Common Limit Range property (CLR) and existence of fixed points in Menger Spaces

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Abstract:*The aim of this paper is to prove some common fixed-point theorems for occasionally weakly compatible mappings for six self maps in Menger spaces satisfying common limit range property(CLR). Some examples are also given which demonstrate the validity of our results. As an application of our main result, we present a common fixed-point theorem for four finite families of self-mappings in Menger spaces. Our result is an improved probabilistic version of the result of Sedghi et al. [Gen. math. 18:3-12, 2010].*

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Keywords*: t-norm; Menger space; occasionally weakly compatible mappings;property (CLR)*

1. Introduction:

In 1922, Banach proved the principal contraction result [1]. As we know, there have been published many works about fixed-point theory for different kinds of contractions on some spaces such as quasi-metric spaces[2], cone metric spaces [3], convex metric spaces [4], partially ordered metric spaces [5-9], G-metric spaces[10-14], partial metric spaces[15,16], quasi-partial metric spaces[17], fuzzy metric spaces [18], and Menger spaces[19]. Also, studies ether on approximate fixed point or on qualitative aspects of numerical procedures for approximating fixed points are available in the literature; see[4,20,21].

Jungck and Rhoades [22] weakened the notion of compatibilty by introducing the notion of weakly compatible mappings (extended by Singh and Jain [23] to probabilistic metric space) and proved common fixed-point theorems without assuming continuity of the involved mappings in metric spaces. In 2002, Aamri and Moutawakil [24] introduced the notion of property (E.A) (extended by Kubiaczyk and Sharma[25] to probabilistic metric space) for self-mappings which contained the class of non compatible mappings due to Pant [26]. Further, Liu *et al.* [27] defined the notion of common property (E.A) (extended by Ali et al. [28] to probabilistic metric space) which contains the property (E.A) and proved several fixed-point theorems under hybrid contractive conditions. Since then, there has been continuous and intense research activity in fixed- point theory and by now there exists an extensive literature (*e.g.* [29-33] and the references therein).

Many mathematicians proved several common fixed-point theorems for contraction mappings in Menger spaces by using different notions *viz.* compatible mappings, weakly compatible mappings, property (E.A), common property (E.A) (see [28,34-51]).

In the present paper, we prove some common fixed-point theorems for two pairs of occasionally weakly compatible mappings for six self mapsin Menger space using the common limit range property (CLR). Some examples are also derived which demonstrate the validity of our results. As an application of our main result, we extend the related results to four finite families of self-mappings in Menger spaces.

2. Preliminaries:

In the sequel, $\mathbb{R}, \mathbb{R}^+$, and $\mathbb{N}$ denote the set of real numbers, the set of nonnegative real numbers, and the set of positive integers, respectively.

Definition 2.1 [52] A triangular norm $*$ (shortly t-norm) is a binary operation on the unit interval [0,1] such that for all $a, b, c, d \in [0,1]$ the following conditions are satisfied:

(1) $a * 1 = a$,
(2) $a * b = b * a$,
(3) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$,
(4) $a * (b * c) = (a * b) * c$.

Examples of t-norms are $a * b = \min\{a, b\}$, $a * b = ab$, and $a * b = \max\{a + b - 1, 0\}$.

Definition 2.2[52] A mappings $F: \mathbb{R} \to \mathbb{R}^+$ is called a distribution function if it is non- decreasing and left continuous with $\inf\{F(t): t \in \mathbb{R}\} = 0$ and $\sup\{F(t): t \in \mathbb{R}\} = 1$. We shall denote the set of all distribution functions on $(-\infty, \infty)$ by $\Im$, while H will always denotes the specific distribution function defined by

$$H(t) = \begin{cases} 0, & if\ t \leq 0; \\ 1, & if\ t > 0. \end{cases}$$

If X is a nonempty set, $\mathcal{F}: X \times X \to \Im$ is called a probabilistic distance on X and *F(x,y)* is usually denoted by $F_{x,y}$.

Definition 2.3 [52] The ordered pair $(X, \mathcal{F})$ is called a probabilistic metric space (shortly, PM-space) if X is a nonempty set and $\mathcal{F}$ is a probabilistic distance satisfying the following conditions:

(1) $F_{x,y}(t) = 1$ for all $t > 0$ if and only if $x = y$,
(2) $F_{x,y}(0) = 0$ for all $x, y \in X$,
(3) $F_{x,y}(t) = F_{y,x}(t)$ for all $x, y \in X$ and for all $t > 0$,
(4) $F_{x,z}(t) = 1, F_{z,y}(s) = 1 \Rightarrow F_{x,y}(t + s) = 1$ for $x, y, z \in X$ and $t, s > 0$.

