On some generalized results of fractional derivatives

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ABSTRACT
The purpose of the present document is to derive a number of key formulas for fractional derivatives of multivariables Aleph-function and generalized multivariable polynomials. Some of the applications of the key formulas provide potentially useful generalizations of know results in the theory of fractional calculus.

KEYWORDS : Aleph-function of several variables,general fractional derivative formulae, special function, general class of polynomials.

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1. Introduction and preliminaries.

The object of this document is to study the fractional derivative formula from the multivariables aleph-function. These function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [6] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c|c} z_1 & \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & \end{array}\right.$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\qquad\cdots\cdots\cdots\cdots\qquad ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]$
$[(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\left.\right)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$a_j,j=1,\cdots,p;b_j,j=1,\cdots,q;$

$c_j^{(k)},j=1,\cdots,n_k;c_{ji^{(k)}}^{(k)},j=n_k+1,\cdots,p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R, i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \quad (1.4)$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1 \cdots, r, i = 1, \cdots, R, i^{(k)} = 1, \cdots, R^{(k)} \quad (1.5)$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1} \ldots |z_r|^{\alpha_r}), max(|z_1| \ldots |z_r|) \rightarrow 0$$

$$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1} \ldots |z_r|^{\beta_r}), min(|z_1| \ldots |z_r|) \rightarrow \infty$$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper

$$V = m_1, n_1; \cdots; m_r, n_r \quad (1.6)$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \quad (1.7)$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{n+1,p_i}\} \quad (1.8)$$

$$B = \{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C = \{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.10}$$

$$D = \{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r) = \aleph_{p_i,q_i,\tau_i;R:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} z_1 & \mathrm{A}:\mathrm{C} \\ \cdot & \cdots \\ \cdot & \\ \cdot & \mathrm{B}:\mathrm{D} \\ z_r & \end{array}\right) \tag{1.12}$$

Srivastava and Garg introduced and defined a general class of multivariable polynomials [9] as follows

$$S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s] = \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} (-L)_{h_1R_1+\cdots+h_sR_s} B(E;R_1,\cdots,R_s)\frac{z_1^{R_1}\cdots z_s^{R_s}}{R_1!\cdots R_s!} \tag{1.13}$$

The fractional derivative of a function f (x) of a complex order μ is defined by Oldham et al[4], (1974 , page 49) in the followin manner :

$$_aD_x^{\mu} f(x) = \frac{1}{\Gamma(-\mu)}\int_a^x (x-y)^{-\mu-1} f(y)\,\mathrm{d}y \ \text{ if } Re(\mu)<0\,;\ \frac{d^m}{dx^m}\ {}_aD_x^{\mu-m} f(x) \ \text{ if } 0\leqslant Re(\mu)<m$$

where m is a positive integer.

For simplicity , the special ense of the fractional derivative operator $_aD_x^{\mu}$ when $a=0$, will be written as D_x^{μ}

Also we have :

$$D_x^{\mu}(x^{\lambda}) = \frac{d^{\mu}}{dx^{\mu}}(x^{\lambda}) = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)}x^{\lambda-\mu} \quad , Re(\lambda)>-1 \tag{1.14}$$

and the binomial expansion

$$(x+\mu)^{\lambda} = \mu^{\lambda}\sum_{m=0}^{\infty}\binom{\lambda}{m}\left(\frac{x}{\mu}\right)^m \ , \ \left|\frac{x}{\mu}\right|<1 \tag{1.15}$$

For $0\leqslant\alpha<1,\beta,\eta,x\in\mathbb{R};m\in\mathbb{N}$, the generalized modified fractional derivative operator due to Saigo is defined in Samko, Kilbas and Marichev [5] as

$$D_{0,x,m}^{\alpha,\beta,\eta} f(x) = \frac{d}{dz}\left(\frac{z^{-m(\beta-\eta)}}{\Gamma(1-\alpha)}\int_a^x (x^m-t^m)^{-\alpha}F(\beta-\alpha,1-\eta;1-\alpha;1-t^m/x^m)\right)f(t)\mathrm{d}t^m \tag{1.16}$$

the multiplicity of t^m-x^m is above equation is removed by requiring $log(t^m-x^m)$ as real for $t^m-x^m>0$ and is assumed to be well defined in the unit disk.

We have . $D_{0,x,1}^{\alpha,\alpha,\eta} f(x) = D_x^{\alpha} f(x)$ (1.17)

Where D_x^{α} is the familiar Riemann-Liouville fractional derivative operator.

For $0\leqslant\alpha<1,\beta,\eta,x\in\mathbb{R};m\in\mathbb{N},\mu>max(0,\beta-\eta)$, the refined form due to Bhatt and Raina [1] is given by.

$$D_{0,x,m}^{\alpha,\beta,\eta}\{x^{(\mu-1)m}\} = \frac{\Gamma(\mu)\Gamma(\mu+\eta-\beta)}{\Gamma(\mu-\beta)\Gamma(\mu+\eta-\alpha)} x^{(\mu-\beta-1)m} \tag{1.18}$$

2.Formulas

In these section, we give three formulas fractional derivatives of multivariable Aleph-function.

