Fourier series involving certain products of generalized class of polynomials, Aleph-function and the multivariable Aleph-function

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ABSTRACT
The aim of the present document is to establish some finite integrals and Fourier serie expansion for the products of class of polynomials, Aleph-function and multivariable Aleph-function. The results established in this paper are of general nature and hence encompass several particular cases.

Keywords :Multivariable Aleph-function, Aleph-function, Fourier serie, general class of polynomials.

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1. Introduction and preliminaries.

The Aleph- function , introduced by Südland [10] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph^{M,N}_{P_i,Q_i,c_i;r}\left(z\ \middle|\ \begin{matrix}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\(\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right)$$

$$=\frac{1}{2\pi\omega}\int_L \Omega^{M,N}_{P_i,Q_i,c_i;r}(s)z^{-s}\mathrm{d}s \tag{1.1}$$

for all z different to 0 and

$$\Omega^{M,N}_{P_i,Q_i,c_i;r}(s)=\frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \tag{1.2}$$

With :

$$|argz|<\frac{1}{2}\pi\Omega \quad \text{Where } \Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i\Big(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji}\Big)>0 \quad \text{with } i=1,\cdots,r$$

For convergence conditions and other details of Aleph-function , see Südland et al [10].

Serie representation of Aleph-function is given by Chaurasia et al [1].

$$\aleph^{M,N}_{P_i,Q_i,c_i;r}(z)=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r}(s)}{B_G g!}z^{-s} \tag{1.3}$$

With $s=\eta_{G,g}=\dfrac{b_G+g}{B_G}, P_i<Q_i, |z|<1$ and $\Omega^{M,N}_{P_i,Q_i,c_i;r}(s)$ is given in (1.2) (1.4)

The Aleph-function of several variables generalize the multivariable h-function defined by H.M. Srivastava and R. Panda [9] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\right.$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]$ $,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
.... $,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]$
$\left.[(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\right)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.5}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.6}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.7}$$

where $j=1$ to r and $k=1$ to r

Suppose , as usual , that the parameters

$a_j,j=1,\cdots,p;b_j,j=1,\cdots,q;$

$c_j^{(k)},j=1,\cdots,n_k;c_{ji^{(k)}}^{(k)},j=n_k+1,\cdots,p_{i^{(k)}};$

$d_j^{(k)},j=1,\cdots,m_k;d_{ji^{(k)}}^{(k)},j=m_k+1,\cdots,q_{i^{(k)}};$

with $k=1\cdots,r,i=1,\cdots,R\ ,i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's,\beta's,\gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}+\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}+\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}-\sum_{j=1}^{m_k}\delta_j^{(k)}$$

$$-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}\leqslant 0 \tag{1.8}$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$
The contour L_k is in the s_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)$ with $j=1$ to m_k are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)}s_k)$ with $j=1$ to n and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)$ with $j=1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by

extension of the corresponding conditions for multivariable H-function given by as :

$$|argz_k| < \frac{1}{2}A_i^{(k)}\pi \text{ , where}$$

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1 \cdots, r, i = 1, \cdots, R \text{ , } i^{(k)} = 1, \cdots, R^{(k)} \tag{1.9}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.
We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1} \ldots |z_r|^{\alpha_r}), max(|z_1| \ldots |z_r|) \rightarrow 0$$

$$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1} \ldots |z_r|^{\beta_r}), min(|z_1| \ldots |z_r|) \rightarrow \infty$$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \text{ ; } V = m_1, n_1; \cdots; m_r, n_r \tag{1.10}$$

$$\text{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.11}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.12}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.13}$$

$$C = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots, \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.14}$$

$$D = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots, \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.15}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots, z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A : C} \\ \cdot & \cdot\ \cdot\ \cdot \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B : D} \end{array}\right) \tag{1.16}$$

The generalized polynomials of multivariables defined by Srivastava [7], is given in the following manner :