Every metric space *(X,d)* can always be realized as a probabilistic metric space defined by $F_{x,y}(t) = H(t - d(x, y))$ for all $x, y \in X$ and $t > 0$. So probabilistic metric spaces offer a wider framework (than that of the metric spaces) and are general enough to cover even wider statistical situations.

Definition 2.4[19] A Menger space $(X, \mathcal{F}, *)$ is a triplet where $(X, \mathcal{F})$ is a probabilistic metric space and $*$ is a t-norm satisfying the following condition:

$$F_{x,y}(t + s) \geq F_{x,z}(t) * F_{z,y}(s),$$

for all $x, y, z \in X$ and $t, s > 0$.

Throughout this paper, $(X, \mathcal{F}, *)$ is considered to be a Menger space with condition $\lim_{t\to\infty} F_{x,y}(t) = 1$ for all $x, y \in X$. Every fuzzy metric space $(X, M, *)$ may be a Menger space by considering $\mathcal{F}: X \times X \to \Im$ defined by $F_{x,y}(t) = M(x, y, t)$ for all $x, y \in X$.

Definition 2.5[52] Let $(X, \mathcal{F}, *)$ be a Menger space and $*$ be a t-norm. Then

(1) a sequence $\{x_n\}$ in X is said to converge to a point x in X if and only if for every $\epsilon > 0$ and $\lambda \in (0,1)$, there exists an integer $N \in \mathbb{N}$ such that $F_{x_n,x}(\epsilon) > 1 - \lambda$ for all $n \geq N$;
(2) a sequence $\{x_n\}$ in X is said to be Cauchy if for every $\epsilon > 0$ and $\lambda \in (0,1)$, there exists an integer $N \in \mathbb{N}$ such that $F_{x_n,x_m}(\epsilon) > 1 - \lambda$ for all $n, m \geq N$.

A Menger space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.6 [53] A pair *(A,S)* of self-mappings of a Menger space$(X, \mathcal{F}, *)$ is said to be compatible if$\lim_{n\to\infty} F_{ASx_n,SAx_n}(t) = 1$ for all$t > 0$,whenever$\{x_n\}$is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Definition 2.7[28] A pair *(A,S)* of self-mappings of a Menger space$(X, \mathcal{F}, *)$ is said to be noncompatible if there exists at least one sequence$\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = z = \lim_{n\to\infty} Sx_n$ for some $z \in X$, but, for some $t > 0$, either$\lim_{n\to\infty} F_{ASx_n,SAx_n}(t) \neq 1$ or the limit does not exist.

Definition 2.8[25] A pair *(A,S)*of self-mappings of a Menger space$(X, \mathcal{F}, *)$ is said to satisfy property (E.A) if there exists a sequence$\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

for some $z \in X$.

Remark 2.9 From Definition 2.8, it is easy to see that any two noncompatible self-mappings of $(X, \mathcal{F}, *)$satisfy property (E.A) but the reverse need not be true (see [40, Example 1]).

Definition 2.10 [34] Two pairs (A,S) and (B,T) of self-mappings of a Menger space$(X, \mathcal{F}, *)$ are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$, $\{y_n\}$in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

for some $z \in X$.

Definition 2.11 [22] A pair *(A,S)* of self-mappings of a nonempty set X is said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if $Az=Sz$ for some $z \in X$, then $ASz=SAz$.

Remark 2.12 If self-mappings A and S of a Menger space $(X,\mathcal{F},*)$ are compatible then they are weakly compatible but the reverse need not be true (see [23, Example 1]).

Remark 2.13 It is noticed that the notion of weak compatibility and the (E.A) property are independent to each other (see[54, Example 2.2]).

Definition 2.14[56]A pair *(A,S)* of self-mappings of a nonempty set X is said to be occasionally weakly compatible(owc) if and only if there is a point $z \in X$ which is a coincidence point of *A* and *S* at which *A* and *S* commute. i.e., there exists a point $z \in X$ such that $Az=Sz$ and $ASz=SAz$.

Definition 2.15[57] A pair (*A*, *S*) of self-mappings of a Menger space $(X,\mathcal{F},*)$ is said to satisfy the common limit range property with respect to mapping S (briefly,(CLR_S) property), if there exists a sequence $\{x_n\}$ in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ where$z \in S(X)$.

Definition 2.16 [58]. Two pairs (A,S) and (B,T) of self-mappings of a Menger space $(X,\mathcal{F},*)$ are said to satisfy the common limit range property with respect to mappings S and T(briefly, (CLR_{ST}) property), if there exists two sequence $\{x_n\},\{y_n\}$ in *X* such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

where $z \in S(X) \cap T(X)$.

