beta^*g\alpha$ Hausdorff space.

Proof: Let $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. Since φ_χ is a soft bijective function, there exist $\gamma, \delta \in F_B$ such that $\alpha = \varphi_\chi^{-1}(\gamma)$, $\beta = \varphi_\chi^{-1}(\delta)$ and $\gamma \neq \delta$. Since (F_B, η) is soft generalized $\beta^*g\alpha$-Hausdorff space, there exist soft $\beta^*g\alpha$ η-open sets F_G, F_H such that $\gamma \in F_G$, $\delta \in F_H$ and $F_G \cap F_H = F_\emptyset$. Then $\varphi_\chi^{-1}(F_G)$ and $\varphi_\chi^{-1}(F_H)$ are soft $\beta^*g\alpha$ μ-open sets, because φ_χ is soft (μ, η) $\beta^*g\alpha$-irresolute. Also $F_G \cap F_H = F_\emptyset$ which implies $\varphi_\chi^{-1}(F_G \cap F_H) = \varphi_\chi^{-1}(F_\emptyset)$ which implies $\varphi_\chi^{-1}(F_G) \cap \varphi_\chi^{-1}(F_H) = F_\emptyset$. Now $\gamma \in F_G$ and $\delta \in F_H$ implies $\varphi_\chi^{-1}(\gamma) \in \varphi_\chi^{-1}(F_G)$ and $\varphi_\chi^{-1}(\delta) \in \varphi_\chi^{-1}(F_H)$ implies $\alpha \in \varphi_\chi^{-1}(F_G)$ and $\beta \in \varphi_\chi^{-1}(F_H)$. Hence (F_A, μ) is soft generalized $\beta^*g\alpha$ Hausdorff space.

V. CONCLUSION

Hence we introduced four new soft μ-closed sets namely soft α μ-closed sets, soft $g\alpha$ μ-closed sets, soft $*g\alpha$ μ-closed sets, soft $\beta^*g\alpha$ μ-closed sets in Soft Generalized Topological Spaces and derived their relationship. By using soft $\beta^*g\alpha\mu$-closed sets, we introduced some seperation axioms and their properties are investigated with the help of continuity and irresoluteness in Soft Generalized Topological Spaces.

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