Some New Families of Face Edge Product Cordial Graphs

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ABSTRACT

In this paper, we investigate the face edge product cordial labeling of the alternative triangular snakes, $K_{1,n} \odot P_m$ and $(P_n \odot K_1) \odot P_m$. Also non face edge product cordial labeling of the graph $DS(P_n)$ is presented.

Keywords : Edge product cordial labeling, Alternate triangular snake, Face edge product cordial labeling, Face edge product cordial graph.

1. Introduction

Throughout this paper we consider only finite, planar, undirected and simple graphs. Let G be a graph with p vertices and q edges. For all terminologies and notations related to graph theory, we follow Harary [2]. For standard terminology and notations related to graph labeling, we refer to Gallian [1]. The concept of edge product cordial labeling of graphs is introduced by Vaidya et al. [5]. In [3], Lawrence et al. introduced the concept of face edge product cordial labeling of graphs and face edge product cordial labeling triangular snake is presented. Face edge product cordial labeling of some corona graphs are investigated by Muthaiyan et al. [4]. In [6], Vaidya et al. presented the product cordial labelings for alternate triangular snake graphs.

The brief summaries of definition which are necessary for the present investigation are provided below.

Definition : 1.1

The corona $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2.

Definition : 1.2

For a graph G = (V(G),E(G)), an edge labeling function f : E(G)→{0,1} induces a vertex labeling function f^* :V(G)→{0,1} defined as $f^*(v) = \prod f(e_i)$ for $\{e_i \in E(G)/e_i$ is incident to v}. Now denoting the number of vertices of G having label i under f^* as $v_f(i)$ and the number of edges of G having label i under f as $e_f(i)$. Then f is called edge product cordial labeling of graph G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called edge product cordial if it admits edge product cordial labeling.

Definition : 1.3

For a planar graph G, the edge labeling function is defined as g : E(G)→{0,1} and g(e) is called the label of the edge e of G under g, induced vertex labeling function g^* : V(G) → {0,1} is given as if $e_1,e_2,\ldots,e_m$ are the edges incident to vertex v, then $g^*(v) = g(e_1)g(e_2)\ldots g(e_m)$ and induced face labeling function g^{**} : F(G) → {0,1} is given as if $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$ are the vertices and edges of face f then $g^{**}(f) = g^*(v_1)g^*(v_2)\ldots g^*(v_n)$ $g(e_1)g(e_2)\ldots g(e_m)$. Let us denote $v_g(i)$ is the number of vertices of G having label i under g^*, $e_g(i)$ is the number of edges of G having label i under g and $f_g(i)$ is the number of interior faces of G having label i under g^{**} for i = 1,2. g is called face edge product cordial labeling of graph G if $|v_g(0)-v_g(1)|\le 1$, $|e_g(0)-e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$. A graph G is face edge product cordial if it admits face edge product cordial labeling.

Definition : 1.4

A triangular snake is obtained from a path $v_1v_2\ldots v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $1 \le i \le n-1$.

Definition : 1.5

An Alternate Triangular snake $A(T_n)$ is obtained from a path $v_1v_2\ldots v_n$ by joining v_i and v_{i+1} alternatively to a new vertex w_i. That is every alternate edge of a path is replaced by C_3.

Definition : 1.6

Let G = (V(G), E(G)) be a graph with vertex set $V = S_1\cup S_2\cup\ldots\cup S_i\cup T$ where each S_i is a set of vertices having at least two vertices of the same degree and $T = V\backslash\cup S_i$. The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, w_3, \ldots, w_t$ and joining to each vertex of S_i for $1 \le i \le t$.

In this paper, we investigate the face edge product cordial labeling of the alternative triangular snakes, $K_{1,n}\odot P_m$ and $(P_n\odot K_1)\odot P_m$. Also non face edge product cordial labeling of the planar graph $DS(P_n)$ is presented.

2. Main Theorems

Theorem 2.1

The graph alternative triangular snake $A(T_n)$ is face edge product cordial graph except $n \equiv 3 \pmod 4$.

Proof.

Let G be a alternative triangular snake $A(T_n)$.

Let $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_{n-1}$ be the vertices and edges of the path P_n.