Definition 2.17[41] Two families of self-mappings $\{A_i\}$ and $\{S_j\}$ are said to be pairwise commuting if:

(1) $A_iA_j = A_jA_i,\ \ i,j \in \{1,2,\dots,m\}$,
(2) $S_kS_l = S_lS_k,\ \ k,l \in \{1,2,\dots,n\}$,
(3) $A_iS_k = S_kA_i,\ \ i \in \{1,2,\dots,m\}, k \in \{1,2,\dots,n\}$.

Lemma 2.18 [56] Let *X* be a set, *S* and *T* be occasionally weakly compatible(owc) self maps on *X*. If *S* and *T* have a unique point of coincidence $w=Sx=Tx$ for $x \in X$, then *w* is the unique common fixed point of *S* and *T*.

3 Main Results:

Let $\Phi$is a set of all increasing and continuous functions $\phi:(0,1]\to(0,1]$, such that $\phi(t)>t$ for every $t \in (0,1)$

Example 3.1 Let $\phi:(0,1]\to(0,1]$ defined by $\phi(t)=t^{1/2}$. It is easy to see that $\phi \in \Phi$

Before proving our main theorems, we begin with the following lemma.

Lemma 3.2 Let A, B, S, T, P and Q be self-mappings of a Menger space $(X,\mathcal{F},*)$, where $*$ is a continuous t-norm. Suppose that

3.2.1 $P(X) \subset ST(X)$ or $Q(X) \subset AB(X)$

3.2.2 The pair *(P,AB)* satisfies the $(CLR)_{AB}$ property or *(Q, ST)* satisfies the property $(CLR)_{ST}$

3.2.3Q(y_n) converges for every sequence $\{y_n\}$ in X whenever $ST(y_n)$ converges or $P(x_n)$ converges

for every sequence$\{x_n\}$ in X whenever AB(x_n) converges,

3.2.4 ST(X) (or AB(X)) is a closed subset of X,

3.2.5 There exists $\phi \in \Phi$ and$1 \le k \le 2$ such that

$$F_{Px,Qy}(t) \geq \phi\left(min \begin{Bmatrix} F_{ABx,STy}(t), \\ Sup_{t_1+t_2=\frac{2}{k}t} \min\{F_{ABx,Px}(t_1), F_{STy,Qy}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t} \max\{F_{ABx,Qy}(t_3), F_{STy,Px}(t_4)\} \end{Bmatrix} \right)$$

Holds for all$x,y \in X, t>0$. Then the pairs (P,AB) and (Q,ST) enjoy the $(CLR)_{(AB)(ST)}$ property.

Proof: Suppose the pair (P,AB) satisfy property. $(CLR)_{(AB)}$ property, then there exists a sequence $\{x_n\}$ in X such that

3.2.6$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = z$

where $z \in AB(X)$. Since $P(X) \subset ST(X)$, hence for each $\{x_n\}$ there corresponds a sequence $\{y_n\}$ such that $Px_n = STy_n$.

Therefore, due to the closedness of ST(X)
3.2.7$\lim_{n\to\infty} STy_n = \lim_{n\to\infty} Px_n = z$where $z \in AB(X) \cap ST(X)$.

Thus in all, we have $x_n \to z,\ ABx_n \to z\ and\ STy_n \to z\ as\ n \to \infty$. By (3), the sequence $\{Qy_n\}$ converges and in all we need to show that $Qy_n \to l(\neq z)$ for $t > 0 as\ n \to \infty$. Then it is enough to show that $z = l$. Suppose that $z \neq l$, then there exists $t_0 > 0$ such that

3.2.8 $F_{z,l}\left(\frac{2}{k}t_0\right) > F_{z,l}(t_0)$

In order to establish the claim embodied in (3.2.8), let us assume that (3.2.8) does not hold. Then we have

$F_{z,l}\left(\frac{2}{k}t\right) \leq F_{z,l}(t)$ for all $t > 0$. Repeatedly using this equality, we obtain

$$F_{z,l}(t) \geq F_{z,l}\left(\frac{2}{k}t\right) \geq \cdots \geq F_{z,l}\left(\left(\frac{2}{k}\right)^n t\right) \to 1,$$

$as\ n \to \infty$. This show that $F_{z,l}(t) = 1$ for all $t > 0$, which contradicts $z \neq l$, and hence (3.2.8) is proved. Using inequality (3.2.5), with $x = x_n, y = y_n$ we get.

