Signed Product Cordial labeling in duplicate graphs of Bistar, Double Star and Triangular Ladder Graph

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Abstract *-In this paper, we prove that the duplicate graph of triangular ladder* $DG(TL_m)$*,* $m \geq 2$*, extended duplicate graph of bistar* $EDG(B_{m,m}), m \geq 2$ *and extended duplicate graph of double star* $EDG(DS_{m,m}), m \geq 2$ *admitsigned product cordial labeling.*

Keywords *-Graph labeling, duplicate graph, triangular ladder, bistar, double star, signed product cordial labeling.*

AMS Subject Classification: 05C78

I. INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967 [6]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling). In the intervening years various labeling of graphs have been investigated in over 2000 papers [4]. The concept of cordial labeling was introduced by I. Cahit[2]. The concept of duplicate graph was introduced by E. Sampathkumar and he proved many results on it [7]. The concept of signed product cordial labeling was introduced by J. BaskarBabujee and he proved that many graphs admit signed product cordial labeling. K. Thirusangu, P.P. Ulaganathan and B. Selvam, have proved that the duplicate graph of a path graph P_m is Cordial [8].K. Thirusangu, P.P. Ulaganathan and P. Vijayakumar have proved that the duplicate graph of Ladder graph $L_m, m \geq 2$, is cordial , total cordial and prime cordial[9].

II. PRELIMINARIES

In this section, we give the basic notions relevant to this paper.

Definition 2.1: A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling).

Definition2.3: Let $G(V,E)$ be a simple graph. A duplicate graph of G is $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V'$ *and* $V \cap V' = \emptyset$ and $f : V \to V'$ is bijective($for\ v \in V$, we write $f(v) = v'$) and the edge set E_1 of DG is defined as : The edge uv is in E if and only if both uv' and $u'v$ are edges in E_1.

Definition2.4:A vertex labeling of graph G $f: V(G) \to \{-1, 1\}$ with induced edge labeling $f^*: E(G) \to \{-1, 1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$, where $v_f(-1)$ is the number of vertices labeled with -1, $v_f(1)$ is the number of vertices labeled with 1, $e_f(-1)$ is the number of edges labeled with -1 and $e_f(1)$is the number of edges labeled with 1. [1].

Definition 2.5: The ladder graph L_m is a planar undirected graph with 2m vertices and 3m – 2 edges. It is obtained as the cartesian product of two path graphs, one of which has only one edge: $L_{m,1} = P_m X P_1$, where m is the number of rungs in the ladder.

Definition 2.6: The triangular ladder graph $TL_m$is obtained from the ladder graph by joining v_i *and* $v_{i+3}, 1 \leq i \leq m+2$.

Definition 2.7: The Star graph S_m is a complete bipartite graph $K_{1,m}$, where m represents the number of vertices and S_m has (m – 1) edges.

Definition 2.8: Double star $DS_{m,m}$ is a tree $K_{1,m,m}$ obtained from the star $K_{1,m}$ by adding a new pendent edge of the existing m pendant vertices. It has 2m + 1 vertices and 2m edges.[11].

Definition 2.9: Bistar$B_{m,m}$ is a graph obtained from K_2 by joining m pendant edges to each end of K_2. The edge K_2 is called the central edge of $B_{m,m}$ and the vertices of K_2 are called the central vertices of $B_{m,m}$. [5].

Definition2.10: The extended duplicate graph of bistar denoted by $EDG(B_{m,m})$, is obtained from the duplicate graph of bistar by joining v_1 and v_1'.

Definition2.11: The extended duplicate graph of double star denoted by $EDG(DS_{m,m})$, is obtained from the duplicate graph of double star by joining v_1 and v_1'.

III. MAIN RESULTS

Algorithm 3.1: Construction of duplicate graph of triangular ladder $DG(TL_m)$.

