Euler type triple integrals involving, general class of polynomials and multivariable Aleph-function II

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ABSTRACT
The aim of the present document is to evaluate three triple Euler type integrals involving general class of polynomials, special functions and multivariable Aleph-function. Importance of our findings lies in the fact that they involve the multivariable Aleph-function, which are the sufficiently general in nature and are capable of yielding a large number of simpler and useful results merely by specializing the parameters in them. Further we establish some special cases.

KEYWORDS : Aleph-function of several variables, triple Euler type integrals, special function, general class of polynomials.

2010 Mathematics Subject Classification. 33C99, 33C60, 44A20

1. Introduction and preliminaries.

The object of this document is to study three triple Eulerian integral involving general class of polynomials, special functions and the multivariables aleph-function. These function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [5] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c} z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\cdots\cdots\cdots\cdots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

where $j=1$ to r and $k=1$ to r

Suppose , as usual , that the parameters

$a_j, j=1,\cdots,p; b_j, j=1,\cdots,q;$

$c_j^{(k)}, j=1,\cdots,n_k; c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,p_{i^{(k)}};$

$d_j^{(k)}, j=1,\cdots,m_k; d_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,q_{i^{(k)}};$

with $k=1\cdots,r, i=1,\cdots,R$, $i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j=1$ to m_k are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)} s_k)$ with $j=1$ to n and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)} s_k)$ with $j=1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k=1\cdots,r, i=1,\cdots,R \text{ , } i^{(k)}=1,\cdots,R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}\,), max(\,|z_1|\ldots|z_r|\,) \rightarrow 0$

$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\,), min(\,|z_1|\ldots|z_r|\,) \rightarrow \infty$

where, with $k=1,\cdots,r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and

$$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \; ; \; V = m_1, n_1; \cdots ; m_r, n_r \tag{1.6}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.7}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.8}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots, \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.10}$$

$$D = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots, \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots, z_r) = \aleph_{U:W}^{0,\mathrm{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A : C} \\ \cdot & \cdot\;\cdot\;\cdot \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B : D} \end{array}\right) \tag{1.12}$$

The generalized polynomials defined by Srivastava [5], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s] = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1, K_1; \cdots; N_s, K_s] y_1^{K_1} \cdots y_s^{K_s} \tag{1.14}$$

Where $M_1, \cdots, M_s$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots; N_s, K_s]$ are arbitrary constants, real or complex.

2 . Results required :

$$\text{a) } \int_0^1 x^{c-1}(1-x)^{-1/2} {}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x = \frac{\pi\Gamma(c)\Gamma(a+b+1/2)\Gamma(c-a-b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma(c-a+1/2)\Gamma(c-b+1/2)} \tag{2.1}$$

Where Re(c) > 0, Re(2c-a-b) > -1 , see Vyas and Rathie [7].

Erdélyi [1] [p.78, eq.(2.4) (1), vol 1]

$$\text{b) } \int_0^1 \int_0^1 t^{b-1} r^{a-1} (1-t)^{c-b-1} (1-r)^{c-a-1} (1-trz)^{-c} \,\mathrm{d}r\mathrm{d}t$$

$$= \frac{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}{[\Gamma(c)]^2} {}_2F_1(a,b;c;z) \tag{2.2}$$

$$Re(a) > 0, Re(b) > 0, Re(c-a) > 0, Re(c-b) > 0$$

Erdélyi [1] [p.230, eq.(5.8.1) (2), vol 1]

c) $\int_0^1\int_0^1 u^{\beta-1}v^{\beta'-1}(1-u)^{\gamma-\beta-1}(1-v)^{\gamma'-\beta'-1}(1-ux-vy)^{-\alpha}\,\mathrm{d}u\mathrm{d}v$

$$=\frac{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta)\Gamma(\gamma'-\beta')}{\Gamma(\gamma)\Gamma(\gamma')}F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y) \tag{2.3}$$

$Re(\beta)>0, Re(\beta')>0, Re(\gamma-\beta)>0, Re(\gamma'-\beta')>0$

Erdélyi [1] [p.230, eq.(5.8.1) (4), vol 1]

d) $\int_0^1\int_0^1 u^{\alpha-1}v^{\beta-1}(1-u)^{\gamma-\alpha-1}(1-v)^{\gamma'-\beta-1}(1-ux)^{\alpha-\gamma-\gamma'+1}(1-vy)^{\beta-\gamma-\gamma'+1}$