Formula 1

$$D_{x_1}^{\mu_1}\cdots D_{x_r}^{\mu_r}[x_1^{m_1}(x_1^{v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r}(x_r^{v_r}+\zeta_r)^{\lambda_r}$$

$$\aleph_{p_i,q_i,\tau_i;R:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 x_1^{\rho_1'}(x_1^{v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{\rho_r'}(x_r^{v_r}+\zeta_r)^{-\sigma_r'} \\ \cdots \\ \cdots \\ \mathrm{z}_n x_1^{\rho_1^n}(x_1^{v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{\rho_r^n}(x_r^{v_r}+\zeta_r)^{-\sigma_r^n} \end{array}\right)\Bigg]$$

$$= \zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r} x_1^{m_1-\mu_1}\cdots x_r^{m_r-\mu_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-x_1^{v_1}/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-x_r^{v_r}/\zeta_r)^{N_r}}{N_r!}$$

$$\aleph_{p_i+2r,q_i+2r,\tau_i;R:W}^{0,\mathfrak{n}+2r:V}\left(\begin{array}{c} \mathrm{z}_1 A_1 \\ \cdot\ \cdot \\ \mathrm{z}_n A_n \end{array}\middle|\begin{array}{c} (1+\lambda_1-N_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r-N_r:\sigma_r',\cdots,\sigma_r^n), \\ \cdots\cdots \\ (1+\lambda_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r:\sigma_r',\cdots,\sigma_r^n), \end{array}\right.$$

$$\left.\begin{array}{c} (-\mathrm{m}_1-v_1N_1:\rho_1',\cdots,\rho_1^n),\cdots,(-m_r-v_rN_r:\rho_r',\cdots,\rho_r^n),A:C \\ \cdots\cdots \\ (\mu_1-m_1-v_1N_1:\rho_1',\cdots,\rho_1^n),\cdots,(\mu_r-m_r-v_rN_r:\rho_r',\cdots,\rho_r^n),B:D \end{array}\right) \tag{2.1}$$

Where $A_i = \dfrac{x_1^{\rho_1^i}\cdots x_r^{\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}, i = 1,\cdots,n$

Provided

a) $min(v_1,\cdots,v_r;\rho_1^i,\cdots,\rho_r^i;\sigma_1^i,\cdots,\sigma_r^i) > 0, i = 1,\cdots,n$

b) $max[|arg(x_1^{v_1}/\zeta_1)|,\cdots,|arg(x_r^{v_r}/\zeta_r)|] < \pi$

c) $Re[m_1+\sum_{i=1}^{n}\rho_1^i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1,\cdots,Re[m_r+\sum_{i=1}^{n}\rho_r^i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1$

Proof of (2.1)

Let $M = = \dfrac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)$

Where $\psi(s_1,\cdots,s_r),\theta_k(s_k)$ are defined respectively by (1.2) and (1.3), thefore

$$D_{x_1}^{\mu_1}\cdots D_{x_r}^{\mu_r}\{M[x_1^{m_1}(x_1^{v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r}(x_r^{v_r}+\zeta_r)^{\lambda_r}\cdots[z_1x_1^{\rho_1'}(x_1^{v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{\rho_r'}(x_r^{v_r}+\zeta_r)^{-\sigma_r'}]^{s_1}$$

$$[z_n x_1^{\rho_1^n}(x_1^{v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{\rho_r^n}(x_r^{v_r}+\zeta_r)^{-\sigma_r^n}]^{s_n}\,\mathrm{d}s_1\cdots\mathrm{d}s_n\}$$

Using the formulas (1.14) and (1.15), we obtain.

$$\Bigg[M\frac{z_1^{s_1}\cdots z_n^{s_n}\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}}{\zeta_r^{\sigma_1' s_1+\cdots+\sigma_1^n s_n}\zeta_1^{\sigma_r' s_1+\cdots+\sigma_r^n s_n}}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-)^{N_1+\cdots+N_n}}{N_1!\cdots N_r!\zeta_1^{N_1}\cdots\zeta_r^{N_r}}\frac{\Gamma(-\lambda_1+\sum_{i=1}^n\sigma_1^i s_i+N_1)}{\Gamma(-\lambda_1+\sum_{i=1}^n\sigma_1^i s_i)}$$

$$\cdots\frac{\Gamma(-\lambda_r+\sum_{i=1}^n\sigma_r^i s_i+N_r)}{\Gamma(-\lambda_r+\sum_{i=1}^n\sigma_r^i s_i)}\frac{\Gamma(h_1+\sum_{i=1}^n\sigma_1^i s_i+v_1N_1+1)}{\Gamma(h_1+\sum_{i=1}^n\sigma_1^i s_i+v_1N_1-\mu_1)}\cdots$$

$$\frac{\Gamma(h_r+\sum_{i=1}^n\sigma_r^i s_i+v_rN_r+1)}{\Gamma(h_r+\sum_{i=1}^n\sigma_r^i s_i+v_rN_r-\mu_r)}\;x_1^{h_1+\sum_{i=1}^n\rho_1^i s_i+v_1N_1-\mu_1}\cdots x_1^{h_1+\sum_{i=1}^n\rho_r^i s_i+v_rN_r-\mu_r}\mathrm{d}s_1\cdots\mathrm{d}s_n\Big]$$