$V \leftarrow \{v_1, v_2, v_3, \dots v_{2m}, v_1', v_2', \dots v_{2m}'\}$

$E \leftarrow \{e_1, e_2, e_3, \dots, e_{4m-3}, e_1', e_2', e_3', \dots e_{4m-3}'\}$

$for\ 1 \le k \le m-1$

$v_{2k-1}v_{2n}' \leftarrow e_{4k-3},\ v_{2k-1}v_{2k+1}' \leftarrow e_{4k-2},$

$v_{2k-1}v_{2k+2}' \leftarrow e_{4k-1},\ v_{2k}v_{2k+2}' \leftarrow e_{4k},$

$v_{2k-1}'v_{2k} \leftarrow e_{4k-3}',\ v_{2k-1}'v_{2k+1} \leftarrow e_{4k-2}',$

$v_{2k-1}'v_{2k+2} \leftarrow e_{4k-1}',\ v_{2k}'v_{2k+2} \leftarrow e_{4k}'.$

$for\ k = m$

$v_{2k-1}v_{2k}' \leftarrow e_{4k-3},\ v_{2k-1}'v_{2k} \leftarrow e_{4k-3}'.$

Illustration

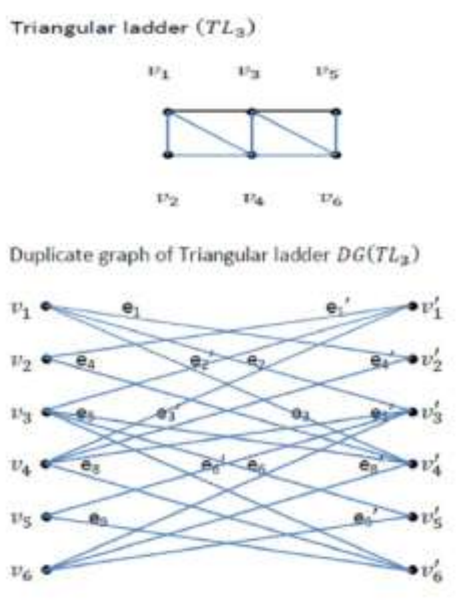


Fig. 1Example of Triangular ladder (TL_3) and its duplicate graph $EDG(TL_3)$

Remark: The Duplicate graph of Triangular ladder $DG(TL_m)$ has 2m vertices and 8m – 6 edges.

Algorithm3.2: Assignment of labels to vertices

$V \leftarrow \{v_1, v_2, v_3, \dots v_{2m}, v_1', v_2', \dots v_{2m}'\}$

$E \leftarrow \{e_1, e_2, e_3, \dots, e_{4m-3}, e_1', e_2', e_3', \dots e_{4m-3}'\}$

$for\ 1 \le k \le m$

$v_{2k-1} \leftarrow 1,\ v_{2k} \leftarrow -1;$

$for\ 1 \le k \le m-1$

$v_{2k-1}' \leftarrow -1,\ v_{2k}' \leftarrow 1;$

$for\ k = m$

$v_{2k-1}' \leftarrow 1,\ v_{2k}' \leftarrow -1.$

Theorem 3.1: The duplicate graph of triangular ladder$DG(TL_m)$, $m \ge 2$, admits signed product cordial labeling.

Proof: Let$V = \{v_1, v_2, \dots v_{2m}, v_1', v_2', \dots v_{2m}'\}$ and $E = \{e_1, e_2, \dots, e_{4m-3}, e_1', e_2', \dots e_{4m-3}'\}$ be the set of vertices and edges of the duplicate graph of triangular ladder.

Using the algorithm 3.2 , each of the 2m vertices receive label 1 and – 1. Using the induced function f^* defined by

$$f^*(uv) = f(u)f(v)$$

The 4m – 3 edges namely e_1, e_3,e_5,e_7,e_9, …, e_{4m-9},e_{4m-7},e_{4m-6},e_{4m-4},e_1^1, e_3',e_5',e_7', …, e_{4m-7}',e_{4m-5}' receive label 1 and the 4m – 3 edges namely e_2, e_4,e_6,e_8,…, e_{4m-8},e_{4m-5},e_{4m-3},e_2', e_4',e_6',e_8', …, e_{4m-6}',e_{4m-4}',e_{4m-3}' receive label – 1 respectively. Thus the number of edges labeled with 1 and – 1 differ at most by one. Hence the duplicate graph of triangular ladder $DG(TL_m)$, $m \ge 2$, admits signed product cordial labeling.