$(1-ux-vy)^{\gamma+\gamma'-\alpha-\beta-1}\mathrm{d}u\mathrm{d}v$

$$=\frac{\Gamma(\beta)\Gamma(\alpha)\Gamma(\gamma-\alpha)\Gamma(\gamma'-\beta)}{\Gamma(\gamma)\Gamma(\gamma')}F_4(\alpha,\beta,\gamma,\gamma';x(1-y),y(1-x)) \tag{2.4}$$

$Re(\beta)>0, Re(\alpha)>0, Re(\gamma-\alpha)>0, Re(\gamma'-\beta)>0$

3. Main results

a) $\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$

$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}(y_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}(1-yzt)^{-\mu_1},\cdots,$

$y_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{\mu_s-\rho^{(s)}}(1-z)^{\mu_s-\zeta^{(s)}}(1-yzt)^{-\mu_s})$

$$\aleph\left(\begin{matrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}(1-yzt)^{-\eta_r}\end{matrix}\ \middle|\ \begin{matrix}\mathrm{A}:\mathrm{C}\\ \mathrm{B}:\mathrm{D}\end{matrix}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{k=0}^{\infty}\frac{t^k}{k!}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s}\aleph^{0,n+6:V}_{U_{64}:W}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\ \middle|\right.$$

$$\begin{matrix}(1-\alpha-\zeta'K_1-\cdots-\zeta^{(s)}K_s-k;\zeta_1,\cdots,\zeta_r),(1/2-\mathrm{c}+\mathrm{a}+\mathrm{b}-\mathrm{c}_1K_1-\cdots-c_sK_s;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\qquad\cdots\\ (1-\lambda-\mu_1K_1-\mu_sK_s-k;\eta_1,\cdots,\eta_r),\quad(1/2-\mathrm{c}+\mathrm{b}-\mathrm{c}_1K_1-\cdots-c_sk_s;\sigma_1,\cdots,\sigma_r),\end{matrix}$$

$$\begin{matrix}(1-\mathrm{c}-\mathrm{c}_1K_1-\cdots-C_sK_s;\sigma_1,\cdots,\sigma_r),\quad(1-\beta-k-\rho'K_1-\cdots-\rho^{(s)}K_s;\rho_1,\cdots,\rho_r),\\ \cdots\qquad\qquad\qquad\cdots\\ (1/2-\mathrm{c}-\mathrm{c}_1K_1-\cdots-c_sK_s+a;\sigma_1,\cdots,\sigma_r),\qquad\qquad \cdots\cdots,\end{matrix}$$

$$(1+\beta-\lambda-(\mu_1-\rho')K_1-\cdots-(\mu_s-\rho^{(s)})K_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1-\lambda-\mu_1K_1-\mu_sK_s;\eta_1,\cdots,\eta_r),$$

$$\left.\begin{matrix}(1+\alpha-\lambda-(\mu_1-\zeta')K_1-\cdots-(\mu_s-\zeta^{(s)})K_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),A:C\\ \cdots\\ \cdots, \mathrm{B:D}\end{matrix}\right) \tag{3.1}$$

Where $U_{64}=p_i+6,q_i+4,\tau_i;R$

Provided that :

$Re(c+c_1K_1+\cdots+c_sK_s+\sigma_1s_1+\cdots+\sigma_rs_r)>0;$

$Re(2(c+c_1K_1+\cdots+c_sK_s+\sigma_1s_1+\cdots+\sigma_rs_r)-a-b)>-1$

$Re(\beta+\rho'K_1+\cdots+\rho^{(s)}K_s+\rho_1s_1+\cdots+\rho_rs_r)>0$

$Re(\alpha+\zeta'K_1+\cdots+\zeta^{(s)}K_s+\zeta_1s_1+\cdots+\zeta_rs_r)>0$

$Re(\lambda-\alpha+(\mu_1-\zeta')K_1+\cdots+(\mu_s-\zeta^{(s)})K_s+(\eta_1-\zeta_1)s_1+\cdots+(\eta_r-\zeta_r)s_r)>0$

$Re(\lambda-\beta+(\mu_1-\rho')K_1+\cdots+(\mu_s-\rho^{(s)})K_s+(\eta_1-\rho_1)s_1+\cdots+(\eta_r-\rho_r)s_r)>0$