$$F_{Px_n,Qy_n}(t_0) \geq \phi\left(min \left\{ \begin{array}{c} F_{ABx_n,STy_n}(t_0), \\ Sup_{t_1+t_2=\frac{2}{k}t_0} \min\{F_{ABx_n,Px_n}(t_1), F_{STy_n,Qy_n}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t_0} \max\{F_{ABx_n,Qy_n}(t_3), F_{STy_n,Px_n}(t_4)\} \end{array} \right\} \right)$$

$$\geq \phi\left(min \left\{ \begin{array}{c} F_{ABx_n,STy_n}(t_0), \\ \min\left\{F_{ABx_n,Px_n}(\epsilon), F_{STy_n,Qy_n}\left(\frac{2}{k}t_0 - \epsilon\right)\right\}, \\ \max\left\{F_{ABx_n,Qy_n}\left(\frac{2}{k}t_0 - \epsilon\right), F_{STy_n,Px_n}(\epsilon)\right\} \end{array} \right\} \right)$$

for all $\epsilon \in \left(0, \frac{2}{k}t_0\right)$. As $n \to \infty$, it follows that

$$F_{z,l}(t_0) \geq \phi\left(min \left\{ \begin{array}{c} F_{z,z}(t_0), \\ \min\left\{F_{z,z}(\epsilon), F_{z,l}\left(\frac{2}{k}t_0 - \epsilon\right)\right\}, \\ \max\left\{F_{z,l}\left(\frac{2}{k}t_0 - \epsilon\right), F_{z,z}(\epsilon)\right\} \end{array} \right\} \right)$$

$$= \phi\left(F_{z,l}\left(\frac{2}{k}t_0 - \epsilon\right)\right)$$

$$> F_{z,l}\left(\frac{2}{k}t_0 - \epsilon\right),$$

as $\epsilon \to 0$, we have

$$F_{z,l}(t_0) \geq F_{z,l}\left(\frac{2}{k}t_0\right),$$

which contradicts (3.2.8). Therefore, $z = l$. Hence the pairs (P, AB) and (Q, ST) share the $(CLR)_{(AB)(ST)}$ property.

Remark 3.3: In general, the converse of Lemma 3.2 is not true (see[28, Example 3.1])

Now we prove a common fixed point theorem for two pairs of mappings in Menger space which is an extension of the main result of Sedghi *et al.* [55] in a version of Menger space.

Theorem 3.4*Let A, B, P, Q, S and T be self-mappings of a Menger space $(X, \mathcal{F}, *)$, where $*$is a continuous t-norm satisfying inequality (3.2.5) of Lemma 3.2. Suppose that the pairs (P,AB) and (Q,ST) satisfy the $(CLR)_{(AB)(ST)}$property, then the pairs (P,AB) and (Q,ST) have a coincidence point each. Moreover, P, Q, AB and ST have a unique common fixed point provided that both pairs (P,AB) and (Q,ST) are occasionally weakly compatible. Further if (A,B), (S,T), (A,P) and (S,Q) are commuting maps then A, B, S, T, P and Q have a unique common fixed point.*

Proof. Since the pairs (P,AB) and (Q,ST) satisfy the $(CLR)_{(AB)(ST)}$ property, then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that.

$$\mathbf{3.4.1}\ \lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} STy_n = z$$

where $z \in AB(X) \cap ST(X)$. Since $z \in AB(X)$, there exists a point $u \in X$ such that $ABu = z$. We show that $Pu = ABu$. Suppose that $Pu \neq ABu$, then there exists $t_0 > 0$ such that

$$\mathbf{3.4.2} \quad F_{Pu,ABu}\left(\frac{2}{k}t_0\right) > F_{Pu,ABu}(t_0)$$

In order to establish the claim embodied in (3.4.2), let us assume that (3.4.2) does not hold.

Then we have $F_{Pu,ABu}\left(\frac{2}{k}t\right) \leq F_{Pu,ABu}(t)$ for all > 0 . Repeatedly using this equality, we obtain

$$F_{Pu,ABu}(t) \geq F_{Pu,ABu}\left(\frac{2}{k}t\right) \geq \cdots$$
$$\geq F_{Pu,ABu}\left(\left(\frac{2}{k}\right)^n t\right) \to 1,$$

$as\ n \to \infty$. This shows that $F_{Pu,ABu}(t) = 1$ for all $t > 0$, which contradicts $Pu \neq ABu$ and hence (3.4.2) is proved. Using inequality (3.2.5), with $x = u, y = y_n$, we get

$$F_{Pu,Qy_n}(t_0)$$
$$\geq \phi\left(min \begin{Bmatrix} F_{ABu,STy_n}(t_0), \\ Sup_{t_1+t_2=\frac{2}{k}t_0} \min\{F_{ABu,Pu}(t_1), F_{STy_n,Qy_n}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t_0} \max\{F_{ABu,Qy_n}(t_3), F_{STy_n,Pu}(t_4)\} \end{Bmatrix}\right)$$