Finally, interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

Formula 2

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}[x_1^{(\mu_1-1)m_1}(x_1^{m_1v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r(\mu_r-1)}(x_r^{m_rv_r}+\zeta_r)^{\lambda_r}$$

$$\aleph_{p_i,q_i,\tau_i;R:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x_1^{m_1\rho_1'}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{m_r\rho_r'}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r'}\\ \cdots\\ \cdots\\ \mathrm{z}_nx_1^{m_1\rho_1^n}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{m_r\rho_r^n}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r^n}\end{array}\right)\Bigg]$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-x_1^{m_1v_1}/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-x_r^{m_rv_r}\zeta_r)^{N_r}}{N_r!}$$

$$\aleph_{p_i+3r,q_i+3r,\tau_i;R:W}^{0,\mathfrak{n}+3r:V}\left(\begin{array}{c}\mathrm{z}_1B_1\\ \cdot\;\cdot\\ \mathrm{z}_nB_n\end{array}\middle|\begin{array}{c}(1\text{-}\mu_1-v_1N_1:\rho_1',\cdots,\rho_1^n),\cdots,(1-\mu_r-v_rN_r:\rho_r',\cdots,\rho_r^n),\\ \cdots\cdots\\ (1+\beta_1-\mu_1-v_1N_1:\rho_1',\cdots,\rho_1^n),\cdots,(1+\beta_r-\mu_r-v_rN_r:\rho_r',\cdots,\rho_r^n),\end{array}\right.$$

$$\begin{array}{c}(1\text{-}\mu_1-\eta_1+\beta_1-v_1N_1:\rho_1',\cdots,\rho_1^n),\cdots,(1-\mu_r-\eta_r+\beta_r-v_rN_r:\rho_r',\cdots,\rho_r^n),\\ \cdots\cdots\\ (1\text{-}\mu_1-\eta_1+\alpha_1-v_1N_1:\rho_1',\cdots,\rho_1^n),\cdots,(1-\mu_r-\eta_r+\alpha_r-v_rN_r:\rho_r',\cdots,\rho_r^n),\end{array}$$

$$\left.\begin{array}{c}(1+\lambda_1-N_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r-N_r:\sigma_r',\cdots,\sigma_r^n),A:C\\ \cdots\cdots\\ (1+\lambda_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r:\sigma_r',\cdots,\sigma_r^n),B:D\end{array}\right) \tag{2.2}$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}\quad i=1,\cdots,n$

Provided that

a) For $0\leqslant\alpha_i<1,\beta_i,\eta_i,x_i\in\mathbb{R};m_i\in\mathbb{N},\mu_i>max(0,\beta_i-\eta_i)$,

b) $min(v_1,\cdots,v_r;\rho_1^i,\cdots,\rho_r^i;\sigma_1^i,\cdots,\sigma_r^i)>0,i=1,\cdots,n$

c) $max[|arg(x_1^{m_1v_1}/\zeta_1)|,\cdots,|arg(x_r^{m_rv_r}/\zeta_r)|]<\pi$

d) $Re[\mu_1 - 1 + \sum_{i=1}^{n} \rho_1^i \min_{1 \leqslant j \leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0, \cdots, Re[\mu_r - 1 + \sum_{i=1}^{n} \rho_r^i \min_{1 \leqslant j \leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

Proof of (2.2)

Let $M = = \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_r) \prod_{k=1}^{r} \theta_k(s_k)$

Where $\psi(s_1, \cdots, s_r), \theta_k(s_k)$ are defined respectively by (1.2) and (1.3)

Therefore

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1} \cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}[M x_1^{(mu_1 + \sum_{i=1}^{n} \rho_1^i n_i) m_1} (x^{m_1 v_1} + \zeta_1)^{\lambda_1 - (\sum_{i=1}^{n} \rho_1^i n_i)}$$

$$[x_1^{(mu_r + \sum_{i=1}^{n} \rho_r^i n_i) m_r} (x^{m_r v_r} + \zeta_r)^{\lambda_r - (\sum_{i=1}^{n} \rho_r^i n_i)}] \mathrm{d}s_1 \cdots \mathrm{d}s_n]$$

Using the formulas (1.15) and (1.18), we obtain.

$$\Bigg[M \frac{z_1^{s_1} \cdots z_n^{s_n} \zeta_1^{\lambda_1} \cdots \zeta_r^{\lambda_r}}{\zeta_r^{\sigma_1' s_1 + \cdots + \sigma_1^n s_n} \zeta_1^{\sigma_r' s_1 + \cdots + \sigma_r^n s_n}} \sum_{N_1, \cdots, N_r = 0}^{\infty} \frac{(-)^{N_1 + \cdots + N_n}}{N_1! \cdots N_r! \zeta_1^{N_1} \cdots \zeta_r^{N_r}} \frac{\Gamma(-\lambda_1 + \sum_{i=1}^{n} \sigma_1^i s_i + N_1)}{\Gamma(-\lambda_1 + \sum_{i=1}^{n} \sigma_1^i s_i)}$$

$$\cdots \frac{\Gamma(-\lambda_r + \sum_{i=1}^{n} \sigma_r^i s_i + N_r)}{\Gamma(-\lambda_r + \sum_{i=1}^{n} \sigma_r^i s_i)} \frac{\Gamma(\mu_1 + \sum_{i=1}^{n} \rho_1^i s_i + v_1 N_1)}{\Gamma(\mu_1 + \sum_{i=1}^{n} \sigma_1^i s_i + v_1 N_1 - \beta_1)} \cdots$$

$$\cdots \frac{\Gamma(\mu_r + \sum_{i=1}^{n} \rho_r^i s_i + v_r N_r)}{\Gamma(\mu_r + \sum_{i=1}^{n} \sigma_r^i s_i + v_1 N_r - \beta_r)} \frac{\Gamma(\mu_1 + \sum_{i=1}^{n} \rho_1^i s_i + v_1 N_1 + \eta_1 - \beta_1)}{\Gamma(\mu_1 + \sum_{i=1}^{n} \sigma_1^i s_i + v_1 N_1 + \eta_1 - \alpha_1)}$$

$$\frac{\Gamma(\mu_r + \sum_{i=1}^{n} \rho_r^i s_i + v_r N_r + \eta_r - \beta_r)}{\Gamma(\mu_r + \sum_{i=1}^{n} \sigma_r^i s_i + v_r N_r + \eta_r - \alpha_r)} \quad x_1^{m_1(\mu_1 - 1 + \sum_{i=1}^{n} \rho_1^i s_i + v_1 N_1 - \beta_1 + \sum_{i=1}^{s} k_1^i R_i)} \cdots$$

$$x_r^{m_r(\mu_r - 1 + \sum_{i=1}^{n} \rho_1^i s_i + v_r N_r - \beta_r + \sum_{i=1}^{s} k_r^i R_i)} \mathrm{d}s_1 \cdots \mathrm{d}s_n \Big]$$