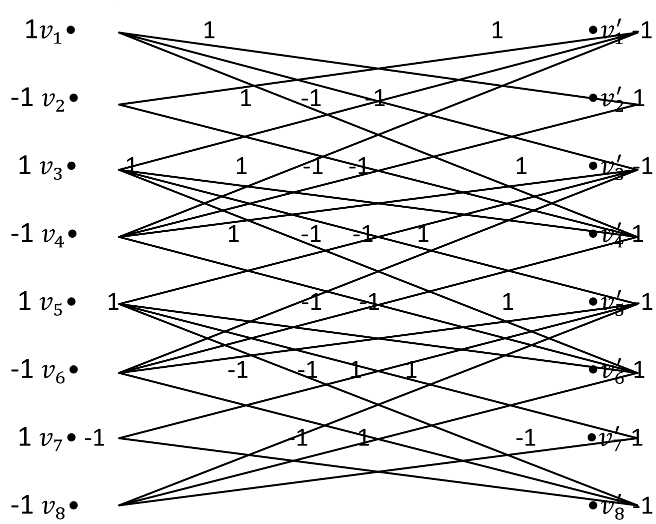


Fig 2. Example of signed product cordial labeling in $DG(TL_3)$

Algorithm 3.3: (Construction of extended duplicate graph of Bistar$DG(B_{m,m})$).

$V \leftarrow \{v_1, v_2, v_3, \dots v_{2m+2}, v'_1, v'_2, \dots v'_{2m+2}\}$
$E \leftarrow \{e_1, e_2, e_3, \dots, e_{2m+2}, e'_1, e'_2, e'_3, \dots e'_{2m+1}\}$
$fix\ v_1 v'_1 \leftarrow e_{2m+2}$
$for\ 2 \le k \le m+2$
$\quad v_1 v'_k \leftarrow e_{k-1},\ v'_1 v_k \leftarrow e'_{k-1}$
$for\ m+2 \le k \le 2m+1$
$\quad v_{m+2} v'_{k+1} \leftarrow e_k,\ v'_{m+2} v_{k+1} \leftarrow e'_k.$

Illustration:

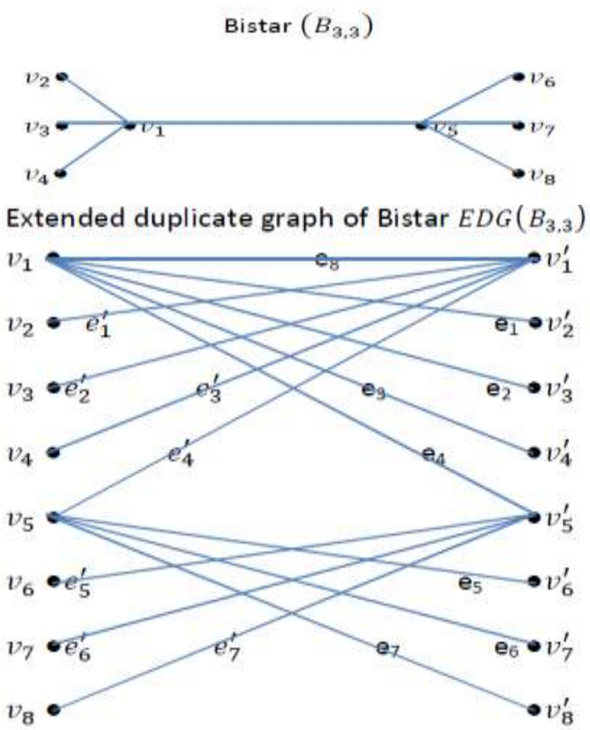


Fig.3 Example of bistar $(B_{3,3})$ and $EDG(B_{3,3})$

Remark: TheExtended duplicate graph of Bistar$EDG(B_{m,m})$ has 4m + 4 vertices and 4m+3 edges.

Algorithm 3.4: Assignment of labels to vertices
$V \leftarrow \{v_1, v_2, v_3, \dots v_{2m+2}, v'_1, v'_2, \dots v'_{2m+2}\}$
$E \leftarrow \{e_1, e_2, e_3, \dots, e_{2m+2}, e'_1, e'_2, e'_3, \dots e'_{2m+1}\}$
$for\ 1 \le k \le m+1$
$\quad v_{2k-1} \leftarrow 1,\ v_{2k} \leftarrow -1;$
$\quad v'_{2k-1} \leftarrow -1,\ v'_{2k} \leftarrow 1:$

Theorem 3.2: The extended duplicate graph of the bistar$EDG(B_{m,m})$, $m \ge 2$, admits signed product cordial labeling.