$$\geq \phi\left(min \begin{Bmatrix} F_{z,STy_n}(t_0), \\ \min\left\{F_{z,Pu}\left(\frac{2}{k}t_0 - \epsilon\right), F_{STy_n,Qy_n}(\epsilon)\right\}, \\ \max\left\{F_{z,Qy_n}(\epsilon), F_{STy_n,Pu}\left(\frac{2}{k}t_0 - \epsilon\right)\right\} \end{Bmatrix}\right)$$

for all $\epsilon \in \left(0, \frac{2}{k}t_0\right)$. As $n \to \infty$, it follows that

$$F_{Pu,z}(t_0)$$
$$\geq \phi\left(min \begin{Bmatrix} F_{z,z}(t_0), \\ \min\left\{F_{z,Pu}\left(\frac{2}{k}t_0 - \epsilon\right), F_{z,z}(\epsilon)\right\}, \\ \max\left\{F_{z,z}(\epsilon), F_{z,Pu}\left(\frac{2}{k}t_0 - \epsilon\right)\right\} \end{Bmatrix}\right)$$

$$= \phi\left(F_{z,Pu}\left(\frac{2}{k}t_0 - \epsilon\right)\right)$$

$$> F_{z,Pu}\left(\frac{2}{k}t_0 - \epsilon\right),$$

as $\epsilon \to 0$, we have

$$F_{Pu,z}(t_0) \geq F_{Pu,z}\left(\frac{2}{k}t_0\right),$$

which contradicts (3.4.2). Therefore $Pu = ABu = z$ and hence u is a coincidence point of the pair (P,AB).

Also $z \in ST(X)$, there exists a point $v \in X$ such that $STv = z$.

Next, we show that $Qv = STv = z$. Let, on the contrary $Qv \neq STv$. As earlier, there exists $t_0 > 0$ such that

3.4.3 $$F_{Qv,STv}\left(\frac{2}{k}t_0\right) > F_{Qv,STv}(t_0)$$

To support the claim, let it be untrue. Then we have $F_{Qv,STv}\left(\frac{2}{k}t\right) \leq F_{Qv,STv}(t)$ for all > 0 . Repeatedly using this equality, we obtain

$$F_{Qv,STv}(t) \geq F_{Qv,STv}\left(\frac{2}{k}t\right) \geq \cdots$$
$$\geq F_{Qv,STv}\left(\left(\frac{2}{k}\right)^n t\right) \to 1,$$

$as\ n \to \infty$. This shows that $F_{Qv,STv}(t) = 1$ for all $t > 0$, which contradicts $Qv \neq STv$ and hence (3.4.3) is proved. Using inequality (3.2.5), with $x = u, y = v$, we have (for $t > 0$)

$$F_{Pu,Qv}(t_0)$$
$$\geq \phi\left(min \begin{Bmatrix} F_{ABu,STv}(t_0), \\ Sup_{t_1+t_2=\frac{2}{k}t_0} \min\{F_{ABu,Pu}(t_1), F_{STv,Qv}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t_0} \max\{F_{ABu,Qv}(t_3), F_{STv,Pu}(t_4)\} \end{Bmatrix}\right) F_{z,Qv}(t_0)$$

$$\geq \phi\left(min \begin{Bmatrix} F_{z,z}(t_0), \\ \min\left\{F_{z,z}(\epsilon), F_{z,Qv}\left(\frac{2}{k}t_0 - \epsilon\right)\right\}, \\ \max\left\{F_{z,Qv}\left(\frac{2}{k}t_0 - \epsilon\right), F_{z,z}(\epsilon)\right\} \end{Bmatrix}\right)$$

for all $\epsilon \in \left(0, \frac{2}{k}t_0\right)$ it follows that

$$F_{z,Qv}(t_0) \geq \phi\left(F_{z,Pu}\left(\frac{2}{k}t_0 - \epsilon\right)\right)$$

$$> F_{z,Qv}\left(\frac{2}{k}t_0 - \epsilon\right),$$

as $\epsilon \to 0$, we have

$$F_{z,Qv}(t_0) \geq F_{z,Qv}\left(\frac{2}{k}t_0\right),$$

which contradicts (3.4.3). Therefore $Qv = STv = z$, which shows that v is a coincidence point of the pair *(Q,ST)*

Since the pair *(P,AB)* is occasionally weakly compatible so by definition there exists a point $u \in X$ such that $Pu = ABu$ and $P(AB)u = (AB)Pu$

Since the pair *(Q,ST)* is occasionally weakly compatible so by definition there exists a point $v \in X$ such that $Qv = STv$ and $Q(ST)v = (ST)Qv$

Moreover if there is another point z such that $Pz = ABz$, then using (3.2.5) it follows that $Pz = ABz = Qv = STv$, or $Pu = Pz$ and $w =$

$Pu = ABu$ is unique point of coincidence of P and AB. By Lemma 2.18, w is the unique common fixed point of P and ABi.e., $w = Pw = ABw$. Similarly there is a unique point $z \in X$ such that $z = Qz = STz$.