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1, \cdots, y_u] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_u)_{\mathfrak{M}_u K_u}}{K_u!}$$

$$A[N_1, K_1; \cdots; N_u, K_u] y_1^{K_1} \cdots y_u^{K_u} \tag{1.17}$$

Where $\mathfrak{M}_1, \cdots, \mathfrak{M}_u$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots; N_u, K_u]$ are arbitrary constants, real or complex.
Srivastava and Garg [8] introduced and defined a general class of multivariable polynomials as follows

$$S_E^{F_1,\cdots,F_v}[z_1,\cdots,z_v]=\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}(-E)_{F_1L_1+\cdots+F_vL_v}B(E;L_1,\cdots,L_v)\frac{z_1^{L_1}\cdots z_v^{L_v}}{L_1!\cdots L_v!} \quad (1.18)$$

2. Formulas

We have the following integrals, see([3],p.16(15),[2],p.480(3.891))

a) $$\int_0^{\pi/2}(cosy)^t(cosxy)\mathrm{d}y=\frac{\pi\Gamma(t+1)}{2^{t+1}\Gamma\left(1+\frac{t\pm x}{2}\right)} \text{ where } Re(t)>-1 \quad (2.1)$$

b) $$\int_0^{\pi}sin(2h+1)y(siny)^t\mathrm{d}y=\frac{\sqrt{\pi}\Gamma\left(\frac{1-t}{2}+h\right)\Gamma(1+\frac{t}{2})}{\Gamma\left(h+\frac{t+3}{2}\right)\Gamma\left(\frac{1-t}{2}\right)} \text{ where } Re(t)>-1 \quad (2.3)$$

c) $$\int_0^{\pi}e^{(2m+1)y}sin(2n+1)y\mathrm{d}y=\frac{i\pi}{2}\delta_{m,n} \text{ where } \delta_{m,n}=1 \text{ if } m=n, 0 \text{ else} \quad (2.3)$$

3.Main integrals

In the document , we note :

$$a=\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_u)_{\mathfrak{M}_uK_u}}{K_u!}A[N_1,K_1;\cdots;N_u,K_u]$$

$$b=\frac{(-E)_{F_1L_1+\cdots+F_vL_v}B(E;L_1,\cdots,L_v)}{L_1!\cdots L_v!}\ ;U_{12}=p_i+1,q_i+2,\tau_i;R\ ,U_{22}=p_i+2,q_i+2,\tau_i;R$$

Integral 1

$$\int_0^{\pi/2}cos(u\theta)(cos\theta)^tS_E^{F_1,\cdots,F_v}[x_1(cos\theta)^{l_1},\cdots,x_v(cos\theta)^{l_v}]S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1(cos\theta)^{k_1},\cdots,y_u(cos\theta)^{k_u}]$$

$$\aleph(z(cos\theta)^h)\,\aleph(z_1(cos\theta)^{h_1},\cdots,z_r(cos\theta)^{h_r})\mathrm{d}\theta=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}$$

$$ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_Gg!}x^{\eta_{G,g}}x_1^{L_1}\cdots x_v^{L_v}y_1^{K_1}\cdots y_u^{K_u}\pi2^{-(1+t+\sum_{i=1}^{v}l_iL_i+\sum_{i=1}^{u}k_iK_i+h\eta_{G,g})}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c|c}2^{-h_1}z_1 & (\text{-t -h}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A:C\\ \cdot & \cdots\\ \cdot & \\ \cdot & ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i)/2;h_1/2,\cdots,h_r/2),B:D\\ 2^{-h_r}z_r & \end{array}\right) \quad (3.1)$$

Provided

a) $Re(\alpha)>0,h_i>0,i=1,\cdots,r\,;h>0$

b) $Re[t+h\min\limits_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{r}h_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

c) $|argz|<\frac{1}{2}\pi\Omega$ Where $\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0$

d) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.9)