Proof:

Let $V = \{v_1, v_2, \dots v_{2m+2}, v'_1, v'_2, \dots v'_{2m+2}\}$ and $E = \{e_1, e_2, \dots, e_{2m+2}, e'_1, e'_2, \dots e'_{2m+1}\}$ be the set of vertices and edges of the $EDG(B_{m,m})$.

Case (i): When m is odd

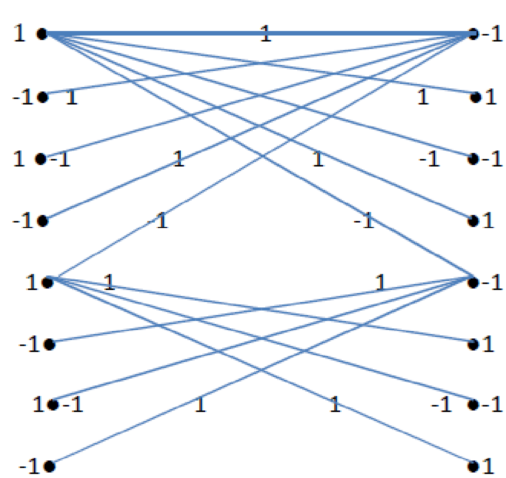
Using the algorithm 3.4 each of the 2m + 2 vertices receive label 1 and – 1. Using the induced function defined in theorem3.1, the 2m + 2 edges namely e_1, e_3,e_5, …, e_{2m-1},e_{2m+1},e'_1, e'_3,e'_7, …, e'_{2m-1},e'_{2m+1} receive label 1 and the 2m + 1 edges namely e_2, e_4, e_6, …, e_{2m-2}, e_{2m},e_{2m+2},e'_2, e'_4, e'_6, …, e'_{2m-2}, e'_{2m} receive label –1 . Thus the number of edges labeled with 1 and – 1 differ at most by one.

Case (ii): When m is even

Using the algorithm 3.4 each of the 2m + 2 vertices receive label 1 and –1.Using the induced function defined in theorem3.1, the 2m + 2 edges namelye_1, e_3,e_5, …, e_{m+1},e_{m+2},e_{m+4},e_{m+6}, …, e_{2m},e'_1, e'_3,e'_5, …, e'_{m+1},e'_{m+2},e'_{m+4},e'_{m+6}, …, e'_{2m} receive label 1 and the 2m + 1 edges namely e_2, e_4,e_6, … , e_m,e_{m+3},e_{m+5}, …, e_{2m-1},e_{2m+1},e_{2m+2},e'_2, e'_4,e'_6, …, e'_m,e'_{m+3},e'_{m+5}, …, e'_{2m-1},e'_{2m+1} receive label –1.Thus the number of edges labeled with 1 and – 1 differ at most by one.

Hence the extended duplicate graph of the bistar$EDG(B_{m,m})$, $m \ge 2$, admits signed product cordial labeling.

Illustration:



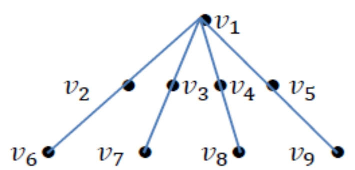
Signed product cordial labeling in Extended duplicate graph of Bistar $EDG(B_{3,3})$

Fig 4 Example of signed product cordial labeling in $EDG(B_{3,3})$

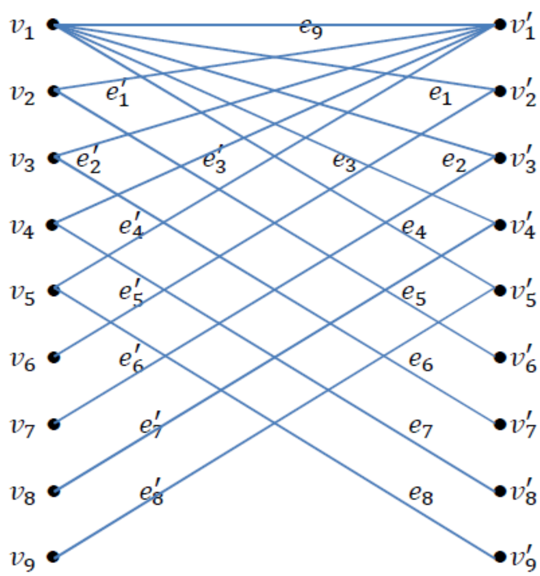
Algorithm 3.5: (Construction of extended duplicate graph of double star $(DS_{m,m})$)