Case 1 : $n \equiv 0 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_n.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i-1}u_i$ and $e'_{2i} = u_i v_{2i}$ for $i = 1,2,\ldots,\frac{n}{2}$ and interior faces $f_i = v_{2i-1}u_i v_{2i}$ for $i = 1,2,\ldots,\frac{n}{2}$.

Then $|V(G)| = \frac{3n}{2}$, $|E(G)| = 2n - 1$ and $|F(G)| = \frac{n}{2}$.

Define edge labeling $g : E(G) \to \{0,1\}$ as follows

$g(e_i) = 1,$ for $1 \le i \le \frac{n}{2}$

$g(e_i) = 0,$ for $\frac{n+2}{2} \le i \le n-1$

$g(e'_i) = 1,$ for $1 \le i \le \frac{n}{2}$

$g(e'_i) = 0,$ for $\frac{n+2}{2} \le i \le n$

In view of the above defined labeling pattern, we have

$$e_g(1) = e_g(0) + 1 = n\ ,\ v_g(0) = v_g(1) = \frac{3n}{4} \text{ and } f_g(0) = f_g(1) = \frac{n}{4}.$$

Therefore $|e_g(0) - e_g(1)| \le 1$, $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face edge product cordial graph, when $n \equiv 0 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_n.

Case 2 : $n \equiv 0 \pmod 4$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i}u_i$ and $e'_{2i} = u_i v_{2i+1}$ for $i = 1,2,\ldots, \frac{n-2}{2}$ and interior faces $f_i = v_{2i}\, u_i\, v_{2i+1}$ for $i = 1,2,\ldots, \frac{n-2}{2}$.

Then $|V(G)| = \frac{3n-2}{2}$, $|E(G)| = 2n - 3$ and $|F(G)| = \frac{n-2}{2}$.

Define edge labeling $g : E(G) \to \{0,1\}$ as follows

$g(e_i) = 1,$ for $1 \le i \le \frac{n}{2}$

$g(e_i) = 0,$ for $\frac{n+2}{2} \le i \le n-1$

$g(e'_i) = 1,$ for $1 \le i \le \frac{n-2}{2}$

$g(e'_i) = 0,$ for $\frac{n}{2} \le i \le n-2$

In view of the above defined labeling pattern, we have

$$e_g(1) = e_g(0) + 1 = n - 1,\ v_g(0) = v_g(1) + 1 = \frac{3n}{4} \text{ and } f_g(0) = f_g(1) + 1 = \frac{n}{4}.$$

Therefore $|e_g(0) - e_g(1)| \le 1$, $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face edge product cordial graph, when $n \equiv 0 \pmod 4$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

Case 3 : $n \equiv 2 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_n.

Let path P_n having vertices $v_1, v_2,\ldots,v_n$ and edges $e_1,e_2,\ldots,e_{n-1}$.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i-1}u_i$ and $e'_{2i} = u_i v_{2i}$ for $i = 1,2,\ldots, \frac{n}{2}$ and interior faces $f_i = v_{2i-1}\, u_i\, v_{2i}$ for $i = 1,2,\ldots, \frac{n}{2}$.

Then $|V(G)| = \frac{3n}{2}$, $|E(G)| = 2n - 1$ and $|F(G)| = \frac{n}{2}$.

Define edge labeling $g : E(G) \rightarrow \{0,1\}$ as follows

$$g(e_i) = 1, \quad \text{for } 1 \le i \le \frac{n}{2}$$

$$g(e_i) = 0, \quad \text{for } \frac{n+2}{2} \le i \le n-1$$

$$g(e'_i) = 1, \quad \text{for } 1 \le i \le \frac{n}{2}$$

$$g(e'_i) = 0, \quad \text{for } \frac{n+2}{2} \le i \le n$$

In view of the above defined labeling pattern, we have

$$e_g(1) = e_g(0) + 1 = n - 1,\ v_g(0) = v_g(1) + 1 = \frac{3n+2}{4} \text{ and } f_g(0) = f_g(1) + 1 = \frac{n+2}{4}.$$

Therefore $|e_g(0) - e_g(1)| \le 1$, $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face edge product cordial graph, when $n \equiv 2 (\text{mod } 4)$ and the first triangle start from v_1 and the last triangle ends with v_n.