Integral 2

$$\int_0^{\pi/2} sin(2h+1)y(siny)^t \aleph(z(sin\theta)^{2k}) S_E^{F_1,\cdots,F_v}[x_1(sin\theta)^{2l_1},\cdots,x_v(sin\theta)^{2l_v}]$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1(sin\theta)^{2k_1},\cdots,y_u(sin\theta)^{2k_u}]\,\aleph(z_1(sin\theta)^{2h_1},\cdots,z_s(sin\theta)^{2h_r})\mathrm{d}\theta$$

$$= \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}x^{\eta_{G,g}}y_1^{K_1}\cdots y_u^{K_u}x_1^{L_1}\cdots x_v^{L_v}$$

$$\sqrt{\pi}\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} z_1 & (\text{-t}/2\text{-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r), \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & (\text{h-(t-1)}/2\text{-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r), \\ z_r & \end{array}\right.$$

$$\left.\begin{array}{c} ((1\text{-t})/2\ \text{-k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A:C \\ \cdots \\ (\text{-(t+1)}/2\ \text{-k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),B:D \end{array}\right) \quad (3.2)$$

Provided

a) $Re(\alpha)>0,h_i>0,i=1,\cdots,r\,;h>0$

b) $Re[t+2k\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+2\sum_{i=1}^{r}h_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

c) $|argz|<\frac{1}{2}\pi\Omega$ Where $\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0$

d) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.9)

Proof of (3.1)

To establish the finite integral (3.1), express the generalized classes of polynomials $S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}$ and $S_E^{F_1,\cdots,F_v}$ occuring on the L.H.S in the series form given by (1.17)and (1.18) respectively , the Aleph-function in serie form given by (1.3) and the multivariable Aleph-function involving there in terms of Mellin-Barnes contour integral by (1.5). We interchange the order of summation and integration (which is permissible under the conditions stated). Now evaluating the θ-integral by using the formula (2.1), after simplifications and on reinterpreting the Mellin-Barnes contour integral, we get the desired result.

The proof of the integral (3.2) can be developed by proceeding on similar method with the help of (2.2).

4. Fourier series

First Fourier serie 1

$$S_E^{F_1,\cdots,F_v}[x_1(cos\theta)^{l_1},\cdots,x_v(cos\theta)^{l_v}]S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1(cos\theta)^{k_1},\cdots,y_u(cos\theta)^{k_u}](cos\theta)^t\aleph(z(cos\theta)^h)$$

$$\aleph(z_1(cos\theta)^{h_1},\cdots,z_r(cos\theta)^{h_r})$$

$$= \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r}(\eta_{G,g})}{B_G g!}\ \frac{x^{\eta_{G,g}}}{2^{t-1+h\eta_{G,g}}}x_1^{L_1}\cdots x_v^{L_v}y_1^{K_1}\cdots y_u^{K_u}$$

$$\aleph^{0,\mathfrak{n}+1:V}_{U_{12}:W}\left(\begin{array}{c}2^{-h_1}z_1\\ \cdot\\ \cdot\\ \cdot\\ 2^{-h_r}z_r\end{array}\middle|\begin{array}{c}(\text{-t -h}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A:C\\ \cdots\\ ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i)/2;h_1/2,\cdots,h_r/2),B:D\end{array}\right)+$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=1}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\frac{(-)^g ab\Omega^{M,N}_{P_i,Q_i,c_i,r}(\eta_{G,g})}{2^{1+t}B_G g!}\frac{\pi x^{\eta_{G,g}}x_1^{K_1}\cdots x_u^{K_u}y_1^{L_1}\cdots y_s^{L_v}}{2^{(\sum_{i=1}^{u}K_ik_i+\sum_{i=1}^{v}L_il_i+h\eta_{G,g})}}$$

$$\aleph^{0,\mathfrak{n}+1:V}_{U_{12}:W}\left(\begin{array}{c}2^{-h_1}z_1\\ \cdot\\ \cdot\\ \cdot\\ 2^{-h_r}z_r\end{array}\middle|\begin{array}{c}(\text{-t -h}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A:C\\ \cdots\\ ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i)/2;h_1/2,\cdots,h_r/2),B:D\end{array}\right)cosn\theta \quad (4.1)$$

which holds true under the same conditions from (3.1)