$|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

b) $\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}$

$(1-uy-vz)^{-n}\,S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}(y_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'},\cdots,$

$y_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}})$

$$\aleph\left(\begin{matrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r}\end{matrix}\;\middle|\;\begin{matrix}\mathrm{A:C}\\ \mathrm{B:D}\end{matrix}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{k,m=0}^{\infty}\frac{u^kv^m}{k!m!}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s}\aleph_{U_{75}:W}^{0,n+7:V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\;\middle|\right.$$

$$(1-\alpha-\zeta'K_1-\cdots-\zeta^{(s)}K_s-m;\zeta_1,\cdots,\zeta_r),(1/2-c+a+b-c_1K_1-\cdots-c_sK_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots\qquad\cdots$$
$$(1-\mu-t'K_1-\cdots-t^{(s)}K_s-m;t_1,\cdots,t_r),\quad(1/2-c+b-c_1K_1-\cdots-c_sK_s;\sigma_1,\cdots,\sigma_r),$$

$$(1 \text{ -c - } c_1 K_1 - \cdots - c_s K_s; \sigma_1, \cdots, \sigma_r),$$
$$\cdots$$
$$(1/2 \text{ -c -} c_1 K_1 - \cdots - c_s K_s + a; \sigma_1, \cdots, \sigma_r),$$

$$(1 - \mu + \alpha - (t' - \zeta')K_1 - \cdots - (t^{(s)} - \zeta^{(s)})K_s; t_1 - \zeta_1, \cdots, t_r - \zeta_r),$$
$$\cdots$$
$$\cdots\cdots$$
$$(1 - \lambda - (e' - \rho')K_1 - \cdots - (e^{(s)} - \rho^{(s)})K_s; \eta_1 - \rho_1, \cdots, \eta_r - \rho_r),$$
$$\cdots$$
$$(1 - \lambda - e'K_1 - \cdots - e^{(s)}K_s - k; \eta_1, \cdots, \eta_r),$$

$$\left.\begin{matrix} (1 \text{ -n -k- } \omega' K_1 - \cdots - \omega^{(s)} K_s - m; \eta_1, \cdots, \eta_r), & (1 - \beta - k - \rho' K_s - \cdots - \rho^{(s)} K_s; \rho_1, \cdots, \rho_r), A : C \\ \cdots & \cdots \\ (1 \text{ -n - } \omega' K_1 - \cdots - \omega^{(s)} K_s; \eta_1, \cdots, \eta_r), & , B : D \end{matrix}\right) \quad (3.2)$$

Where $U_{75} = p_i + 7, q_i + 5, \tau_i; R$

Provided that :

$Re(c + c_1 K_1 + \cdots + C_s K_s + \sigma_1 s_1 + \cdots + \sigma_r s_r) > 0;$

$Re(2(c + c_1 K_1 + \cdots + c_s K_s + \sigma_1 s_1 + \cdots + \sigma_r s_r) - a - b) > -1$

$Re(\beta + \rho' K_1 + \cdots + \rho^{(s)} K_s + \rho_1 s_1 + \cdots + \rho_r s_r) > 0$

$Re(\alpha + \zeta' K_1 + \cdots + \zeta^{(s)} K_s + \zeta_1 s_1 + \cdots + \zeta_r s_r) > 0$

$Re(\lambda - \beta + (e' - \rho')K_1 + \cdots + (e^{(s)} - \rho^{(s)})K_s + (\eta_1 - \rho_1)s_1 + \cdots + (\eta_r - \rho_r)s_r) > 0$

$Re(\mu - \alpha + (t' - \zeta')K_1 + \cdots + (t^{(s)} - \zeta^{(s)})K_s + (t_1 - \zeta_1)s_1 + \cdots + (t_r - \zeta_r)s_r) > 0$

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)}$ is given in (1.5)

c) $\int_0^1 \int_0^1 \int_0^1 x^{c-1}(1-x)^{-1/2} {}_2F_1(a, b; a+b+1/2; x) y^{\alpha-1} z^{\beta-1} (1-y)^{\lambda-\alpha-1} (1-z)^{\mu-\beta-1}$

$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}$

$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}(y_1 x^{\sigma'} y^{\rho'} z^{\zeta'} (1-y)^{\eta'-\rho'} (1-z)^{t'-\zeta'} (1-uy-vz)^{\zeta'-\eta'-t'}, \cdots,$