Finally, interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

Formula 3

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1} \cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r} \{ x_1^{(\mu_1 - 1) m_1} (x_1^{m_1 v_1} + \zeta_1)^{\lambda_1} \cdots x^{m_r(\mu_r - 1)} (x_r^{m_r v_r} + \zeta_r)^{\lambda_r}$$

$$\aleph_{p_i,q_i,\tau_i;R:W}^{0,\mathfrak{n}:V} \left(\begin{array}{c} \mathrm{z}_1 x_1^{m_1 \rho_1'} (x_1^{m_1 v_1} + \zeta_1)^{-\sigma_1'} \cdots x_r^{m_r \rho_r'} (x_r^{m_r v_r} + \zeta_r)^{-\sigma_r'} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_n x_1^{m_1 \rho_1^n} (x_1^{m_1 v_1} + \zeta_1)^{-\sigma_1^n} \cdots x_r^{m_r \rho_r^n} (x_r^{m_r v_r} + \zeta_r)^{-\sigma_r^n} \end{array} \right)$$

$$S_L^{F_1, \cdots, F_s} [w_1 x_1^{k_1' m_1} \cdots x_r^{k_r' m_r}, \cdots, w_s x_1^{k_1^s m_1} \cdots x_r^{k_r^s m_r}] \}$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-1/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-1/\zeta_r)^{N_r}}{N_r!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$(-L)_{h_1R_1+\cdots+h_sR_s}B(L;R_1,\cdots,R_s)\frac{w_1^{R_1}\cdots w_s^{R_s}}{R_1!\cdots R_s!}x_1^{m_1(v_1N_r+\sum_{i=1}^s k_1^iR_i)}\cdots x_r^{m_r(v_rN_r+\sum_{i=1}^s k_r^iR_i)}$$

$$\aleph_{p_i+3r,q_i+3r,\tau_i;R:W}^{0,\mathfrak{n}+3r:V}\left(\begin{array}{c}z_1B_1\\ \cdot\ \cdot\\ z_nB_n\end{array}\middle|\begin{array}{c}(1+\lambda_1-N_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r-N_r:\sigma_r',\cdots,\sigma_r^n),\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\\ (1+\lambda_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r:\sigma_r',\cdots,\sigma_r^n),\end{array}\right.$$

$$\begin{array}{c}(1\text{-}\mu_1-\eta_1+\beta_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots,\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\\ (1\text{-}\mu_1-\eta_1+\alpha_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots,\end{array}$$

$$\begin{array}{c}(1\text{-}\mu_r-\eta_r+\beta_r-v_rN_r-\sum_{i=1}^s k_r^iR_i:\rho_r',\cdots,\rho_r^n),\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\\ (1\text{-}\mu_r-\eta_r+\alpha_r-v_rN_r-\sum_{i=1}^s k_r^iR_i:\rho_r',\cdots,\rho_r^n),\end{array}$$

$$\begin{array}{c}(1\text{-}\mu_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots,\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\\ (1\text{-}\mu_1+\beta_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots,\end{array}$$

$$\left.\begin{array}{c}(1\text{-}\mu_r-v_rN_r-\sum_{i=1}^s k_r^iR_i:\rho_r',\cdots,\rho_r^n),A:C\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\\ (1\text{-}\mu_r+\beta_r-v_rN_r-\sum_{i=1}^s k_r^iR_i:\rho_r',\cdots,\rho_r^n),B:D\end{array}\right) \tag{2.3}$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}\ i=1,\cdots,n$

Provided that

a) For $0\leqslant\alpha_i<1,\beta_i,\eta_i,x_i\in\mathbb{R};m_i\in\mathbb{N},\mu_i>max(0,\beta_i-\eta_i)$,

b) $min(v_1,\cdots,v_r;\rho_1^i,\cdots,\rho_r^i;\sigma_1^i,\cdots,\sigma_r^i)>0,i=1,\cdots,n$

c) $max[|arg(x_1^{m_1v_1}/\zeta_1)|,\cdots,|arg(x_r^{m_rv_r}/\zeta_r)|]<\pi$

d) $Re[\mu_1-1+\sum_{i=1}^n\rho_1^i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0,\cdots,Re[\mu_r-1+\sum_{i=1}^n\rho_r^i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

Proof of (2.3)

Let $M==\dfrac{1}{(2\pi\omega)^r}\displaystyle\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)$

Where $\psi(s_1,\cdots,s_r),\theta_k(s_k)$ are defined respectively by (1.2) and (1.3)