Second Fourier serie

$$(sin\theta)^t\,\aleph(z(sin\theta)^{2k})S_E^{F_1,\cdots,F_v}[x_1(sin\theta)^{2l_1},\cdots,x_v(sin\theta)^{2l_v}]$$
$$S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}_{N_1,\cdots,N_u}[y_1(sin\theta)^{2k_1},\cdots,y_u(sin\theta)^{2k_u}]\ \aleph(z_1(sin\theta)^{2h_1},\cdots,z_s(sin\theta)^{2h_r})$$

$$= \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r}(\eta_{G,g})}{B_G g!}x^{\eta_{G,g}}y_1^{K_1}\cdots y_u^{K_u}$$

$$x_1^{L_1}\cdots x_v^{L_v}\frac{2}{i\sqrt{\pi}}\aleph^{0,\mathfrak{n}+2:V}_{U_{22}:W}\left(\begin{array}{c}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\middle|\begin{array}{c}(\text{-t/2-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r),\\ \cdots\\ \cdots\\ (\text{h-(t-1)/2-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r),\end{array}\right.$$

$$\left.\begin{array}{c}((\text{1-t})/2\text{ -k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A:C\\ \cdots\\ (\text{-(t+1)}/2\text{ -k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),B:D\end{array}\right)e^{(2n+1)i\theta} \quad (4.2)$$

Proof of (4.1)
To establish (4.1), let

$$f(\theta)\ =cos(u\theta)(cos\theta)^tS_E^{F_1,\cdots,F_v}[x_1(cos\theta)^{l_1},\cdots,x_v(cos\theta)^{l_v}]$$
$$S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}_{N_1,\cdots,N_u}[y_1(cos\theta)^{k_1},\cdots,y_u(cos\theta)^{k_u}]\aleph(z_1(cos\theta)^{h_1},\cdots,z_r(cos\theta)^{h_r})=\frac{1}{2}A_0+\sum_{n=1}^{\infty}A_ncos(n\theta) \quad (4.3)$$

The equation (4.3) is valid since $f(\theta)$ is continuous and of bounded variation in the open interval $(0,\pi)$, multiplying both the sides of (4.3) by $cos(n\theta)$ and integrate with respect to y from 0 to π and use the orthogonal property of cosinus function and the integral (2.1), with substitution we get

$$A_0 = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{\pi(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{2^{t-1+h\eta_{G,g}}B_G g!}x^{\eta_{G,g}}x_1^{L_1}\cdots x_v^{L_v}$$

$$\pi 2^{-(t-1+h\eta_{G,g})}y_1^{K_1}\cdots y_u^{K_u}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c|c} 2^{-h_1}z_1 & (\text{-t -h}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A:C \\ \cdot & \cdots \\ \cdot & \\ 2^{-h_r}z_r & ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i)/2;h_1/2,\cdots,h_r/2),B:D \end{array}\right) \quad (4.4)$$

Putting the value of A_n in (4.3), we get the formula (4.1). To establish (4.2), let

$$f(\theta) = (sin\theta)^t\,\aleph(z(sin\theta)^{2k})S_E^{F_1,\cdots,F_v}[x_1(sin\theta)^{2l_1},\cdots,x_v(sin\theta)^{2l_v}]$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u,}[y_1(sin\theta)^{2k_1},\cdots,y_u(sin\theta)^{2k_u}]\,\aleph(z_1(sin\theta)^{2h_1},\cdots,z_s(sin\theta)^{2h_r})$$

$$=\sum_{-\infty}^{\infty}B_ne^{(2n+1)iy},0<y<\infty \quad (4.5)$$

The equation (4.5) is valid since $f(\theta)$ is continuous and of bounded variation in the open interval $(0,\pi)$, multiplying both the sides of (4.5) by $sin((2h+1)\theta)$ and integrate with respect to y from 0 to π and use the integral (2.3), with substitution,we get