Case 4 : $n \equiv 2$ (mod 4) and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i}u_i$ and $e'_{2i} = u_i v_{2i+1}$ for $i = 1,2,\ldots, \frac{n-2}{2}$ and interior faces $f_i = v_{2i}\, u_i\, v_{2i+1}$ for $i = 1,2,\ldots, \frac{n-2}{2}$.

Then $|V(G)| = \frac{3n-2}{2}$, $|E(G)| = 2n - 3$ and $|F(G)| = \frac{n-2}{2}$.

Define edge labeling $g : E(G) \rightarrow \{0,1\}$ as follows

$$g(e_i) = 1, \quad \text{for } 1 \le i \le \frac{n}{2}$$

$$g(e_i) = 0, \quad \text{for } \frac{n+2}{2} \le i \le n-1$$

$$g(e'_i) = 1, \quad \text{for } 1 \le i \le \frac{n-2}{2}$$

$$g(e'_i) = 0, \quad \text{for } \frac{n}{2} \le i \le n-2$$

In view of the above defined labeling pattern, we have

$$e_g(1) = e_g(0) + 1 = n - 1,\ v_g(0) = v_g(1) = \frac{3n-2}{4} \text{ and } f_g(0) = f_g(1) = \frac{n-2}{4}.$$

Therefore $|e_g(0) - e_g(1)| \le 1$, $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face edge product cordial graph, when $n \equiv 2 (\text{mod } 4)$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

Case 5 : $n \equiv 1 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_{n-1}.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i-1}u_i$ and $e'_{2i} = u_i v_{2i}$ for $i = 1,2,\ldots, \frac{n-1}{2}$ and interior faces $f_i = v_{2i-1}\, u_i\, v_{2i}$ for $i = 1,2,\ldots, \frac{n-1}{2}$.

Then $|V(G)| = \frac{3n-1}{2}$, $|E(G)| = 2n - 2$ and $|F(G)| = \frac{n-1}{2}$.

Define edge labeling $g : E(G) \to \{0,1\}$ as follows

$g(e_i) = 1,$ for $1 \le i \le \frac{n-1}{2}$

$g(e_i) = 0,$ for $\frac{n+1}{2} \le i \le n-1$

$g(e'_i) = 1,$ for $1 \le i \le \frac{n-1}{2}$

$g(e'_i) = 0,$ for $\frac{n+1}{2} \le i \le n-1$

In view of the above defined labeling pattern, we have

$$e_g(1) = e_g(0) = n-1\ ,\ v_g(0) = v_g(1) + 1 = \frac{3n+1}{4} \text{ and } f_g(0) = f_g(1) = \frac{n-1}{4}.$$

Therefore $|e_g(0) - e_g(1)| \le 1$, $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face edge product cordial graph, when $n \equiv 1 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_{n-1}.

Case 6 : $n \equiv 1 \pmod 4$ and the first triangle start from v_2 and the last triangle ends with v_n.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i}u_i$ and $e'_{2i} = u_i v_{2i+1}$ for $i = 1,2,\ldots, \frac{n-1}{2}$ and interior faces $f_i = v_{2i}\, u_i\, v_{2i+1}$ for $i = 1,2,\ldots, \frac{n-1}{2}$.

Then $|V(G)| = \frac{3n-1}{2}$, $|E(G)| = 2n - 2$ and $|F(G)| = \frac{n-1}{2}$.

Define edge labeling $g : E(G) \to \{0,1\}$ as follows

$g(e_i) = 0,$ for $1 \le i \le \frac{n-1}{2}$

$g(e_i) = 1,$ for $\frac{n+1}{2} \le i \le n-1$

$g(e'_i) = 0,$ for $1 \le i \le \frac{n-1}{2}$

$g(e'_i) = 1,$ for $\frac{n+1}{2} \leq i \leq n-1$

In view of the above defined labeling pattern, we have

$$e_g(1) = e_g(0) = n - 1,\ v_g(0) = v_g(1) + 1 = \frac{3n+1}{4} \text{ and } f_g(0) = f_g(1) = \frac{n-1}{2}.$$

Therefore $|e_g(0) - e_g(1)| \leq 1$, $|v_g(0) - v_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$

Thus, the alternative triangular snake $A(T_n)$ is face edge product cordial graph, when $n \equiv 1(\text{mod } 4)$ and the first triangle start from v_1 and the last triangle ends with v_n.