$y_s x^{\sigma^{(s)}} y^{\rho^{(s)}} z^{\zeta^{(n)}} (1-y)^{\eta^{(s)}-\rho^{(s)}} (1-z)^{t^{(s)}-\zeta^{(s)}} (1-uy-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}})$

$$\aleph\begin{pmatrix} z_1 x^{\sigma_1} y^{\rho_1} z^{\zeta_1} (1-y)^{\eta_1-\rho_1} (1-z)^{t_1-\zeta_1} (1-uy)^{\rho_1-\eta_1-t_1} (1-vz)^{\zeta_1-\eta_1-t_1} (1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1} \\ \cdots \\ z_r x^{\sigma_r} y^{\rho_r} z^{\zeta_r} (1-y)^{\eta_r-\rho_r} (1-z)^{t_s-\zeta_s} (1-uy)^{\rho_r-\eta_r-t_r} (1-vz)^{\zeta_r-\eta_r-t_r} (1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r} \end{pmatrix}$$

$\mathrm{d}x\mathrm{d}y\mathrm{d}z$

$$= \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{k,m=0}^{\infty} \frac{u^k(1-v)^k v^m (1-u)^m}{k!m!}$$

$$\frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!} A[N_1,K_1;\cdots;N_s,K_s] y_1^{K_1} \cdots y_s^{K_s} \aleph_{U_{64}:W}^{0,n+6:V} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\cdot\cdot \\ \mathrm{z}_r \end{array} \middle| \right.$$

$$(1\text{ -c -}\sigma' K_1 - \cdots - \sigma^{(k)} K_s; \sigma_1, \cdots, \sigma_r), \quad (1/2\text{ -c - }\sigma' K_1 - \cdots - \sigma^{(s)} K_s + a + b; \sigma_1, \cdots, \sigma_r),$$

$$\cdots \qquad\qquad \cdots$$

$$(1/2\text{ -c- }\sigma' K_1 - \cdots - \sigma^{(k)} K_s + a; \sigma_1, \cdots, \sigma_r), \quad (1/2\text{ -c - }\sigma' K_1 - \cdots - \sigma^{(s)} K_s + b; \sigma_1, \cdots, \sigma_r),$$

$$(1\text{ -}\mu - (t' - \zeta')K_1 + \cdots + (t^{(s)} - \zeta^{(s)})K_s + \beta; t_1 - \zeta_1, \cdots, t_r - \zeta_r),$$

$$\cdots$$

$$(1\text{-}\lambda - (\eta' - \rho')K_1 - \cdots - (\eta^{(s)} - \rho^{(s)})K_s - k; \eta_1, \cdots \eta_r)$$

$$(1\text{ -}\lambda - (\eta' - \rho')K_1 - \cdots - (\eta^{(s)} - \rho^{(s)})K_s; \eta_1 - \rho_1, \cdots, \eta_r - \rho_r),$$

$$\cdots$$

$$(1\text{ -}\mu - (t' - \zeta')K_1 - \cdots - (t^{(s)} - \zeta^{(s)})K_s - m; t_1, \cdots, t_r),$$

$$(1\text{ -}\alpha - k - \rho' K_1 - \cdots - \rho^{(s)} K_s - m; \rho_1, \cdots, \rho_r),$$

$$\cdots$$

$$, \cdots\cdots$$

$$\left. \begin{array}{c} (1\text{ -}\beta - k - \zeta' K_1 - \cdots - \zeta^{(s)} K_s; \zeta_1, \cdots, \zeta_r), A : C \\ \cdots \\ , \ldots ., \mathrm{B} : \mathrm{D} \end{array} \right) \tag{3.3}$$

Where $U_{64} = p_i + 6, q_i + 4, \tau_i; R$

Provided that :

$$Re(c + \sigma' K_1 + \cdots + \sigma_s^{(s)} K_s + \sigma_1 s_1 + \cdots + \sigma_r s_r) > 0;$$

$$Re(2(c + \sigma_1' K_1 + \cdots + \sigma^{(s)} K_s + \sigma_1 s_1 + \cdots + \sigma_r s_r) - a - b) > -1$$

$$Re(\alpha + \rho' K_1 + \cdots + \rho^{(s)} K_s + \rho_1 s_1 + \cdots + \rho_r s_r) > 0$$

$$Re(\beta + \zeta' K_1 + \cdots + \zeta^{(s)} K_s + \zeta_1 s_1 + \cdots + \zeta_r s_r) > 0$$

$$Re(\lambda - \alpha + (\eta' - \rho')K_1 + \cdots + (\eta^{(s)} - \rho^{(s)})K_s + (\eta_1 - \rho_1)s_1 + \cdots + (\eta_r - \rho_r)s_r) > 0$$

$$Re(\mu - \beta + (t' - \zeta')K_1 + \cdots + (t^{(s)} - \zeta^{(s)})K_s + (t_1 - \zeta_1)s_1 + \cdots + (t_r - \zeta_r)s_r) > 0$$