$V \leftarrow \{v_1, v_2, v_3, \dots v_{2m+1}, v'_1, v'_2, \dots v'_{2m+1}\}$
$E \leftarrow \{e_1, e_2, e_3, \dots, e_{2m+1}, e'_1, e'_2, e'_3, \dots e'_{2m}\}$
$fix\ v_1 v'_1 \leftarrow e_{2m+1}$
$for\ 1 \le k \le m$
$\quad v_1 v'_{k+1} \leftarrow e_k,\ v'_1 v_{k+1} \leftarrow e'_k$
$for\ 2 \le k \le m+1$
$\quad v_k v'_{k+4} \leftarrow e_{k+3},\ v'_1 v_{k+4} \leftarrow e'_{k+3}.$

Illustration:



$Double\ Star\ DS_{4,4}$



$EDG(DS_{4,4})$

Fig 5 Example of double star $(DS_{4,4})$ and $EDG(DS_{4,4})$

Algorithm 3.6: (Assignment of labels to vertices)

$V \leftarrow \{v_1, v_2, v_3, \dots v_{2m+1}, v'_1, v'_2, \dots v'_{2m+1}\}$
$E \leftarrow \{e_1, e_2, e_3, \dots, e_{2m+1}, e'_1, e'_2, e'_3, \dots e'_{2m}\}$

$\boldsymbol{Case(i): when\ m \equiv 0 (mod\ 4)}$

$fix\ v_{m+1} \leftarrow 1,\ v'_{m+1} \leftarrow -1$
$for\ 1 \le k \le \frac{m}{4}$
$\quad v_{4k-3} \leftarrow 1, v_{4k-2} \leftarrow 1, v_{4k-1} \leftarrow -1,\ v_{4k} \leftarrow -1;$
$\quad v'_{4k-3} \leftarrow -1,\ v'_{4k-2} \leftarrow -1,\ v'_{4k-1} \leftarrow 1,\ v'_{4k} \leftarrow 1:$
$for\ \frac{m}{4} + 1 \le k \le \frac{m}{2}$
$\quad v_{4k-3} \leftarrow -1,\ v_{4k-2} \leftarrow -1,\ v_{4k-1} \leftarrow 1,\ v_{4k} \leftarrow 1;$
$\quad v'_{4k-3} \leftarrow 1,\ v'_{4k-2} \leftarrow 1,\ v'_{4k-1} \leftarrow -1,\ v'_{4k} \leftarrow -1:$

$\boldsymbol{Case(ii): when\ m \equiv 1 (mod\ 4)}$

$for\ 1 \le k \le \frac{m-1}{2}$
$\quad v_{4k-3} \leftarrow 1,\ v_{4k-2} \leftarrow 1,\ v_{4k-1} \leftarrow -1,\ v_{4k} \leftarrow -1;$
$\quad v'_{4k-3} \leftarrow -1,\ v'_{4k-2} \leftarrow -1,\ v'_{4k-1} \leftarrow 1,\ v'_{4k} \leftarrow 1:$
$for k = \frac{m+1}{2}$
$\quad v_{4k-3} \leftarrow 1, v_{4k-2} \leftarrow 1, v_{4k-1} \leftarrow -1;$
$\quad v'_{4k-3} \leftarrow -1, v'_{4k-2} \leftarrow -1, v'_{4k-1} \leftarrow 1:$