$$B_n = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}x^{\eta_{G,g}}y_1^{K_1}\cdots y_u^{K_u}$$

$$x_1^{L_1}\cdots x_v^{L_v}\frac{2}{i\sqrt{\pi}}\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} z_1 & (\text{-t/2-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r), \\ \cdot & \cdots \\ \cdot & \cdots \\ z_r & (\text{h-(t-1)/2-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r), \end{array}\right.$$

$$\left.\begin{array}{c} ((\text{1-t})/2\text{ -k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A:C \\ \cdots \\ (\text{-(t+1)}/2\text{ -k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),B:D \end{array}\right)e^{(2n+1)i\theta} \quad (4.6)$$

5. Multivariable I-function

In these section, we get two formulas concerning Fourier series and multivariable I-function defined by Sharma et al [4]

Let $\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$

First Fourier serie

$$S_E^{F_1,\cdots,F_v}[x_1(cos\theta)^{l_1},\cdots,x_v(cos\theta)^{l_v}]S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u,}[y_1(cos\theta)^{k_1},\cdots,y_u(cos\theta)^{k_u}](cos\theta)^t\aleph(z(cos\theta)^h)$$

$$I(z_1(cos\theta)^{h_1},\cdots,z_r(cos\theta)^{h_r})$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}\frac{x^{\eta_{G,g}}}{2^{t-1+h\eta_{G,g}}}x_1^{L_1}\cdots x_v^{L_v}y_1^{K_1}\cdots y_u^{K_u}$$

$$I_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c}2^{-h_1}z_1\\ \cdot\\ \cdot\\ \cdot\\ 2^{-h_r}z_r\end{array}\middle|\begin{array}{c}(\text{-t -h}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A':C'\\ \cdots\\ ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i)/2;h_1/2,\cdots,h_r/2),B':D'\end{array}\right)+$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=1}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\frac{(-)^g ab\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{2^{1+t}B_G g!}\frac{\pi x^{\eta_{G,g}}x_1^{K_1}\cdots x_u^{K_u}y_1^{L_1}\cdots y_s^{L_v}}{2^{(\sum_{i=1}^{u}K_ik_i+\sum_{i=1}^{v}L_il_i+h\eta_{G,g})}}$$

$$I_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c}2^{-h_1}z_1\\ \cdot\\ \cdot\\ \cdot\\ 2^{-h_r}z_r\end{array}\middle|\begin{array}{c}(\text{-t -h}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A':C'\\ \cdots\\ ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i)/2;h_1/2,\cdots,h_r/2),B':D'\end{array}\right)cos\theta \quad (5.1)$$

which holds true under the same conditions from (3.1)

Second Fourier serie

$$(siny)^t\aleph(z(sin\theta)^{2k})S_E^{F_1,\cdots,F_v}[x_1(sin\theta)^{2l_1},\cdots,x_v(sin\theta)^{2l_v}]$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1(sin\theta)^{2k_1},\cdots,y_u(sin\theta)^{2k_u}]\;I(z_1(sin\theta)^{2h_1},\cdots,z_s(sin\theta)^{2h_r})$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}x^{\eta_{G,g}}y_1^{K_1}\cdots y_u^{K_u}$$

$$x_1^{L_1}\cdots x_v^{L_v}\frac{2}{i\sqrt{\pi}}I_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\middle|\begin{array}{c}(\text{-t/2-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r),\\ \cdots\\ \cdots\\ (\text{h-(t-1)/2-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r),\end{array}\right.$$

$$\left.\begin{array}{c}(\text{(1-t)/2 -k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A':C'\\ \cdots\\ (\text{-(t+1)/2 -k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),B':D'\end{array}\right)e^{(2n+1)i\theta} \quad (5.2)$$

which holds true under the same conditions from (3.2)

6. Multivariable H-function

If $\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$ and $r=r^{(1)}=\cdots=r^{(r)}=1$, then the multivariable Aleph-function degenere to the multivariable H-function defined by Srivastava et al [9]. And we have the following results.

