# An Inventory Ordering Interval, for Deterioration as a Constant Rate and Demand as Stock Dependent, with Discrete in Time

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**ABSTRACT:**

*In this paper ,we consider with the constant rate of deterioration having stock dependent demand. Demand is a linear function of lot size q. Shortage is not considered and rate of replenishment is instantaneous. A numerical example is considered to illustrate the model taken.*

**KEYWORDS:**

*Inventory control, constant deterioration, stock dependant demand time a discrete change.*

**INTRODUCTION**:

In recent years ,a good number of authors have discussed models with different kinds of decay rates. Cohen (1) considered joint pricing and ordering policy for exponentially decaying inventory with demand rate as a known function of price p of unit Mukherjee (2) considered an extension of cohen's model to the case where the decay rate is a function of time. Gupta and jauhari (3) further generalized Mukharjee`s model considering both the decay rate and the demand rate as function of time. In all these models, time is treated as a continuous variable, which is not exactly the case in practice. In real life situations time is always treated as a discrete variable. Gupta and jauhari (3) extend their model considering demand rate as a function of price P with constant decay rate of inventory and time variable is assumed to be discrete. The deterioration is a constant fraction of the on-hand inventory where as the replenishment is instantaneous.

**ASSUMPTION & NOTATIONS**

    I.      The demand rate (A+Bq) is known where A and B are constants and q is lot size.

   II.      Deterioration $\lambda$ is a constant fraction of the on-hand inventory.

 III.      The lead time is zero and shortages are not allowed.

 IV.      There is no replacement or repair under consideration.

   V.      The replenishment rate is infinite.

 VI.      The unit purchasing cost C, the holding cost h per unit per time unit and the ordering cost K per order are known and constant during the scheduling period T.

VII.      I(t) and $I_w$ (t) are inventories at any time t with and without decay respectively.

**THE MODEL:**

The difference equation describing the inventory level of the system at time t is

$$\Delta\, l(t) = -\lambda l(t) - (A - Bq)$$

$$\Rightarrow l(t+1) - l(t) + \lambda l(t) = -(A+Bq)$$

$$\Rightarrow l(t+1) + (\lambda-1)l(t) = -(A+Bq)$$

$$\text{Or } l(t+1) + (l-\lambda)l(t) = -(A+Bq)$$

Multiplying by $(1-\lambda)^{-(t+1)}$ both sides

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$$(1 - \lambda)^{-(t+1)}I(t + 1) - (1 - \lambda)^{-t}I(t) = -(1 - \lambda)^{-(t+1)}(A + Bq)$$

$$\Delta[(1 - \lambda)^{-t}I(t)] = -(1 - \lambda)^{-(t+1)}(A + Bq)$$

$$(1 - \lambda)^{-t}l(t) = -\Delta^{-1}(1 - \lambda)^{-(t+1)}(A + Bq) + C$$

$$= \frac{-(A+Bq)(1-\lambda)^{(t+1)}}{1} + C$$

$$\frac{1-\lambda^{-1}}{\qquad\qquad} - \quad |$$

$$= \frac{(A+Bq)(1-\lambda)^{-(t+1)}(1-\lambda)}{\lambda} + C$$

At t=0,l(t)=I(0)

$$\therefore l(0) = -\frac{A+Bq}{\lambda} + C$$

$$\therefore l(t)(1-\lambda)^t = l(0) + \frac{A+Bq}{\lambda} - \frac{(A+Bq)(1-\lambda)^{-1}}{\lambda} \qquad (2)$$

In case of non decay, the difference equation defining the decrease in inventory level due to demand only is

$$I_w(t) = -(A+Bq) \qquad (3)$$

$$I_w(t) = -(A+Bq)t + I(0) \qquad (4)$$

$$\because t=0 \ I_w(t) = I(0)$$

$$Z(t) = l_w(t) - l(t) \qquad (5)$$

Or $\quad z(t) = l(0) - (A+Bq)t - I(t) \qquad (6)$

$$Z(t) = l(t)(1-\lambda)^{-t} - \frac{(A+Bq)}{\lambda} + \frac{(A+Bq)(1-\lambda)^{-t}}{\lambda}$$

$$-(A+Bq)t - I(t)$$

The stock loss due to decay is

After putting the value of I(0) from (2) for t=T

$$Z(t) = \frac{A+Bq(1-\lambda)^{-T}}{\lambda} - \frac{(A+Bq)}{\lambda} + l(T)(1 - \lambda)^{-T}$$

$$-(A-Bq)T - I(T)$$

I(T) being zero in case of instantaneous replenishment.