Case 7 : $n \equiv 3$ (mod 4) and the first triangle start from v_1 and the last triangle ends with v_{n-1}.

In order to satisfy the edge condition for G, it is essential to assign label 0 and 1 to exactly n–1 edges. Any pattern of edge labeling which satisfies the edge condition will induce vertex labels for $\frac{3n-1}{2}$ number of vertices in such a way that $| v_g(0) - v_g(1) | \geq 2$.

Therefore the vertex condition for G is violated. Thus the graph G under this consideration is not a face edge product cordial graph.

Hence, the alternative triangular snake $A(T_n)$ is not face edge product cordial graph, when $n \equiv 3(\text{mod } 4)$ and the first triangle start from v_1 and the last triangle ends with v_{n-1}.

Case 8 : $n \equiv 3(\text{mod } 4)$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

In order to satisfy the edge condition for G, it is essential to assign label 0 and 1 to exactly n–1 edges. Any pattern of edge labeling which satisfies the edge condition will induce vertex labels for $\frac{3n-1}{2}$ number of vertices in such a way that $| v_g(0) - v_g(1) | \geq 2$.

Therefore the vertex condition for G is violated. Thus the graph G under this consideration is not a face edge product cordial graph.

Hence, the alternative triangular snake $A(T_n)$ is not face edge product cordial graph, when $n \equiv 3$ (mod 4) and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

Therefore, the alternative triangular snake $A(T_n)$ is face edge product cordial graph except $n \equiv 3(\text{mod } 4)$.

Example : 2.1

(i). The alternative triangular snake $A(T_6)$ with first triangle start from first vertex and its face edge product cordial labeling are shown in Figure 2.1(a).

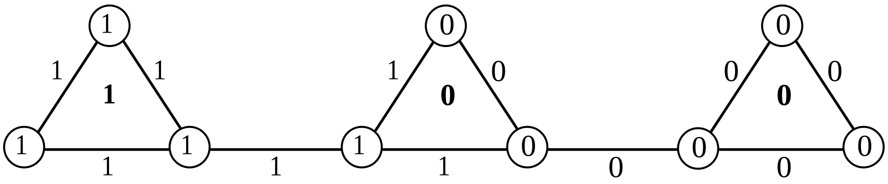

Figure 2.1(a)

(ii). The alternative triangular snake $A(T_6)$ with first triangle start from second vertex and its face edge product cordial labeling are shown in Figure 2.1(b).

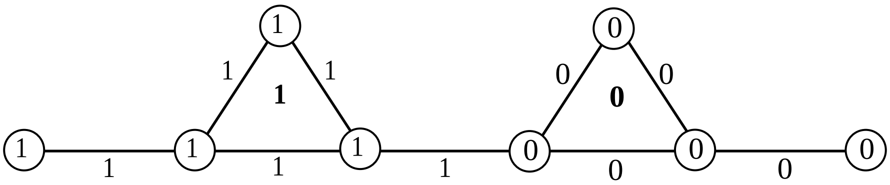

Figure 2.1(b)

Theorem : 2.2

$K_{1,n} \odot P_m$ is face edge product cordial graph except n is even and m > 2.

Proof.

Let $u_1, u_2, \ldots, u_{n+1}$ and $e_1, e_2, \ldots, e_n$ be the vertices and edges of $K_{1,n}$.

Let G be the graph $K_{1,n} \odot P_m$.

The vertex set $V(G) = \{u_i, v_{ij} : 1 \leq i \leq n+1, 1 \leq j \leq m\}$, edge set $E(G) = \{e_i, e_{jk}: 1 \leq i \leq n, 1 \leq j \leq n+1, 1 \leq k \leq 2m-1\}$ and interior face set $F(G) = \{ f_i : 1 \leq i \leq (n+1)(m-1)\}$ of G, where $e_i = u_i u_{n+1}$ for $1 \leq i \leq n$, $e_{jk} = u_j v_{jk}$ for $1 \leq j \leq n+1$ and $1 \leq k \leq m$, $e_{j(m+k)} = v_{jk} v_{j(k+1)}$ for $1 \leq j \leq n+1$ and $1 \leq k \leq m-1$, $f_i = u_i\, v_{ik}\, v_{i(k+1)}\, u_i$ for $1 \leq i \leq n+1$ and $1 \leq k \leq m-1$.