Uniqueness: Suppose that $w \neq z$. Using inequality (3.2.5) with $x = w$, $y = z$, we get

$$F_{Pw,Qz}(t_0) \geq \phi\left(min \begin{Bmatrix} F_{ABw,STz}(t_0), \\ Sup_{t_1+t_2=\frac{2}{k}t_0} \min\{F_{ABw,Pw}(t_1), F_{STz,Qz}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t_0} \max\{F_{ABw,Qz}(t_3), F_{STz,Pw}(t_4)\} \end{Bmatrix} \right)$$

$$F_{Pw,Qz}(t_0) \geq \phi\left(min \begin{Bmatrix} F_{w,z}(t_0), \\ \min\left\{F_{w,w}(\epsilon), F_{z,z}\left(\frac{2}{k}t_0 - \epsilon\right)\right\}, \\ \max\left\{F_{w,z}(\epsilon), F_{z,w}\left(\frac{2}{k}t_0 - \epsilon\right)\right\} \end{Bmatrix} \right)$$

for all $\epsilon \in \left(0, \frac{2}{k}t_0\right)$. As $\epsilon \to 0$, we have

$$F_{w,z}(t_0) \geq \phi\left(min\left\{ F_{w,z}(t_0), F_{z,w}\left(\frac{2}{k}t_0 - \epsilon\right)\right\}\right)$$

$$= \phi\left(F_{w,z}(t_0)\right) > F_{w,z}(t_0)$$

which is a contradiction. Therefore $z = w$ and z is a common fixed point. By the preceding argument it is clear that z is unique. z is the unique common fixed point of P, Q, AB, ST.

Finally we need to show that z is a common fixed point of A, B, P, Q, S and T

Since (A, B), (A, P) are commutative

$Az=A(ABz)=A(BAz)=(AB)Az$;

$Az=APz=PAz$

$Bz=B(ABz)=(BA)Bz=(AB)Bz$;

$Bz=BPz=PBz$

Which shows that Az, Bz are common fixed point of (AB, P) yielding then by

$Az=z=Bz=Pz=ABz$ in the view of uniqueness of common fixed point of the pairs (P, AB)

Similarly using the commutativity of (S,T) and (S,Q) it can be shown that. $Sz=z=Tz=Qz=Az=Bz=Pz$.

which shows that z is a common fixed point of A, B, P, Q, S and T.

We can easily prove the uniqueness of z from **(3.2.5)**

Remark 3.5 Theorem 3.4 improves the corresponding results contained in Sunny Chauhan et.al [59] , Theorem 3.1] as closedness of the underlying subspaces is not required.

The following example illustrates Theorem 3.4.

Example 3.6 Let $(X, \mathcal{F}, *)$ be a Menger space, where X=[2,19], with continuous t-norm $*$ is defined by $a * b = ab$ for all $a, b \in [0,1]$ and

$$F_{x,y}(t) = \left(\frac{t}{t+1}\right)^{|x-y|}$$

For all $x, y \in X$. The function ϕ is defined as in Example 3.1. Define the self-mappings A, B, S, and T by

$$P(X) = \begin{cases} 2, & if \quad x \in \{2\} \cup (3,19]; \\ 3, & if \quad x \in (2,3], \end{cases}$$

$$Q(X) = \begin{cases} 2, & if \quad x \in \{2\} \cup (3,19]; \\ 2.5 & if \quad x \in (2,3], \end{cases}$$

$$A(X) = \begin{cases} 2, & if \quad x = 2; \\ 10, & if \quad x \in (2,3]; \\ \frac{x+77}{40}, & if \quad x \in (3,19], \end{cases}$$

$$S(x) = \begin{cases} 2, & if \ x = 2; \\ 13, & if \ x \in (2,3); \\ 14, & if \ x = 3; \\ \frac{x+77}{40}, & if \ x \in (3,19]. \end{cases}$$

$Bx = x \quad \forall \quad x \in [2,19]$ and

$Tx = x \quad \forall \quad x \in [2,19]$

Wetake $\{x_n\} = \{3 + \frac{1}{n}\}, \{y_n\} = \{2\}$ or $\{x_n\} = \{2\}, \{y_n\} = \{3 + \frac{1}{n}\}$.

We have

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} STy_n = 2 \in \text{AB(X)} \cap \text{ST(X)}.$$

Therefore, both pairs (P,AB) and (Q,ST) enjoy the $\left(CLR_{(AB)(ST)}\right)$ property.