5) $|argz_k| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)}$ is given in (1.5)

Proof de (3.1) : We fisrt express the multivariable Aleph-function involving in the left hand side of (2.1) in terms of Mellin-Barnes contour integral with the help of (1.1) and then interchanching the order of integration. We get L.H.S.

$$= \frac{1}{(2\pi\omega)^r}\left(\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\right.$$
$$\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s}$$

$$\left(\int_0^1 x^{c+c_1K_1+\cdots+c_sK_s+\sigma_1s_1+\cdots+\sigma_rs_r-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x\right)$$
$$\times\left(\int_0^1\int_0^1 y^{\beta+\rho'K_1+\cdots+\rho^{(s)}K_s+\rho_1s_1+\cdots+\rho_rs_r}z^{\alpha+\zeta'K_1+\cdots+\zeta^{(s)}K_s+\zeta_1s_1+\cdots+\zeta_rs_r-1}\right.$$
$$(1-yzt)^{-(\lambda+\mu_1K_1+\cdots+\mu_sK_s+\eta_1s_1+\cdots+\eta_rs_r)}$$
$$\times(1-y)^{(\lambda+\mu_1K_1+\cdots+\mu_sK_s+\eta_1s_1+\cdots+\eta_rs_r)-(\beta+\rho'K_1+\cdots\rho^{(s)}K_s+\rho_1s_1+\cdots+\rho_rs_r)-1}$$
$$\left.\times(1-z)^{(\lambda+\mu_1K_1+\cdots+\mu_sK_s+\eta_1s_1+\cdots+\eta_rs_r)-(\alpha+\zeta'K_1+\cdots+\zeta^{(s)}K_s+\zeta_1s_1+\cdots+\zeta_rs_r)-1}\mathrm{d}y\mathrm{d}z\right)\mathrm{d}s_1\cdots\mathrm{d}s_r$$

Now using the result (2.1), (2.2) and (1.1) we get right hand side of (3.1). Similarly we can prove (3.2) and (3.3) with help of the results (2.3) and (2.4).

4. Particular cases

Our main the results provided unification and extensions of various (known or new) results. For the sake illustration, we mention the following few special cases :

i) If take s = 1, we get the results obtained by Garg et al [3]

ii) If we take $a=-n, b=n$ in ${}_2F_1(a,b;a+b+1/2;x)$ and using the relationship [2,p.18]

${}_2F_1(a,b;a+b+1/2;x)$= ${}_2F_1(-n,n;1/2;[1-(1-2x)]/2)=T_n(1-2x)$, we get the results involving

Tchebcheff polynomial.

(iii) If we take $a=-n, b=k+n$ in ${}_2F_1(a,b;a+b+1/2;x)$ and using the relationship [2,p.18]

${}_2F_1(a,b;a+b+1/2;x)$= ${}_2F_1(-n,k+n;k+1/2;x)=P_n^{k,k+1/2}(x)$, we get the results involving

Jacobi polynomial.

(iV) If we take M = 1 and $A_{N,K}=\binom{N+\alpha'}{N}\frac{1}{(\alpha'+1)_{K_1}}$, then general class of polynomial reduces to

Laguerre polynomial and we get the results involving Laguerre polynomial.

Remarks : If $\tau_i=\tau_{i^{(k)}}=1$, then the Aleph-function of several variables degenere in the I-function of several variables defined by Sharma and Ahmad [4].

And if $R=R^{(1)}=,\cdots,R^{(r)}=1$, the multivariable I-function degenere in the multivariable H-function defined by srivastava et al [6].

5. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as I-function of several variables defined by Sharma and Ahmad [4], multivariable H-function, see Srivastava et al [6], and the h-function of two variables , see Srivastava et a[6].

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