On an expansion formula for the multivariable aleph-function involving generalized Legendre's associated function

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ABSTRACT
We have established a new expansion formula for the multivariable Aleph-function in terms of product of the multivariable Aleph-function and the generalized Legendre's function due to Meulenbeld [2]. Some special cases are given in the last.

Keywords : Multivariable Aleph-function , Multivariable I-function , generalized Legendre's associated function.

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1. Introduction and preliminaries.

The object of this document is to study a expansions involving the multivariables aleph-function and generalized Legendre's associated function. These function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [5] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $$\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \end{array}\right.$$

$$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}] \ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}] :$$
$$\cdots\cdots\cdots\cdots \ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}] :$$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}] \\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\phi_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega = \sqrt{-1}$

$$\psi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $\phi_k(s_k) = \frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)} - \delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{l=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{l=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]}$ (1.3)

where $j=1$ to r and $k=1$ to r

Suppose , as usual , that the parameters

$a_j, j=1,\cdots,p; b_j, j=1,\cdots,q;$

$c_j^{(k)}, j=1,\cdots,n_k; c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,p_{i^{(k)}};$

$d_j^{(k)}, j=1,\cdots,m_k; d_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,q_{i^{(k)}};$

with $k=1$ to r , $i=1$ to R , $i^{(k)}=1$ to $R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} + \tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} + \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} - \sum_{j=1}^{m_k}\delta_j^{(k)}$$

$$-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)}s_k)$ with $j=1$ to m_k are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)}s_k)$ with $j=1$ to n and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)$ with $j=1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg x_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} - \tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)} - \tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0 \quad \text{, with } k=1 \text{ to } r \text{ , } i=1 \text{ to } R \text{ , } i^{(k)}=1 \text{ to } R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\alpha_1}\dots|z_r|^{\alpha_r}\,), max(\,|z_1|\dots|z_r|\,) \to 0$

$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\,)\,,min(\,|z_1|\ldots|z_r|\,)\rightarrow\infty$

where, with $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})],j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U=p_i,q_i,\tau_i;R\ ;\ V=m_1,n_1;\cdots;m_r,n_r \tag{1.6}$$

$$\mathrm{W}=p_{i^{(1)}},q_{i^{(1)}},\tau_{i(1)};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i(r)};R^{(r)} \tag{1.7}$$

$$A=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.8}$$

$$B=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.10}$$

$$D=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r)=\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}:\mathrm{C}\\ \cdot & \cdots\\ \cdot & \\ \cdot & \mathrm{B}:\mathrm{D}\\ \mathrm{z}_r & \end{array}\right) \tag{1.12}$$

2. Integral

The integral to be evaluated is :

$$\int_{-1}^{1}(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}P_{k-\frac{u-v}{2}}^{u,v}(x)\,\aleph\begin{pmatrix}(\ 1\text{ - x })^{\alpha_1}(1+x)^{\beta_1}z_1\\ \cdots\\ (\ 1\text{ - x })^{\alpha_r}(1+x)^{\beta_r}z_r\end{pmatrix}\mathrm{d}x$$

$$=2^{\rho-u+v+\sigma+1}\sum_{t=0}^{\infty}\frac{(-k)_t(v-u+k+1)_t}{\Gamma(1-u+t)t!}\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} 2^{\alpha_1+\beta_1}z_1 & (\text{ - }\sigma-v;\beta_1,\cdots,\beta_r),\\ \cdots & \cdots\\ 2^{\alpha_r+\beta_r}z_r & \cdots, \end{array}\right.$$

$$\left.\begin{array}{c}(\text{ u - }\rho-t;\alpha_1,\cdots,\alpha_r),A:C\\ \cdots\\ (\text{ u - v - }\rho-\sigma-t;\alpha_1+\beta_1,\cdots,\alpha_r+\beta_r),B:D\end{array}\right) \tag{2.1}$$

Where $U_{21}=p_i+2,q_i+1,\tau_i;R$

Integral (2.1) is valid under the following condition provided $:k-\dfrac{u-v}{2},k$ are positive integers

a) $\alpha_i,\beta_i>0,i=1,\cdots,r$

b) $Re[\rho-u+\sum_{i=1}^{r}\alpha_i\min_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

c) $Re[\sigma+v+\sum_{i=1}^{r}\beta_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

d) $|argx_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

Proof : On expressing the multivariable Aleph-function in the integrand as a multiple Mellin-Barnes type integral (1.1) and changing the order of integrations , wich is justified due to the absolute convergence of the integrals involving in the process , we get :

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\phi_k(s_k)z_k^{s_k}\left[\int_{-1}^{1}(1-x)^{\rho-\frac{u}{2}+\alpha_1 s_1+\cdots+\alpha_r s_r}\right.$$

$$\left.\times(1+x)^{\sigma+\frac{v}{2}+\beta_1 s_1+\cdots+\beta_r s_r}\,P^{u,v}_{k-\frac{u-v}{2}}(x)\mathrm{d}x\right]\mathrm{d}s_1\cdots\mathrm{d}s_r$$