Or $\quad Z(T) = \frac{A+Bq}{\lambda}\{(1 - \lambda)^{-T} - 1\} - (A + Bq)T \qquad (7)$

Since shortages are not allowed, the order quantity $Q_T$ must be sufficiently large to satisty the the demand during a cycle of length T. so $Q_1$ must be given by:

$$Q_T = Z(T) + (A+Bq)T$$

Or $\quad Q_T = \frac{A+Bq}{\lambda}\{(1 - \lambda)^{-T} - 1\} \qquad (8)$

In case of instantaneous replenishment

$$I(0)=Q_T$$

From (2), putting value of I(0)

$$I(t)(1-\lambda)^{-t}=\frac{(A+Bq)}{\lambda}\{(1-\lambda)^{-T}-1\}+\frac{(A+Bq)}{\lambda}-\frac{(A+Bq)}{\lambda}(1-\lambda)^{-t}$$

$$I(t)=\frac{A+Bq}{\lambda}\{(1-\lambda)^{t-T}-1\} \tag{9}$$

Also the average number of units in inventory per time unit during a cycle is

$$I_A(T)=\frac{(A+Bq)}{\lambda(T+l)}\sum_{t=0}^{T}\{(1-\lambda)^{t-T}-1\}$$

Or $l_A(T)=\frac{(A+Bq)}{\lambda(T+l)}[\{(1-\lambda)^{t-T}-1\}-\lambda T]$

Finally the total average cost per unit time of the system C(T,q) is

$$C(T,q)=\frac{k}{T}+\frac{CQ_T}{T}+h.l_A(T)$$

$$)=\frac{k}{T}+\frac{C}{\lambda T}(A+Bq)\{(1-\lambda)^{-T}-1\}$$

$$+\frac{h(A+Bq)}{\lambda^2(T+l)}[\{(1-\lambda)^{t-T}-1\}-\lambda T] \tag{10}$$

Since T must be a non-negative integer,the condition for C(T,q) to have absolute minimum at T=T* is

$$\Delta C(T^*-1,q)\leq0\leq\Delta C(T^*,q) \tag{11}$$

For a fixed lot size q,

$$\Delta C(T,q)=C(T+1,q)-C(T,q)$$

Using (10),(11) becomes at T=T*

$$\Delta C(T,q)=k[\frac{1}{T+1}-\frac{1}{T}]+\frac{C(A+Bq)}{\lambda}$$

$$[\frac{\{(1-\lambda)^{-(T+1)}-1\}}{T+1}-\frac{\{(1-\lambda)^{-T}-1\}}{T}]+\frac{h(A+Bq)}{\lambda^2(T+l)}$$

$$[\frac{\{(1-\lambda)^{t-T-1}-1\}-\lambda(T+1)}{T+2}-\frac{\{(1-\lambda)^{t-T}}{(T+1)}-1\}-\lambda T]$$

At T=T*

$$\phi(T^*)\leq\frac{k}{A+Bq}\leq\phi(T*+1) \tag{12}$$

If we take fixed T,for C(T,q)to have absolute minimum at q=q*

$$=C(T,q+1)-C(T,q)$$

$$\Delta C(T,q)=A(cons\tan t)$$

So from (12) we take absolute min at T=T*for a fixed q

Where

$$\phi(T) = \frac{c\,\Psi(T)}{\lambda(1-\lambda)^T} + \frac{h(T-1)\,\Psi(T+1)}{\lambda^2(T-1)(1-\lambda)^T}$$