Then $|V(G)| = (n+1)(m+1)$, $|E(G)| = 2(n+1)m-1$ and $|F(G)| = (n+1)(m-1)$.

Define edge labeling $g : E(G) \to \{0,1\}$ as follows

Case 1 : n is odd

$$g(e_i) = 1, \quad \text{for } 1 \leq i \leq \frac{n+1}{2}$$

$$g(e_i) = 0, \quad \text{for } \frac{n+3}{2} \leq i \leq n$$

$$g(e_{jk}) = 1, \quad \text{for } 1 \leq j \leq \frac{n+1}{2} \text{ and } 1 \leq k \leq 2m-1$$

$$g(e_{jk}) = 0, \quad \text{for } \frac{n+3}{2} \leq j \leq n+1 \text{ and } 1 \leq k \leq 2m-1$$

In view of the above defined labeling pattern we have

$$e_g(1) = e_g(0) + 1 = (n+1)m,\ v_g(0) = v_g(1) = \frac{(n+1)(m+1)}{2} \text{ and } f_g(0) = f_g(1) = \frac{(n+1)(m-1)}{2}.$$

Then $|v_g(0) - v_g(1)| \leq 1$, $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Therefore, $K_{1,n} \odot P_m$ is face edge product cordial graph for n is odd.

Case 2 : n is even

Sub case 2.1 : n is even and m = 2.

$g(e_i) = 1,\quad$ for $1 \le i \le \frac{n}{2}$

$g(e_i) = 0,\quad$ for $\frac{n+3}{2} \le i \le n$

$g(e_{jk}) = 1,\quad$ for $1 \le j \le \frac{n+1}{2}$ and $1 \le k \le 2m-1$

$g(e_{jk}) = 0,\quad$ for $\frac{n+3}{2} \le j \le n+1$ and $1 \le k \le 2m-1$

$g(e_{jk}) = 1,\quad$ for $j = n+1$ and $k = 1,3$.

$g(e_{jk}) = 0,\quad$ for $j = n+1$ and $k = 2$.

In view of the above defined labeling pattern we have

$$e_g(1) = e_g(0)+1 = 2n+2,\ v_g(0) = v_g(1)+1 = \frac{3n+4}{2} \text{ and } f_g(0) = f_g(1) + 1 = \frac{n+2}{2}.$$

Then $|v_g(0) - v_g(1)| \le 1$, $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Therefore, $K_{1,n} \odot P_m$ is face edge product cordial graph for n is even and m = 2.

Sub case 2.2 : n is even and m > 2.

In order to satisfy the edge condition for G, it is essential to assign label 1 to atmost (n+1)m edges out of 2(n+1)m–1 edges. Assigning any pattern of edge labels which satisfying the edge condition will induce face labels for (n+1)(m–1) number of faces in such a way that $|f_g(0)-f_g(1)| \ge m-1$, that is face condition for G is violated. Thus the graph G under consideration is not a face edge product cordial graph when n is even and m > 2. Therefore, the graph $K_{1,n} \odot P_m$ is not a face edge product cordial graph for n is even and m > 2.

Hence, the graph $K_{1,n} \odot P_m$ is face edge product cordial graph except n is even and m > 2.

Example : 2.2

The graph $K_{1,5} \odot P_5$ and its face edge product cordial labeling is given in figure 2.2.