$\boldsymbol{Case(iii): when\ m \equiv 2 (mod\ 4)}$

$fix\ v_{m+1} \leftarrow -1, v'_{m+1} \leftarrow 1$

$for\ 1 \le k \le \frac{m+2}{4}$
$\quad v_{4k-3} \leftarrow 1,\ v_{4k-2} \leftarrow 1;\ v'_{4k-3} \leftarrow -1,\ v'_{4k-2} \leftarrow -1:$
$for\ 1 \le k \le \frac{m-2}{4}$
$\quad v_{4k-1} \leftarrow -1, v_{4k} \leftarrow -1; v'_{4k-1} \leftarrow 1, v'_{4k} \leftarrow 1:$
$for\ \frac{m+2}{4} \le k \le \frac{m-2}{2}$
$\quad v_{4k} \leftarrow 1,\ v_{4k+1} \leftarrow 1,\ v_{4k+2} \leftarrow -1,\ v_{4k+3} \leftarrow -1;$
$\quad v'_{4k} \leftarrow -1,\ v'_{4k+1} \leftarrow -1,\ v'_{4k+2} \leftarrow 1,\ v'_{4k+3} \leftarrow 1:$
$for\ k = \frac{m}{2}$
$\quad v_{4k} \leftarrow 1, v_{4k+1} \leftarrow 1, v'_{4k} \leftarrow -1, v'_{4k+1} \leftarrow -1:$

$\boldsymbol{Case(iv): when\ m \equiv 3 (mod\ 4)}$

$for\ 1 \le k \le \frac{m-1}{2}$
$\quad v_{4k-3} \leftarrow 1,\ v_{4k-2} \leftarrow 1,\ v_{4k-1} \leftarrow -1,\ v_{4k} \leftarrow -1;$
$\quad v'_{4k-3} \leftarrow -1,\ v'_{4k-2} \leftarrow -1,\ v'_{4k-1} \leftarrow 1,\ v'_{4k} \leftarrow 1:$
$fork = \frac{m-1}{2}$
$\quad v_{4k+1} \leftarrow 1, v_{4k+2} \leftarrow 1, v_{4k+3} \leftarrow -1;$
$\quad v'_{4k+1} \leftarrow -1, v'_{4k+2} \leftarrow -1, v'_{4k+3} \leftarrow 1:$

Theorem 3.3: The extended duplicate graph of double star $EDG(DS_{m,m})$, $m \geq 2$, admits signed product cordial labeling.

Proof:

Let $V = \{v_1, v_2, \ldots v_{2m+1}, v_1', v_2', \ldots v_{2m+1}'\}$ and $E = \{e_1, e_2, \ldots, e_{2m+1}, e_1', e_2', \ldots e_{2m}'\}$ be the set of vertices and the set of edges of $EDG(DS_{m,m})$.

$\boldsymbol{Case(i): when\ m \equiv 0 (mod\ 4)}$

Using the algorithm 3.6, each of the 2m + 1 vertices receive label –1 and 1. Using the induced function f^* as in theorem 3.1, the 2m edges namely e_2, e_3,e_6,e_7, …, e_{m-6},e_{m-5},e_{m-2},e_{m-1},e_{m+1},e_{m+3}, …, e_{2m-3},e_{2m-1},e_2', e_3',e_6',e_7', …, e_{m-6}',e_{m-5}',e_{m-2}',e_{m-1}',e_{m+1}',e_{m+3}', …, e_{2m-3}',e_{2m-1}' receive label 1 and the 2m + 1 edges namely, e_1,e_4,e_5, …, e_{m-3},e_{m-1},e_m,e_{m+2},e_{m+4}, …, e_{2m},e_{2m+1},e_1', e_4',e_5', …, e_{m-3}',e_{m-1}',e_m',e_{m+2}',e_{m+4}', …, e_{2m}' receive label –1 respectively. Thus the number of edges labeled with –1 and 1 differ by one.

$\boldsymbol{Case(ii): when\ m \equiv 1 (mod\ 4)}$

Using the algorithm 3.6, each of the 2m + 1 vertices receive label –1 and 1. Using the induced function f^* as in theorem 3.1, the 2m edges namelye_2, e_3,e_6,e_7,…, e_{m-3},e_{m-2},e_{m+1},e_{m+3}, …, e_{2m-2},e_{2m},e_2', e_3',e_6',e_7', …, e_{m-3}',e_{m-2}',e_{m+1}',e_{m+3}', …, e_{2m-2}', e_{2m}' receive label 1 and the 2m + 1 edges namely e_1, e_4,e_5,e_8,e_9, …, e_{m-1},e_m,e_{m+2},e_{m+4}, …, e_{2m-3},e_{2m-1},e_{2m+1},e_1', e_4',e_5',e_8',e_9', …, e_{m-1}',e_m',e_{m+2}',e_{m+4}', …, e_{2m-3}',e_{2m-1}' receive label –1.Thus the number of edges labeled with –1 and 1 differ by one.