And

$$\Psi(T) = (1-\lambda)^T + \lambda T - 1$$

When there is no decay, i.e. $\lambda = 0$

$$C(T,q) = \frac{k}{T} + c(A+Bq) + \frac{h(A+Bq)T}{2} \tag{13}$$

Special case:

(1)    If we put A=0 and B=R then q=1 then (13) reduce to

$$C(T,1) = \frac{k}{T} + CR + \frac{hRT}{2}$$

And

$$(T*-1, T*) \le \frac{2k}{hR} \le T^*(T^*+1)$$

Which is Rajendra jauhari's result for non-decaying inventory.

On maximizing profit function for a fix-period length we should find the optimum lot size decision.

Where

f(T,q)=q(A+Bq)-C(T,q)

The profit maximizing lot size q*, is obtained from

$$\Delta f(T,q^*) \le 0 \le \Delta f(T,q^*-1)$$

where

$$\Delta f(T,q^*) = f(T,q^*+1) - f(T,q^*)$$

$$\Rightarrow (q^*+1)[A+B(q^*+1)] - c(T,q^*+1)$$

$$-[q^*(A+Bq^*) - C(T,q^*)]$$

$$\le 0 \le [q(A+Bq^*) - C(T,q^*)]$$

$$-[(q^*-1)\{A+B(q^*-1) - C(T,q^*-1)]$$

$$\Rightarrow (Aq^*+A+Bq^*+2Bq^*+B-Aq^*-Bq^{*2})$$

$$-[(\tfrac{K}{T} - \tfrac{K}{T}) + \tfrac{c}{\lambda T}\{(1-\lambda)^{-T}-1\}\{A+B(q^*+1-q^*)\}]$$

$$+\frac{h(T+1)}{\lambda^2}[\{(1-\lambda)^{t-T} - 1\} - \lambda T]\{A+B(q*+1-q*)\}]$$

$$\le 0 \le [Aq^*+Bq^{*2}-Aq^*+A-Bq^*+2Bq^*-B]$$

$$-[(\tfrac{K}{T} - \tfrac{K}{T}) + \tfrac{c}{\lambda T}\{(1-\lambda)^{-T}-1\}\{A+B(q^*+1-q^*)\}]$$

$$+\frac{h}{\lambda^2(T+1)}[\{(1-\lambda)^{t-T}-1\}-\lambda T]\{A+B(q*+1-q*)]]$$

Or

$(A+2Bq*+B)-[\frac{C}{\lambda T}\{(1-\lambda)^{-T}-1\}\{A+B\}+\frac{h}{\lambda^2(T+1)}\{(1-\lambda)^{t-T}-1\}-\lambda T]$

$(A+B)]\leq 0\leq[A+2Bq*-B]-\{\frac{C}{\lambda T}\{(1-\lambda)^{-T}-1\}(A+B)$

$+[\frac{C}{\lambda T}\{(1-\lambda)^{-T}-1\}(A+B)+\frac{h}{\lambda^2(T+1)}\{(1-\lambda)^{t-T}-1\}-\lambda T]](A+B)\}$

Or

$(A+2Bq*+B)\leq\frac{B}{\lambda}[\frac{C}{T}\{(1-\lambda)^T-1\}+\frac{h}{\lambda^2(T+1)}[\{(1-\lambda)^{t-T}-1\}-\lambda T]]$

$\leq(A+2Bq*-B)$    -------(14)

For profit max. time $T*\Delta f(T*,q)=-\Delta C(T^*,q)$

If C (T,q) to have absolute minmum at T=T* then f(T,q) have absolute maximum at T=T*

Example

If k=300 per order,h=50 per unit per week

A=10,B=5,A+Bq=10+5q

For T*

| q | λ=.02 | λ=.06 | C |
|---|-------|-------|---|
| 10 | -1.2 | -.45 | 5 |

## CONCLUSION

For the numerical example taken above we have found the optional values of T for a fixed set of values of K and h and varying the values of parameters λ and q. The table indicates that the length of the optimal cyclic period T* decreases when λ increases.

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