It is noted that $P(X) = \{2,3\} \not\subseteq \{2,13,14\} \cup (2,2.4] = ST(X)$ and $Q(X) = \{2,2.5\} \not\subseteq \{2,10\} \cup (2,2.4] = AB(X)$. Also, the pairs *(P,AB)* and *(Q,ST)* commute at 2 which is their common coincidence point. Thus all the conditions of Theorem 3.1 are satisfied and 2 is the unique common fixed point of the pairs *(P,AB)* and *(Q,ST)* which also remains a point of coincidence as well. Also, notice that some mappings in this example are even discontinuous at their unique common fixed point 2.

Theorem 3.7.*Let A, B, P, Q, S and T be self-mappings of a Menger space $(X,\mathcal{F},*)$, where $*$is a continuous t-norm satisfying all the hypotheses of Lemma 3.2.Then the pairs (P,AB) and (Q,ST) have a coincidence point each. Moreover, P, Q, AB and ST have a unique common fixed pint provided that both pairs (P,AB) and (Q,ST) are occasionally weakly compatible. Further if (A,B), (S,T), (A,P) and (S,Q) are commuting maps then A, B, S, T, P and Q have a unique common fixed point.*

Proof. In view of Lemma 3.2, the pairs *(P,AB)* and *(Q,ST)* enjoy the $(CLR)_{(AB)(ST)}$ property, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} STy_n = z$$

where $z \in AB(X) \cap ST(X)$. The rest of the proof runs on the lines of the proof of Theorem 3.4.

Remark 3.8 Theorem 3.7 is also a partial improvement of Theorem 3.4 besides relaxing the closedness of the subspaces.

Taking $T=B=I_x$in Theorem 3.4 we get the following corollary.

Corollary 3.9*Let $(X,\mathcal{F},*)$, be a Menger space, where $*$ is a continuous t-norm. Let A, S, P and Q be mappings from X into itself and satisfying the following conditions:*

3.9.1 *The pairs (P,A) and (Q,S) satisfy the $(CLR)_{AS}$ property*

3.9.2 *There exists* $\phi \in \Phi$ and $1 \le k \le 2$ *such that*

3.9.3 $F_{Px,Qy}(t) \ge$

$$\phi\left(min \left\{ \begin{array}{c} F_{Ax,Sy}(t), \\ Sup_{t_1+t_2=\frac{2}{k}t} \min\{F_{Ax,Px}(t_1), F_{Sy,Qy}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t} \max\{F_{Ax,Qy}(t_3), F_{Sy,Px}(t_4)\} \end{array} \right\} \right)$$

holds for all $x,y \in X$ and $t > 0$. Then the pairs (P,A) and (Q,S) has a coincidence point. Moreover, P,Q, A and S have a unique common fixed point provided that the pair (P,A) and (Q,S) are occasionally weakly compatible.

Taking $P=Q$and $T=B=I_x$in Theorem 3.4 we get the following corollary.

Corollary 3.10*Let $(X,\mathcal{F},*)$, be a Menger space, where $*$ is a continuous t-norm. Let A, S and P be mappings from X into itself and satisfying the following conditions:*

3.10.1 *The pairs (P,A) and (P,S) satisfy the $(CLR)_{AS}$ property*

3.10.2 *There exists* $\phi \in \Phi$ and $1 \le k \le 2$ *such that*

3.10.3 $F_{Px,Py}(t) \ge$

$$\phi\left(min \left\{ \begin{array}{c} F_{Ax,Sy}(t), \\ Sup_{t_1+t_2=\frac{2}{k}t} \min\{F_{Ax,Px}(t_1), F_{Sy,Py}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t} \max\{F_{Ax,Py}(t_3), F_{Sy,Px}(t_4)\} \end{array} \right\} \right)$$

holds for all $x,y \in X$ and $t > 0$. Then the pairs (P,A) and (P,S) has a coincidence point. Moreover, P, A and S have a unique common fixed point provided that the pair (P,A)and (P,S) are occasionally weakly compatible.

Taking $A=S$and $T=B=I_x$in Theorem 3.4 we get the following corollary.

Corollary 3.11*Let $(X,\mathcal{F},*)$, be a Menger space, where $*$ is a continuous t-norm. Let A, P and Q be mappings from X into itself and satisfying the following conditions:*

3.11.1 *The pairs (P,A) and (Q,A) satisfy the $(CLR)_{A}$ property*

3.11.2*There exists* $\phi \in \Phi$ and $1 \le k \le 2$ *such that*

3.11.3 $F_{Px,Qy}(t) \geq$
$$\phi\left(\min\left\{\begin{array}{c} F_{Ax,Ay}(t), \\ Sup_{t_1+t_2=\frac{2}{k}t}\min\{F_{Ax,Px}(t_1), F_{Ay,Qy}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t}\max\{F_{Ax,Qy}(t_3), F_{Ay,Px}(t_4)\} \end{array}\right\}\right)$$

holds for all $x, y \in X$ and $t > 0$. Then the pairs (P,A) and (Q,A) has a coincidence point. Moreover, P,Q and A have a unique common fixed point provided that the pair (P,A) and (Q,A) are occasionally weakly compatible.