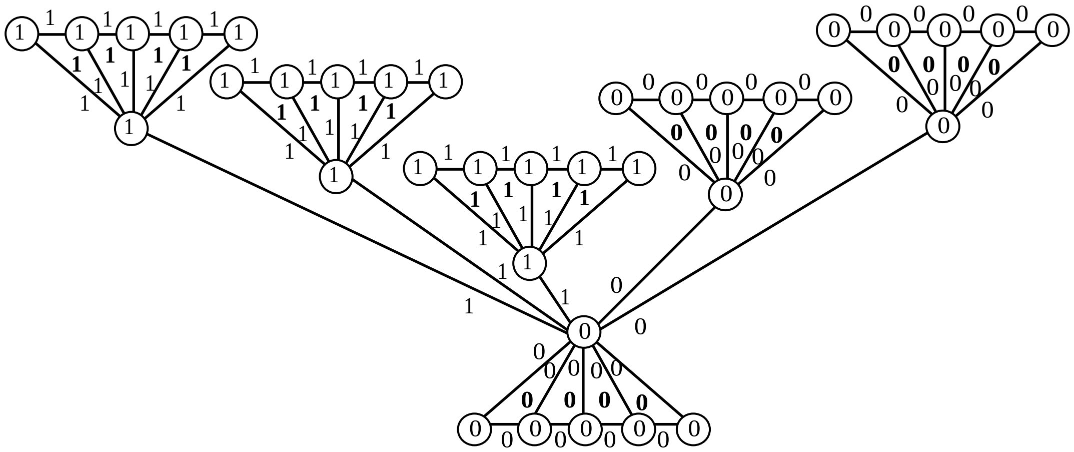

Figure 2.2

Theorem : 2.3

$(P_n \odot K_1) \odot P_m$ is face edge product cordial graph.

Proof.

Let $u_1, u_2, \ldots, u_{2n}$ and $e_1, e_2, \ldots, e_{2n-1}$ be the vertices and edges of the comb graph $P_n \odot K_1$.

Let G be the graph $(P_n \odot K_1) \odot P_m$.

The vertex set $V(G) = \{u_i, v_{ij} : 1 \le i \le 2n, 1 \le j \le m\}$, edge set $E(G) = \{e_i, e_{jk} : 1 \le i \le 2n-1, 1 \le j \le 2n$ and $1 \le k \le 2m-1\}$ and interior face set $F(G) = \{f_i : 1 \le i \le 2n(m-1)\}$, where $e_i = u_i u_{i+1}$ for $1 \le i \le n-1$, $e_{(n-1)+i} = u_i u_{n+i}$ for $1 \le i \le n$, $e_{jk} = u_j v_{jk}$ for $1 \le j \le 2n$ and $1 \le k \le m$, $e_{j(m+k)} = v_{jk} v_{j(k+1)}$ for $1 \le j \le 2n$ and $1 \le k \le m-1$, $f_i = u_i v_{ik} v_{i(k+1)} u_i$ for $1 \le i \le 2n$ and $1 \le k \le m-1$.

Then $|V(G)| = 2n(m+1)$, $|E(G)| = 4nm-1$ and $|F(G)| = 2n(m-1)$.

Define edge labeling $g : E(G) \to \{0,1\}$ as follows

Case 1 : n is odd

$g(e_i) = 1$, for $1 \le i \le \frac{n-1}{2}$

$g(e_i) = 0$, for $\frac{n+1}{2} \le i \le n-1$

$g(e_{n-1+i}) = 1$, for $1 \le i \le \frac{n+1}{2}$

$g(e_{n-1+i}) = 0$, for $\frac{n+3}{2} \le i \le n$

$g(e_{jk}) = 1$, for $1 \le j \le \frac{n-1}{2}$ and $1 \le k \le 2m-1$

$g(e_{jk}) = 0$, for $\frac{n+1}{2} \le j \le n$ and $1 \le k \le 2m-1$

$g(e_{(n+i)k}) = 1$, for $1 \le j \le \frac{n+1}{2}$ and $1 \le k \le 2m-1$

$g(e_{(n+i)k}) = 0$, for $\frac{n+3}{2} \le j \le n$ and $1 \le k \le 2m-1$

In view of the above defined labeling pattern we have

$e_g(1) = e_g(0) + 1 = 2nm$, $v_g(0) = v_g(1) = n(m+1)$ and $f_g(0) = f_g(1) = n(m-1)$.