First Fourier serie

$$S_E^{F_1,\cdots,F_v}[x_1(cos\theta)^{l_1},\cdots,x_v(cos\theta)^{l_v}]S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1(cos\theta)^{k_1},\cdots,y_u(cos\theta)^{k_u}](cos\theta)^t\aleph(z(cos\theta)^h)$$
$$H(z_1(cos\theta)^{h_1},\cdots,z_r(cos\theta)^{h_r})$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_Gg!}\frac{x^{\eta_{G,g}}}{2^{t-1+h\eta_{G,g}}}x_1^{L_1}\cdots x_v^{L_v}y_1^{K_1}\cdots y_u^{K_u}$$

$$H_{p+1,q+2:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix}2^{-h_1}z_1\\ \cdot\\ \cdot\\ \cdot\\ 2^{-h_r}z_r\end{matrix}\left|\begin{matrix}(\text{-t -h}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A'':C''\\ \cdots\\ ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i)/2;h_1/2,\cdots,h_r/2),B'':D''\end{matrix}\right.\right)+$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=1}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\frac{(-)^gab\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{2^{1+t}B_Gg!}\frac{\pi x^{\eta_{G,g}}x_1^{K_1}\cdots x_u^{K_u}y_1^{L_1}\cdots y_s^{L_v}}{2^{(\sum_{i=1}^{u}K_ik_i+\sum_{i=1}^{v}L_il_i+h\eta_{G,g})}}$$

$$H_{p+1,q+2:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix}2^{-h_1}z_1\\ \cdot\\ \cdot\\ \cdot\\ 2^{-h_r}z_r\end{matrix}\left|\begin{matrix}(\text{-t -h}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A'':C''\\ \cdots\\ ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i)/2;h_1/2,\cdots,h_r/2),B'':D''\end{matrix}\right.\right)cos\theta \quad (6.1)$$

which holds true under the same conditions from (3.1)

Second Fourier serie

$$(siny)^t\,\aleph(z(sin\theta)^{2k})S_E^{F_1,\cdots,F_v}[x_1(sin\theta)^{2l_1},\cdots,x_v(sin\theta)^{2l_v}]$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1(sin\theta)^{2k_1},\cdots,y_u(sin\theta)^{2k_u}]\,H(z_1(sin\theta)^{2h_1},\cdots,z_s(sin\theta)^{2h_r})$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_Gg!}x^{\eta_{G,g}}y_1^{K_1}\cdots y_u^{K_u}$$

$$x_1^{L_1}\cdots x_v^{L_v}\frac{2}{i\sqrt{\pi}}H_{p+2,q+2:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{matrix}\left|\begin{matrix}(\text{-t/2-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r),\\ \cdots\\ \cdots\\ (\text{h-(t-1)/2-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,\cdots,h_r),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}((\text{1-t})/2\text{ -k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),A'':C''\\ \cdots\\ (\text{-(t+1)}/2\text{ -k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,\cdots,h_r),B'':D''\end{matrix}\right)e^{(2n+1)i\theta} \quad (6.2)$$

which holds true under the same conditions from (3.2)

7. Aleph-function of two variables

In these section, we get the two formulas of Fourier series concerning the Aleph-function of two variables defined by K. Sharma [6].