$\boldsymbol{Case(iii): when\ m \equiv 2 (mod\ 4)}$

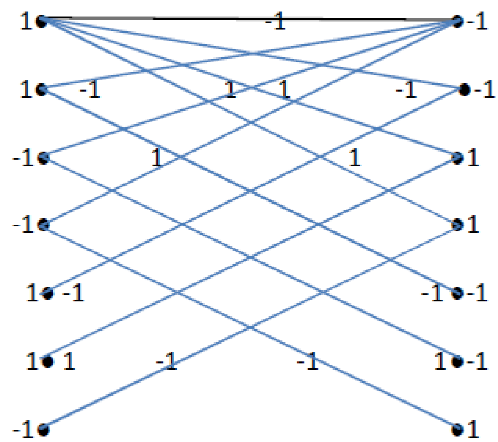
Using the algorithm 3.6, each of the 2m + 1 vertices receive label –1 and 1. Using the induced function f^*defined as in theorem 3.1, the 2m edges namelye_2, e_3,e_6,e_7, …, e_{m-4},e_{m-3},e_m,e_{m+2},e_{m+4}, …, e_{2m},e_2', e_3',e_6',e_7', …, e_{m-4}',e_{m-3}',e_m',e_{m+2}',e_{m+4}', …, e_{2m}' receive label 1 and the 2m + 1 edges namely e_1, e_4,e_5,e_8,e_9, …, e_{m-2},e_{m-1},e_{m+1},e_{m+3}, …, e_{2m-1},e_{2m+1},e_1', e_4',e_5',e_8',e_9', …, e_{m-2}',e_{m-1}',e_{m+1}',e_{m+3}', …., e_{2m-1}' receive label –1. Thus the number of edges labeled with –1 and 1 differ by one.

$\boldsymbol{Case(iv): when\ m \equiv 3 (mod\ 4)}$

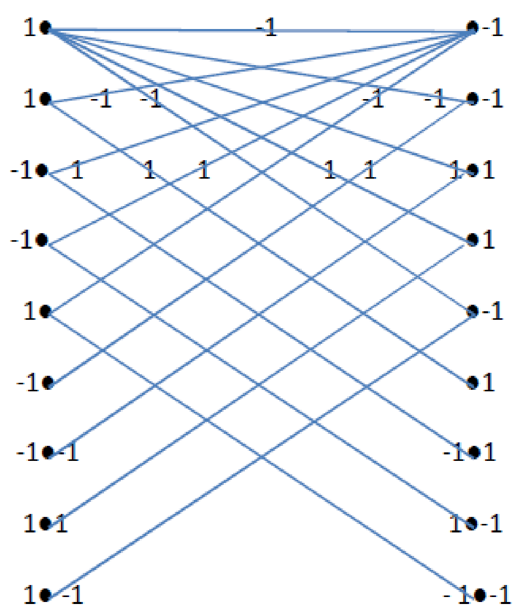
Using the algorithm 3.6, each of the 2m + 1 vertices receive label –1 and 1. Using the induced function f^* as in theorem 3.1, the 2m edges namely, e_2, e_3,e_6,e_7, …, e_{m-1},e_m,e_{m+2},e_{m+4}, …, e_{2m-3},e_{2m-1},e_2', e_3',e_6',e_7', …, e_{m-1}',e_m',e_{m+2}',e_{m+4}', … e_{2m-3}',e_{2m-1}' receive label 1 and the 2m + 1 edges namely e_1, e_4,e_5, …, e_{m-3},e_{m-2},e_{m+1},e_{m+3}, …, e_{2m-2},e_{2m},e_{2m+1},e_1', e_4',e_5', …, e_{m-3}',e_{m-2}',e_{m+1}',e_{m+3}', …, e_{2m-2}',e_{2m}' receive label – 1. Thus the number of edges labeled with –1 and 1 differ by one.

Hence, the extended duplicate graph of double star $EDG(DS_{m,m})$, $m \geq 2$, admits signed product cordial labeling.

Illustration:



Signed product cordial labeling in $EDG(DS_{3,3})$



Signed product Cordial labeling in $EDG(DS_{4,4})$

Fig 6 Example of signed product cordial labeling in $EDG(DS_{4,4})$

IV. Conclusion:

We proved that the duplicate graph of triangular ladder, the extended duplicate graph of bistar and the extended duplicate gtraph of the double star admit signed product cordial labeling.

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