Taking $P=Q$ and $A=S$ and $T=B=I_x$ in Theorem 3.4 we get the following corollary.

Corollary 3.12*Let $(X,\mathcal{F},*)$, be a Menger space, where $*$ is a continuous t-norm. Let A and P be mappings from X into itself and satisfying the following conditions:*

3.12.1*The pair (P,A) enjoys the $(CLR)_A$property,*

3.12.2*There exists* $\phi \in \Phi$ and $1 \leq k \leq 2$ *such that*

3.12.3 $F_{Px,Py}(t) \geq$
$$\phi\left(\min\left\{\begin{array}{c} F_{Ax,Ay}(t), \\ Sup_{t_1+t_2=\frac{2}{k}t}\min\{F_{Ax,Px}(t_1), F_{Ay,Py}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t}\max\{F_{Ax,Py}(t_3), F_{Ay,Px}(t_4)\} \end{array}\right\}\right)$$

holds for all $x, y \in X$ and $t > 0$. Then the pair (A,S) has a coincidence point. Moreover, A and S have a unique common fixed point provided that the pair (A,S) is occasionally weakly compatible.

Corollary 3.13*Let $(X,\mathcal{F},*)$, be a Menger space, where $*$ is a continuous t-norm. Let $\{A_i\}_{i=1}^m$, $\{B_r\}_{r=1}^n$, $\{S_k\}_{k=1}^p$,* and *$\{T_g\}_{g=1}^q$ be four finite families from X into itself such that $A = A_1A_2 \ldots A_m$, $B = B_1B_2 \ldots B_n$, $S = S_1S_2 \ldots S_p$ and $T = T_1T_2 \ldots T_q$, which satisfy the inequality (3.1). If the pairs (A,S) and (B,T) enjoy the(CLR_{ST}) property then (A,S) and (B,T) have a coincidence point each.*

Moreover, $\{A_i\}_{i=1}^m$, $\{B_r\}_{r=1}^n$, $\{S_k\}_{k=1}^p$, and $\{T_g\}_{g=1}^q$ *have a unique common fixed point provided the pairs of families $(\{A_i\},\{S_k\})$ and $(\{B_r\},\{T_g\})$ commute pairwise, where $i \in \{1,2,\ldots, m\}$, $k \in \{1,2,\ldots,p\}$, $r \in \{1,2,\ldots, n\}$* and *$g \in \{1,2,\ldots, q\}$.*

Proof. Th proof of this theorem is similar to that of Theorem 3.1 contained in Imdad *et al.*[41], hence details are omitted.

Remark 3.14 Corollary 3.13 extends the result of Sedghi *et al.* [55, Theorem 2] to four finite families of self-mappings.

By setting $A_1 = A_2 = \cdots A_m = A, B_1 = B_2 = \cdots B_n = B, S_1 = S_2 = \cdots S_p = S, and\ T_1 = T_2 = \cdots T_q = T$ in Corollary 3.5, we deduce the following.

Corollary 3.15*Let $(X,\mathcal{F},*)$, be a Menger space, where $*$ is a continuous t-norm. Let A, B, S and T be mappings from X into itself such that the pairs (A^m, S^p) and (B^n, T^q) (where in m, n, p, q are fixed positive intergers) satisfy the (CLR_{S^p,T^q}) property. Suppose that there exist* $\phi \in \Phi$ and $1 \leq k \leq 2$ *such that*

3.15.1 $F_{A^mx,B^ny}(t) \geq$
$$\phi\left(\min\left\{\begin{array}{c} F_{S^px,T^qy}(t), \\ Sup_{t_1+t_2=\frac{2}{k}t}\min\{F_{S^px,A^mx}(t_1), F_{T^qy,B^ny}(t_2)\}, \\ Sup_{t_3+t_4=\frac{2}{k}t}max\{F_{S^px,B^ny}(t_3), F_{T^qy,A^mx}(t_4)\} \end{array}\right\}\right)$$

holds for all$x, y \in X$ and $t > 0$. Then the pairs (A,S) and (B,T) have a point of coincidence each. Further, A, /B, S, and T have a unique common fixed point provided both thepairs (A^m, S^p) and (B^n, T^q) commute pairwise.

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