Then $|v_g(0) - v_g(1)| \le 1$, $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$

Case 2 : n is even

$g(e_i) = 1$, for $1 \le i \le \frac{n}{2}$

$g(e_i) = 0$, for $\frac{n+2}{2} \le i \le n-1$

$g(e_{n-1+i}) = 1$, for $1 \le i \le \frac{n}{2}$

$g(e_{n-1+i}) = 0$, for $\frac{n+2}{2} \le i \le n$

$g(e_{ij}) = 1$, for $1 \le j \le \frac{n}{2}$ and $1 \le k \le 2m-1$

$g(e_{ij}) = 0$, for $\frac{n+2}{2} \le j \le n$ and $1 \le k \le 2m-1$

$g(e_{(n+i)j}) = 1$, for $1 \le j \le \frac{n}{2}$ and $1 \le k \le 2m-1$

$g(e_{(n+i)j}) = 0,$ for $\frac{n+2}{2} \le j \le n$ and $1 \le k \le 2m-1$

In view of the above defined labeling pattern we have

$e_g(1) = e_g(0) + 1 = 2nm$, $v_g(0) = v_g(1) = n(m+1)$ and $f_g(0) = f_g(1) = n(m-1)$.

Then $|v_g(0) - v_g(1)| \le 1$, $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$

Therefore, the graph $(P_n \odot K_1) \odot P_m$ is face edge product cordial graph.

Example : 2.3

The graph $(P_4 \odot K_1) \odot P_5$ and its face edge product cordial labeling is given in figure 2.3.

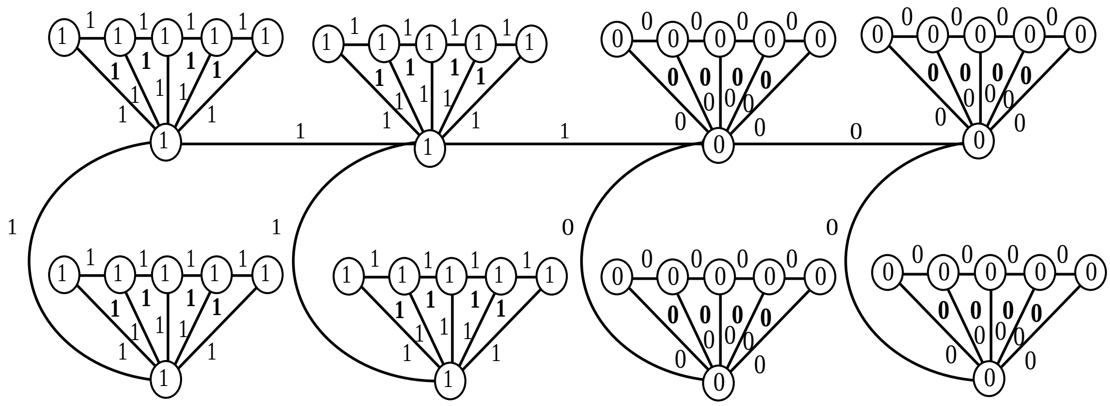

Figure 2.3

Theorem : 2.4

The graph $DS(P_n)$ is not face edge product cordial graph for $n \ge 3$.

Proof.

Let G be the graph $DS(P_n)$

Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n.

Now in order to obtain $DS(P_n)$ from P_n, add the vertices are w_1 and w_2 and add edges are v_1w_1, v_nw_1 and v_iw_2 for $i = 2, 3, \ldots, n-1$.

Then $|V(G)| = n+2$, $E(G) = 2n-1$ and $|F(G)| = n-1$.

To define $g : V(G) \to \{0, 1\}$ as follows

Case 1: n = 3

The graph $DS(P_3) \cong C_4$.

C_4 is not an edge product cordial graph.

Therefore, $DS(P_3)$ is not face edge product cordial graph.

Case 2: $n \ge 4$

In order to satisfy the edge condition for face edge product cordial graph it is essential to assign label 0 to $\left\lfloor \frac{2n-1}{2} \right\rfloor$ edges out of $2n - 1$ edges. The edges with label 0 will give rise all the $n-2$ faces with label 0.

Therefore, $|f_g(0) - f_g(1)| > 2$.

Hence, the graph $DS(P_n)$ is not face edge product cordial graph for $n \ge 3$.

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