On evaluating the x-integral with the help of the integral ([3] , page343 , eq. (38)) :

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}\,P^{m,n}_{k-\frac{m-n}{2}}(x)\,\mathrm{d}x=\frac{2^{\rho+\sigma-\frac{m-n}{2}}\Gamma(\rho-\frac{m}{2}+1)\Gamma(\sigma+\frac{n}{2}+1)}{\Gamma(1-m)\Gamma(\rho+\sigma-\frac{m-n}{2}+2)}$$

$$\times\,{}_3F_2\left(-k,n-m+k+1,\rho-\frac{m}{2}+1;1-m,\rho-\sigma-\frac{m-n}{2}+2;1\right)\tag{2.2}$$

Provided that $Re(\rho-\frac{m}{2})>-1;Re(\sigma+\frac{n}{2})>-1$ and interpreting the result with the help of (1.1) , the integral (2.1) is established.

3. Expansion formula

Let the following conditions be established :

a) $Re(u)>-1,Re(v)>-1$

b) $\alpha_i,\beta_i>0,i=1,\cdots,r$

c) $Re[\rho-u+\sum_{i=1}^{r}\alpha_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

d) $Re[\sigma+v+\sum_{i=1}^{r}\beta_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

e) $|argx_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

Then the following expansion formula holds :

$$(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}\aleph\begin{pmatrix}(1-x)^{\alpha_1}(1+x)^{\beta_1}z_1\\ \cdot\ \cdot\ \cdot\\ (1-x)^{\alpha_r}(1+x)^{\beta_r}z_r\end{pmatrix}$$

$$= 2^{\rho+\sigma}\sum_{n=0}^{\infty}\sum_{p=0}^{n}\frac{(2n-u+v+1)\Gamma(n-u+1)\Gamma(1+v-u+n+p)(-n)_p}{n!p!\Gamma(1+v+n)\Gamma(1-u+p)}\ P^{u,v}_{n-\frac{u-v}{2}}(x)$$

$$\times\aleph^{0,\mathfrak{n}+2:V}_{U_{21}:W}\left(\begin{array}{c}2^{\alpha_1+\beta_1}z_1\\ \cdots\\ 2^{\alpha_r+\beta_r}z_r\end{array}\middle|\begin{array}{c}(\text{-}\ \sigma-v;\beta_1,\cdots,\beta_r),\quad (\text{ u - }\rho-p;\alpha_1,\cdots,\alpha_r),A:C\\ \cdots\qquad\cdots\\ \cdots,\quad(\text{ u - v - }\rho-\sigma-p-1;\alpha_1+\beta_1,\cdots,\alpha_r+\beta_r),B:D\end{array}\right) \quad (3.1)$$

Where $U_{21} = p_i + 2, q_i + 1, \tau_i; R$

Proof : Let : $f(x) = (1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}\aleph\left(\begin{array}{c}(\text{ 1 - x })^{\alpha_1}(1+x)^{\beta_1}z_1\\ \cdots\\ (\text{ 1 - x })^{\alpha_r}(1+x)^{\beta_r}z_r\end{array}\right) = \sum_{n=0}^{\infty}a_n\,P^{u,v}_{n-\frac{u-v}{2}}(x)$ (3.2)

The equation (3.2) is valid since $f(x)$is continuous and bounded variation in the interval (-1,1). Now , multiplying both the sides of (3.2) by$P^{m,n}_{t-\frac{m-n}{2}}(x)$and integrating with respect to x from -1 to 1 , evaluating the L.H.S. with the help of (2.1) and on the R.H.S. interchanging the order of summation using ([1] , p ,176 , eq;(75)) :

$$\int_{-1}^{1}P^{u,v}_{n-\frac{u-v}{2}}(x)P^{u,v}_{k-\frac{u-v}{2}}(x)\mathrm{d}x = \frac{2^{u-v+1}k!\Gamma(k+v+1)}{(2k-u+v+1)\Gamma(k-u+1)\Gamma(k-u+v+1)}\,\delta_{kn} \quad (3.3)$$

where $\delta_{kn} = 1$ if $k = n$, 0 else.

Provided that $Re(u) > -1, Re(v) > -1$, we get :

$$a_n = \frac{2^{\rho+\sigma}(2n-u+v+1)\Gamma(n-u+1)}{n!\Gamma(n+v+1)}\sum_{p=0}^{n}\frac{\Gamma(n+v-u+p+1)(-n)_p}{p!\Gamma(n-u+p)}$$

$$\times\aleph^{0,\mathfrak{n}+2:V}_{U_{21}:W}\left(\begin{array}{c}2^{\alpha_1+\beta_1}z_1\\ \cdots\\ 2^{\alpha_r+\beta_r}z_r\end{array}\middle|\begin{array}{c}(\text{-}\ \sigma-v;\beta_1,\cdots,\beta_r),\quad (\text{ u - }\rho-p;\alpha_1,\cdots,\alpha_r),A:C\\ \cdots\qquad\cdots\\ \cdots,\quad(\text{ u - v - }\rho-\sigma-p-1;\alpha_1+\beta_1,\cdots,\alpha_r+\beta_r),B:D\end{array}\right) \quad (3.4)$$

Where $U_{33} = p_i + 2, q_i + 1, \tau_i; R$

Now on substituting the value of a_n in (3.2) ,the result follows.