First Fourier serie

$$S_E^{F_1,\cdots,F_v}[x_1(cos\theta)^{l_1},\cdots,x_v(cos\theta)^{l_v}]S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u,}[y_1(cos\theta)^{k_1},\cdots,y_u(cos\theta)^{k_u}](cos\theta)^t\aleph(z(cos\theta)^h)$$

$$\aleph(z_1(cos\theta)^{h_1},z_2(cos\theta)^{h_2})$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_Gg!}\frac{x^{\eta_{G,g}}}{2^{t-1+h\eta_{G,g}}}x_1^{L_1}\cdots x_v^{L_v}y_1^{K_1}\cdots y_u^{K_u}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c}2^{-h_1}z_1\\ \cdot\\ \cdot\\ 2^{-h_2}z_2\end{array}\middle|\begin{array}{c}(\text{-t -h}\eta_{G,g}-\sum_{i=1}^vL_il_i-\sum_{i=1}^uK_ik_i;h_1,h_2),A_2:C_2\\ \cdots\\ ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^vL_il_i-\sum_{i=1}^uK_ik_i)/2;h_1/2,h_2/2),B_2:D_2\end{array}\right)+$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=1}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\frac{(-)^gab\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{2^{1+t}B_Gg!}\frac{\pi x^{\eta_{G,g}}x_1^{K_1}\cdots x_u^{K_u}y_1^{L_1}\cdots y_s^{L_v}}{2^{(\sum_{i=1}^uK_ik_i+\sum_{i=1}^vL_il_i+h\eta_{G,g})}}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c}2^{-h_1}z_1\\ \cdot\\ \cdot\\ 2^{-h_2}z_2\end{array}\middle|\begin{array}{c}(\text{-t -h}\eta_{G,g}-\sum_{i=1}^vL_il_i-\sum_{i=1}^uK_ik_i;h_1,h_2),A_2:C_2\\ \cdots\\ ((\text{-t }\pm u-h\eta_{G,g}-\sum_{i=1}^vL_il_i-\sum_{i=1}^uK_ik_i)/2;h_1/2,h_2/2),B_2:D_2\end{array}\right)cos\theta \quad (7.1)$$

which holds true under the same conditions from (3.1)

Second Fourier serie

$$(siny)^t\,\aleph(z(sin\theta)^{2k})S_E^{F_1,\cdots,F_v}[x_1(sin\theta)^{2l_1},\cdots,x_v(sin\theta)^{2l_v}]$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u,}[y_1(sin\theta)^{2k_1},\cdots,y_u(sin\theta)^{2k_u}]\,\aleph(z_1(sin\theta)^{2h_1},z_2(sin\theta)^{2h_2})$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_Gg!}x^{\eta_{G,g}}y_1^{K_1}\cdots y_u^{K_u}$$

$$x_1^{L_1}\cdots x_v^{L_v}\frac{2}{i\sqrt{\pi}}\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c}\text{z}_1\\ \cdot\\ \cdot\\ \text{z}_2\end{array}\middle|\begin{array}{c}(\text{-t/2-k}\eta_{G,g}-\sum_{i=1}^uK_ik_i-\sum_{i=1}^vL_il_i;h_1,h_2),\\ \cdots\\ \cdots\\ (\text{h-(t-1)/2-k}\eta_{G,g}-\sum_{i=1}^uK_ik_i-\sum_{i=1}^vL_il_i;h_1,h_2),\end{array}\right.$$

$$\left.\begin{matrix} ((1\text{-t})/2\ \text{-k}\eta_{G,g} - \sum_{i=1}^{v} L_i l_i - \sum_{i=1}^{u} K_i k_i; h_1, h_2), A_2 : C_2 \\ \cdots \\ (\text{-(t+1)}/2\ \text{-k}\eta_{G,g} - \sum_{i=1}^{v} L_i l_i - \sum_{i=1}^{u} K_i k_i; h_1, h_2), B_2 : D_2 \end{matrix}\right) e^{(2n+1)i\theta} \tag{7.2}$$

which holds true under the same conditions from (3.2)

8. I-function of two variables

In these section, we get two results of double series concerning the I-function of two variables defined by Sharma and Mishra [5]. Let $\tau = \tau' = \tau'' = 1$