Use the formula (1.15), the left hand side of (2.3) is given by

$$\Big[M\frac{z_1^{s_1}\cdots z_n^{s_n}\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}}{\zeta_r^{\sigma_1's_1+\cdots+\sigma_1^ns_n}\zeta_1^{\sigma_r's_1+\cdots+\sigma_r^ns_n}}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-)^{N_1+\cdots+N_n}}{N_1!\cdots N_r!\zeta_1^{N_1}\cdots\zeta_r^{N_r}}(-\lambda_1+\sum_{i=0}^{n}\sigma_1^is_i,N_1)\cdots$$

$$(-\lambda_r+\sum_{i=0}^{n}\sigma_r^is_i,N_r)\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{w_1^{R_1}\cdots w_s^{R_s}}{R_1!\cdots R_s!}$$

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}\{x_1^{(\mu_1-1+\sum_{i=1}^n\rho_1^is_i+\sum_{i=1}^sk_1^iR_i)m_1}\cdots x_r^{(\mu_r-1+\sum_{i=1}^n\rho_r^is_i+\sum_{i=1}^sk_r^iR_i)m_r}\}\mathrm{d}s_1\cdots\mathrm{d}s_n]$$

Use the formula (1.18), we get

$$\Big[M\frac{z_1^{s_1}\cdots z_n^{s_n}\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}}{\zeta_r^{\sigma_1's_1+\cdots+\sigma_1^ns_n}\zeta_1^{\sigma_r's_1+\cdots+\sigma_r^ns_n}}\sum_{N_1,\cdots,N_r=0}^{\infty}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\frac{(-)^{N_1+\cdots+N_n}}{N_1!\cdots N_r!\zeta_1^{N_1}\cdots\zeta_r^{N_r}}$$

$$(-\lambda_1+\sum_{i=0}^{n}\sigma_1^is_i,N_1)\cdots(-\lambda_r+\sum_{i=0}^{n}\sigma_r^is_i,N_r)(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{w_1^{R_1}\cdots w_s^{R_s}}{R_1!\cdots R_s!}$$

$$\frac{\Gamma(-\lambda_1+\sum_{i=1}^n\sigma_1^is_i+N_1)}{\Gamma(-\lambda_1+\sum_{i=1}^n\sigma_1^is_i)}\cdots\frac{\Gamma(-\lambda_r+\sum_{i=1}^n\sigma_r^is_i+N_r)}{\Gamma(-\lambda_r+\sum_{i=1}^n\sigma_r^is_i)}$$

$$\frac{\Gamma(\mu_1+\sum_{i=1}^n\rho_1^is_i+v_1N_1+\sum_{i=1}^sk_i'R_i)}{\Gamma(\mu_1+\sum_{i=1}^n\sigma_1^is_i+v_1N_1-\beta_1+\sum_{i=1}^sk_i'R_i)}\cdots\frac{\Gamma(\mu_r+\sum_{i=1}^n\rho_r^is_i+v_1N_r+\sum_{i=1}^sk_i'R_i)}{\Gamma(h_r+\sum_{i=1}^n\sigma_1^is_i+v_rN_r-\beta_r+\sum_{i=1}^sk_i'R_i)}$$

$$\cdots\frac{\Gamma(\mu_1+\sum_{i=1}^n\rho_1^is_i+v_1N_1+\sum_{i=1}^sk_1^sR_i+\eta_1-\beta_1)}{\Gamma(\mu_1+\sum_{i=1}^n\sigma_1^is_i+v_1N_1+\eta_1-\alpha_1+\sum_{i=1}^sk_1^sR_i)}\cdots$$

$$\frac{\Gamma(\mu_r+\sum_{i=1}^n\rho_r^is_i+v_rN_r+\sum_{i=1}^sk_i'R_i)}{\Gamma(\mu_r+\sum_{i=1}^n\sigma_r^is_i+v_1N_1-\beta_r+\sum_{i=1}^sk_i'R_i)}x_1^{m_1(\mu_1-1+\sum_{i=1}^n\rho_1^is_i+v_1N_1-\beta_1+\sum_{i=1}^sk_1^iR_i)}\cdots$$

$$x_r^{m_r(\mu_r-1+\sum_{i=1}^n\rho_1^is_i+v_rN_r-\beta_r+\sum_{i=1}^sk_r^iR_i)}\mathrm{d}s_1\cdots\mathrm{d}s_n]$$

Finally, interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

3. Particular case

a) If $B(L;R_1,\cdots,R_s) = \frac{\prod_{j=1}^{A}(a_j)_{R_1\theta_j'+\cdots+R_s\theta_j^{(s)}}\prod_{j=1}^{B'}(b_j')_{R_1\phi_j'}\cdots\prod_{j=1}^{B^{(s)}}(b_j^{(v)})_{R_s\phi_j^{(s)}}}{\prod_{j=1}^{C}(c_j)_{m_1\psi_j'+\cdots+m_s\psi_j^{(s)}}\prod_{j=1}^{D'}(d_j')_{R_1\delta_j'}\cdots\prod_{j=1}^{D^{(s)}}(d_j^{(v)})_{R_s\delta_j^{(s)}}}$ (3.1)

then the general class of multivariable polynomial $S_L^{R_1,\cdots,R_s}[x_1,\cdots,x_s]$ reduces to generalized Lauricella function defined by Srivastava et al [8].

$$F_{C:D';\cdots;D^{(s)}}^{1+A:B';\cdots;B^{(s)}}\left(\begin{array}{c}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_v\end{array}\middle|\begin{array}{c}[(\text{-L}):\mathrm{R}_1,\cdots,R_s],[(a):\theta',\cdots,\theta^{(s)}];[(b'):\phi'];\cdots;[(b^{(s)}):\phi^{(s)}]\\ [(\mathrm{c}):\ \psi',\cdots,\psi^{(s)}];[(d'):\delta'];\cdots;[(\mathrm{b})^{(s)}:\delta^{(s)}]\end{array}\right)\quad(3.2)$$