4. Particular cases

Remarks : If $\tau_i = \tau_{i^{(k)}} = 1$, then the Aleph-function of several variables degenere in the I-function of several variables defined by Sharma and Ahmad [5].

We have :

$$(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}I\left(\begin{array}{c}(\text{ 1 - x })^{\alpha_1}(1+x)^{\beta_1}z_1\\ \cdots\\ (\text{ 1 - x })^{\alpha_r}(1+x)^{\beta_r}z_r\end{array}\right)$$

$$= 2^{\rho+\sigma}\sum_{n=0}^{\infty}\sum_{p=0}^{n}\frac{(2n-u+v+1)\Gamma(n-u+1)\Gamma(1+v-u+n+p)(-n)_p}{n!p!\Gamma(1+v+n)\Gamma(1-u+p)}\ P^{u,v}_{n-\frac{u-v}{2}}(x)$$

$$\times I_{U_{21}:W}^{0,\mathrm{n}+2:V}\left(\begin{array}{c} 2^{\alpha_1+\beta_1}z_1 \\ \cdots \\ 2^{\alpha_r+\beta_r}z_r \end{array}\middle| \begin{array}{cc} (\text{ - }\sigma-v;\beta_1,\cdots,\beta_r), & (\text{ u - }\rho-p;\alpha_1,\cdots,\alpha_r),A:C \\ \cdots & \cdots \\ \cdots, & (\text{ u - v - }\rho-\sigma-p-1;\alpha_1+\beta_1,\cdots,\alpha_r+\beta_r),B:D \end{array}\right) \quad (4.1)$$

Where $U_{33}=p_i+2,q_i+1;R$

If $R=R^{(1)}=,\cdots,R^{(r)}=1$, the multivariable I-function degenere in the multivariable H-function and we have :

$$(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}H\left(\begin{array}{c} (\text{ 1 - x })^{\alpha_1}(1+x)^{\beta_1}z_1 \\ \cdots \\ (\text{ 1 - x })^{\alpha_r}(1+x)^{\beta_r}z_r \end{array}\right)$$

$$=2^{\rho+\sigma}\sum_{n=0}^{\infty}\sum_{p=0}^{n}\frac{(2n-u+v+1)\Gamma(n-u+1)\Gamma(1+v-u+n+p)(-n)_p}{n!p!\Gamma(1+v+n)\Gamma(1-u+p)}\; P_{n-\frac{u-v}{2}}^{u,v}(x)$$

$$H_{p+2,q+1:(p',q');\cdots;(p^{(r)},q^{(r)})}^{0,n+2:(m',n');\cdots;(m^{(r)},n^{(r)})}\left(\begin{array}{c} 2^{\alpha_1+\beta_1}z_1 \\ \cdots \\ 2^{\alpha_r+\beta_r}z_r \end{array}\middle| \begin{array}{c} (\text{ u - }\rho-p;\alpha_1,\cdots,\alpha_r), \\ \cdots \\ (\text{ u - v - }\rho-\sigma-p-1;\alpha_1+\beta_1,\cdots,\alpha_r+\beta_r), \end{array}\right.$$

$$\left.\begin{array}{c} ,(\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha_{rj}^{(r)})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}} \\ ,(\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta_{rj}^{(r)})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}} \end{array}\right) \quad (4.2)$$

for more details, see Saxena and Ramawat[4]

5 Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as I-function of several variables defined by Sharma and Ahmad [5] , multivariable H-function , see Srivastava et al [6]

REFERENCES

[1] Carslaw, H.S.; Introduction to the Theory of Fourier's Series and Integrals, Dover Publication Inc., New York (1950).

[2] Meulendbeld, B.; Generalized Legendre's associated functions for real values of the argument numerically less than unity, Nederl. Akad. Wetensch Proc. Ser. A61 (1958), page 557-563.

[3] Meulendbeld, B. and Robin, L.; Nouveaux resultats aux functions de Legendre's generalisees, Nederl. Akad. Wetensch. Proc. Ser. A 64(1961), page 333-347

[4] Saxena, R.K. and Ramawat, A.; An expansion formula for the multivariable H-function invoving generalized Legendre's associated function, Jnanabha, vol. 22(1992), 157-164.

[5] Sharma C.K. and Ahmad S.S. : On the multivariable I-function. Acta ciencia Indica Math , 1992 vol 19

[6] Srivastava, H.M., Gupta, K.C. and Goyal, S.P.; The H-functions of One and Two Variables with Applications, South Asian Publishers, New Delhi and Madras (1982).

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