First Fourier serie

$$S_E^{F_1,\cdots,F_v}[x_1(cos\theta)^{l_1},\cdots,x_v(cos\theta)^{l_v}]S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1(cos\theta)^{k_1},\cdots,y_u(cos\theta)^{k_u}](cos\theta)^t\aleph(z(cos\theta)^h)$$

$$I(z_1(cos\theta)^{h_1}, z_2(cos\theta)^{h_2})$$

$$= \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}\frac{x^{\eta_{G,g}}}{2^{t-1+h\eta_{G,g}}}x_1^{L_1}\cdots x_v^{L_v}y_1^{K_1}\cdots y_u^{K_u}$$

$$I_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix} 2^{-h_1}z_1 \\ \cdot \\ \cdot \\ 2^{-h_2}z_2 \end{matrix}\middle|\begin{matrix} (\text{-t -h}\eta_{G,g} - \sum_{i=1}^{v} L_i l_i - \sum_{i=1}^{u} K_i k_i; h_1, h_2), A_2' : C_2' \\ \cdots \\ ((\text{-t} \pm u - h\eta_{G,g} - \sum_{i=1}^{v} L_i l_i - \sum_{i=1}^{u} K_i k_i)/2; h_1/2, h_2/2), B_2' : D_2' \end{matrix}\right) +$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=1}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\frac{(-)^g ab\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{2^{1+t}B_G g!}\frac{\pi x^{\eta_{G,g}}x_1^{K_1}\cdots x_u^{K_u}y_1^{L_1}\cdots y_s^{L_v}}{2^{(\sum_{i=1}^{u} K_i k_i + \sum_{i=1}^{v} L_i l_i + h\eta_{G,g})}}$$

$$I_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix} 2^{-h_1}z_1 \\ \cdot \\ \cdot \\ 2^{-h_2}z_2 \end{matrix}\middle|\begin{matrix} (\text{-t -h}\eta_{G,g} - \sum_{i=1}^{v} L_i l_i - \sum_{i=1}^{u} K_i k_i; h_1, h_2), A_2' : C_2' \\ \cdots \\ ((\text{-t} \pm u - h\eta_{G,g} - \sum_{i=1}^{v} L_i l_i - \sum_{i=1}^{u} K_i k_i)/2; h_1/2, h_2/2), B_2' : D_2' \end{matrix}\right) cos\theta \tag{8.1}$$

which holds true under the same conditions from (3.1)

Second Fourier serie

$$(siny)^t\,\aleph(z(sin\theta)^{2k})S_E^{F_1,\cdots,F_v}[x_1(sin\theta)^{2l_1},\cdots,x_v(sin\theta)^{2l_v}]$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1(sin\theta)^{2k_1},\cdots,y_u(sin\theta)^{2k_u}]\,\aleph(z_1(sin\theta)^{2h_1}, z_2(sin\theta)^{2h_2})$$

$$= \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E} ab\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}x^{\eta_{G,g}}y_1^{K_1}\cdots y_u^{K_u}$$

$$x_1^{L_1}\cdots x_v^{L_v}\frac{2}{i\sqrt{\pi}}\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} \mathrm{z}_1 & (\text{-t}/2\text{-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,h_2), \\ \cdot & \cdots \\ \cdot & \cdots \\ \mathrm{z}_2 & (\text{h-(t-1)}/2\text{-k}\eta_{G,g}-\sum_{i=1}^{u}K_ik_i-\sum_{i=1}^{v}L_il_i;h_1,h_2), \end{array}\right.$$

$$\left.\begin{array}{c} ((1\text{-t})/2\ \text{-k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,h_2),A_2':C_2' \\ \cdots \\ (\text{-(t+1)}/2\ \text{-k}\eta_{G,g}-\sum_{i=1}^{v}L_il_i-\sum_{i=1}^{u}K_ik_i;h_1,h_2),B_2':D_2' \end{array}\right)e^{(2n+1)i\theta} \quad (8.2)$$

which holds true under the same conditions from (3.2)

7. Conclusion

Due to the nature of the multivariable Aleph-function and the general classesof polynomials $S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}$ and $S_E^{F_1,\cdots,F_v}$, we can get general product of Laguerre, Legendre, Jacobi and other polynomials, the special functions of one and several variables

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