The formula (2.3) write

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}\{x_1^{(\mu_1-1)m_1}(x_1^{m_1v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r(\mu_r-1)}(x_r^{m_rv_r}+\zeta_r)^{\lambda_r}$$

$$\aleph_{p_i,q_i,\tau_i;R:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 x_1^{m_1\rho_1'}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{m_r\rho_r'}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r'} \\ \cdots \\ \cdots \\ \mathrm{z}_n x_1^{m_1\rho_1^n}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{m_r\rho_r^n}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r^n} \end{array}\right)$$

$$F_{C:D';\cdots;D^{(s)}}^{1+A:B';\cdots;B^{(s)}}\left(\begin{array}{c} \mathrm{w}_1 x_1^{k_1'm_1}\cdots x_r^{k_r'm_r} \\ \cdots \\ \mathrm{w}_s x_s^{k_1^s m_1}\cdots x_r^{k_r^s m_r} \end{array}\middle| \begin{array}{c} [\text{(-L)}: \mathrm{R}_1,\cdots,R_s],[(a):\theta',\cdots,\theta^{(s)}];[(b'):\phi'];\cdots;[(b^{(s)}):\phi^{(s)}] \\ [\text{(c)}:\ \psi',\cdots,\psi^{(s)}];[(d'):\delta'];\cdots;[(b)^{(s)}:\delta^{(s)}] \end{array}\right)\Bigg\}$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-1/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-1\zeta_r)^{N_r}}{N_r!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$(-L)_{h_1R_1+\cdots+h_sR_s}B(L;R_1,\cdots,R_s)\frac{w_1^{R_1}\cdots w_s^{R_s}}{R_1!\cdots R_s!}x_1^{m_1(v_1N_r+\sum_{i=1}^s k_1^iR_i)}\cdots x_r^{m_r(v_rN_r+\sum_{i=1}^s k_r^iR_i)}$$

$$\aleph_{p_i+3r,q_i+3r,\tau_i;R:W}^{0,\mathfrak{n}+3r:V}\left(\begin{array}{c} \mathrm{z}_1 B_1 \\ \cdot\ \cdot \\ \mathrm{z}_n B_n \end{array}\middle| \begin{array}{c} (1+\lambda_1-N_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r-N_r:\sigma_r',\cdots,\sigma_r^n), \\ \cdots\cdots \\ (1+\lambda_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r:\sigma_r',\cdots,\sigma_r^n), \end{array}\right.$$

$$\begin{array}{c} (1\text{-}\mu_1-\eta_1+\beta_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots, \\ \cdots\cdots \\ (1\text{-}\mu_1-\eta_1+\alpha_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots, \end{array}$$

$$\begin{array}{c} (1\text{-}\mu_r-\eta_r+\beta_r-v_rN_r-\sum_{i=1}^s k_1^iR_i:\rho_r',\cdots,\rho_r^n),\quad (1\text{-}\mu_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots, \\ \cdots\cdots \qquad\qquad \cdots\cdots \\ (1\text{-}\mu_r-\eta_r+\alpha_r-v_rN_r-\sum_{i=1}^s k_1^iR_i:\rho_r',\cdots,\rho_r^n),(1\text{-}\mu_1+\beta_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots, \end{array}$$

$$\left.\begin{array}{c} (1\text{-}\mu_r-v_rN_r-\sum_{i=1}^s k_1^iR_i:\rho_r',\cdots,\rho_r^n),A:C \\ \cdots\cdots \\ (1\text{-}\mu_r+\beta_r-v_rN_r-\sum_{i=1}^s k_1^iR_i:\rho_r',\cdots,\rho_r^n),B:D \end{array}\right) \tag{3.3}$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}$ $i=1,\cdots,n$ and $B(E;R_1,\cdots,R_s)$ is defined by (3.1)

which holds true under the same conditions as needed in (2.3)

b) If $x_2=\cdots,x_s=0$, then $S_L^{R_1,\cdots,R_s}[x_1,\cdots,x_s]$ degenere to $S_N^M(x)$ defined by Srivastava [7] and we have

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}\{x_1^{(\mu_1-1)m_1}(x_1^{m_1v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r(\mu_r-1)}(x_r^{m_rv_r}+\zeta_r)^{\lambda_r}$$

$$\aleph_{p_i,q_i,\tau_i;R:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 x_1^{m_1\rho_1'}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{m_r\rho_r'}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r'} \\ \cdots \\ \cdots \\ \mathrm{z}_n x_1^{m_1\rho_1^n}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{m_r\rho_r^n}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r^n}\end{array}\right)S_N^M[wx_1^{k_1'm_1}\cdots x_r^{k_r'm_r}]\Bigg\}$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\sum_{K=0}^{[N/M]}\frac{(-1/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-1\zeta_r)^{N_r}}{N_r!}$$

$$\frac{(-N)_{MK}}{K!}A_{N,K}\frac{w^K}{K!}\;x_1^{m_1(v_1N_r+k_1'K)}\cdots x_r^{m_r(v_rN_r+k_r'K)}$$

$$\aleph_{p_i+3r,q_i+3r,\tau_i;R:W}^{0,\mathfrak{n}+3r:V}\left(\begin{array}{c}\mathrm{z}_1B_1\\ \cdot\,\cdot \\ \mathrm{z}_nB_n\end{array}\left|\begin{array}{c}(1+\lambda_1-N_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r-N_r:\sigma_r',\cdots,\sigma_r^n),\\ \cdots\cdots \\ (1+\lambda_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r:\sigma_r',\cdots,\sigma_r^n),\end{array}\right.\right.$$

$$(1\text{-}\mu_1-\eta_1+\beta_1-v_1N_1-k_1K:\rho_1',\cdots,\rho_1^n),\cdots,(1\text{-}\mu_r-\eta_r+\beta_r-v_rN_r-k_rK:\rho_r',\cdots,\rho_r^n),$$
$$\cdots\cdots \qquad \cdots\cdots$$
$$(1\text{-}\mu_1-\eta_1+\alpha_1-v_1N_1-k_1K:\rho_1',\cdots,\rho_1^n),\cdots,(1\text{-}\mu_r-\eta_r+\alpha_r-v_rN_r-k_rK:\rho_r',\cdots,\rho_r^n),$$

$$\left.\begin{array}{c}(1\text{-}\mu_1-v_1N_1-k_1K:\rho_1',\cdots,\rho_1^n),\cdots,\quad(1\text{-}\mu_r-v_rN_r-k_rK:\rho_r',\cdots,\rho_r^n),A:C\\ \cdots\cdots \qquad \cdots\cdots \\ (1\text{-}\mu_1+\beta_1-v_1N_1-k_1K:\rho_1',\cdots,\rho_1^n),\cdots,(1\text{-}\mu_r+\beta_r-v_rN_r-k_rK:\rho_r',\cdots,\rho_r^n),B:D\end{array}\right) \quad (3.4)$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}\; i=1,\cdots,n$

which holds true under the same conditions as needed in (2.3)

4. Multivariable I-function

If $\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$, the Aleph-function of several variables degenere to the I-function of several variables. In these section, we give three formulas fractional derivatives of multivariable I-function defined by Sharma and Ahmad [6].

Formula 1

$$D_{x_1}^{\mu_1}\cdots D_{x_r}^{\mu_r}[x_1^{m_1}(x_1^{v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r}(x_r^{v_r}+\zeta_r)^{\lambda_r}$$

$$I_{p_i,q_i;R:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x_1^{\rho_1'}(x_1^{v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{\rho_r'}(x_r^{v_r}+\zeta_r)^{-\sigma_r'}\\ \cdots \\ \cdots \\ \mathrm{z}_nx_1^{\rho_1^n}(x_1^{v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{\rho_r^n}(x_r^{v_r}+\zeta_r)^{-\sigma_r^n}\end{array}\right)\Bigg]$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{m_1-\mu_1}\cdots x_r^{m_r-\mu_r}\sum_{m_1,\cdots,m_r=0}^{\infty}\frac{(-x_1^{v_1}/\zeta_1)^{m_1}}{m_1!}\cdots\frac{(-x_r^{v_r}/\zeta_r)^{m_r}}{m_r!}$$

$$I^{0,\mathfrak{n}+2r:V}_{p_i+2r,q_i+2r;R:W}\left(\begin{array}{c|c} z_1A_1 & (1+\lambda_1-m_1:\sigma'_1,\cdots,\sigma^n_1),\cdots,(1+\lambda_r-m_r:\sigma'_r,\cdots,\sigma^n_r), \\ \cdot\ \cdot & \cdots\cdots \\ z_nA_n & (1+\lambda_1:\sigma'_1,\cdots,\sigma^n_1),\cdots,(1+\lambda_r:\sigma'_r,\cdots,\sigma^n_r), \end{array}\right.$$

$$\left.\begin{array}{c} (\text{-h}_1-v_1m_1:\rho'_1,\cdots,\rho^n_1),\cdots,(-h_r-v_rm_r:\rho'_r,\cdots,\rho^n_r),A:C \\ \cdots\cdots \\ (\mu_1-h_1-v_1m_1:\rho'_1,\cdots,\rho^n_1),\cdots,(\mu_r-h_r-v_rm_r:\rho'_r,\cdots,\rho^n_r),B:D \end{array}\right) \tag{4.1}$$

Where $A_i=\dfrac{x_1^{\rho_1^i}\cdots x_r^{\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}},i=1,\cdots,n$

which holds true under the same conditions as needed in (2.3)

Formula 2

$$D^{\alpha_1,\beta_1,\eta_1}_{0,x_1,m_1}\cdots D^{\alpha_r,\beta_r,\eta_r}_{0,x_r,m_r}[x_1^{(\mu_1-1)m_1}(x_1^{m_1v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r(\mu_r-1)}(x_r^{m_rv_r}+\zeta_r)^{\lambda_r}$$

$$I^{0,\mathfrak{n}:V}_{p_i,q_i;R:W}\left(\begin{array}{c} z_1x_1^{m_1\rho'_1}(x_1^{m_1v_1}+\zeta_1)^{-\sigma'_1}\cdots x_r^{m_r\rho'_r}(x_r^{m_rv_r}+\zeta_r)^{-\sigma'_r} \\ \cdots \\ \cdots \\ z_nx_1^{m_1\rho^n_1}(x_1^{m_1v_1}+\zeta_1)^{-\sigma^n_1}\cdots x_r^{m_r\rho^n_r}(x_r^{m_rv_r}+\zeta_r)^{-\sigma^n_r} \end{array}\right)\Bigg]$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-x_1^{m_1v_1}/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-x_r^{m_rv_r}/\zeta_r)^{N_r}}{N_r!}$$

$$I^{0,\mathfrak{n}+3r:V}_{p_i+3r,q_i+3;R:W}\left(\begin{array}{c|c} z_1B_1 & (1\text{-}\mu_1-v_1N_1:\rho'_1,\cdots,\rho^n_1),\cdots,(1-\mu_r-v_rN_r:\rho'_r,\cdots,\rho^n_r), \\ \cdot\ \cdot & \cdots\cdots \\ z_nB_n & (1+\beta_1-\mu_1-v_1N_1:\rho'_1,\cdots,\rho^n_1),\cdots,(1+\beta_r-\mu_r-v_rN_r:\rho'_r,\cdots,\rho^n_r), \end{array}\right.$$

$$\begin{array}{c} (1\text{-}\mu_1-\eta_1+\beta_1-v_1N_1:\rho'_1,\cdots,\rho^n_1),\cdots,(1-\mu_r-\eta_r+\beta_r-v_rN_r:\rho'_r,\cdots,\rho^n_r), \\ \cdots\cdots \\ (1\text{-}\mu_1-\eta_1+\alpha_1-v_1N_1:\rho'_1,\cdots,\rho^n_1),\cdots,(1-\mu_r-\eta_r+\alpha_r-v_rN_r:\rho'_r,\cdots,\rho^n_r), \end{array}$$

$$\left.\begin{array}{c} (1+\lambda_1-N_1:\sigma'_1,\cdots,\sigma^n_1),\cdots,(1+\lambda_r-N_r:\sigma'_r,\cdots,\sigma^n_r),A:C \\ \cdots\cdots \\ (1+\lambda_1:\sigma'_1,\cdots,\sigma^n_1),\cdots,(1+\lambda_r:\sigma'_r,\cdots,\sigma^n_r),B:D \end{array}\right) \tag{4.2}$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}\ i=1,\cdots,n$

which holds true under the same conditions as needed in (2.3)

Formula 3

$$D^{\alpha_1,\beta_1,\eta_1}_{0,x_1,m_1}\cdots D^{\alpha_r,\beta_r,\eta_r}_{0,x_r,m_r}[x_1^{(\mu_1-1)m_1}(x_1^{m_1v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r(\mu_r-1)}(x_r^{m_rv_r}+\zeta_r)^{\lambda_r}$$

$$I_{p_i,q_i;R:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 x_1^{m_1\rho_1'}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{m_r\rho_r'}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r'} \\ \cdots \\ \cdots \\ \mathrm{z}_n x_1^{m_1\rho_1^n}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{m_r\rho_r^n}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r^n} \end{array}\right)$$

$$S_L^{F_1,\cdots,F_s}[w_1x_1^{k_1'm_1}\cdots x_r^{k_r'm_r},\cdots,w_sx_1^{k_1^sm_1}\cdots x_r^{k_r^sm_r}]\}$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-1/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-1\zeta_r)^{N_r}}{N_r!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{w_1^{R_1}\cdots w_s^{R_s}}{R_1!\cdots R_s!}x_1^{v_1N_r+m_1\sum_{i=1}^s k_1^iR_i}\cdots x_r^{v_rN_r+m_r\sum_{i=1}^s k_r^iR_i}$$

$$I_{p_i+3r,q_i+3r;R:W}^{0,\mathfrak{n}+3r:V}\left(\begin{array}{c} \mathrm{z}_1B_1 \\ \cdot\,\cdot \\ \mathrm{z}_nB_n \end{array}\right|\begin{array}{c}(1+\lambda_1-N_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r-N_r:\sigma_r',\cdots,\sigma_r^n),\\ \cdots\cdots \\ (1+\lambda_1:\sigma_1',\cdots,\sigma_1^n),\cdots,(1+\lambda_r:\sigma_r',\cdots,\sigma_r^n),\end{array}$$

$$\begin{array}{c}(1\text{-}\mu_1-\eta_1+\beta_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots,\\ \cdots\cdots \\ (1\text{-}\mu_1-\eta_1+\alpha_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots,\end{array}$$

$$\begin{array}{cc}(1\text{-}\mu_r-\eta_r+\beta_r-v_rN_r-\sum_{i=1}^s k_1^iR_i:\rho_r',\cdots,\rho_r^n), & (1\text{-}\mu_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots,\\ \cdots\cdots & \cdots\cdots \\ (1\text{-}\mu_r-\eta_r+\alpha_r-v_rN_r-\sum_{i=1}^s k_1^iR_i:\rho_r',\cdots,\rho_r^n), & (1\text{-}\mu_1+\beta_1-v_1N_1-\sum_{i=1}^s k_1^iR_i:\rho_1',\cdots,\rho_1^n),\cdots,\end{array}$$

$$\left.\begin{array}{c}(1\text{-}\mu_r-v_rN_r-\sum_{i=1}^s k_1^iR_i:\rho_r',\cdots,\rho_r^n),A:C\\ \cdots\cdots \\ (1\text{-}\mu_r+\beta_r-v_rN_r-\sum_{i=1}^s k_1^iR_i:\rho_r',\cdots,\rho_r^n),B:D\end{array}\right) \tag{4.3}$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}\ i=1,\cdots,n$

which holds true under the same conditions as needed in (2.3)

Remark : If an$\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$d $R=R^{(1)}=,\cdots,R^{(r)}=1$,the Aleph-function of several variables degenere to the H-function of several variables defined by Srivastava et al [10] . For more details, see Chandel et al [2] and [3].

5.Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as, multivariable I-function defined by Sharma et al [6], multivariable H-function defined by Srivastava et al [10].

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