Eulerianity of Some Graph Valued Functions

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Abstract *In this paper, we establish a necessary and sufficient condition for line splitting graphs of connected graphs to be eulerian. Also we discuss some properties and eulerianity of total blitact graph, full graph and middle blict graph of a graph.*

Keywords *eulerian, line splitting graph, total blitact graph, full graph, middle blict graph.*

I. INTRODUCTION

All graphs considered here are connected, finite, undirected and without loops or multiple lines. We use the terminology of [3]. A connected graph G is called eulerian if it has a closed path, which contains every line of G exactly once and contains every point of G. Such a path is referred to as an eulerian path. It is well known that a connected graph G is eulerian if and only if each point of G has an even degree. In [1], graphs whose line graphs are eulerian or hamiltonian are investigated and characterizations of these graphs are given. In [2], graphs whose middle graphs are eulerian or hamiltonian are investigated. The aim of this paper is to establish characterizations of graphs whose line splitting graphs are eulerian. Also we discuss some properties and eulerianity of total blitact graph, full graph and middle blict graph of a graph**.** Many other graph valued functions in graph theory were studied, for example, in [7-23].

The following will be useful in the proof of our results.

Theorem A. [3, p. 29] *Every nontrivial connected graph has at least two points which are not cutpoints.*

Theorem B. [3, p. 14] *In any graph, the number of points of odd degree is even.*

Let c be a cutpoint. Then $deg_B c$ represents the number of blocks incident with c.

Theorem C. [5] *The middle blict graph $M_n(G)$ of a graph G is eulerian if and only if G satisfies the following conditions;*

(1) *degree of every point of G is odd*

(2) *for a cutpoint c of G, $deg_B c$ is also odd*

and (3) *if B is a block of G, then the number of blocks adjacent with B and the number of points incident to B are either all even or odd.*

2. Eulerianity of line splitting graph of a graph

The open-neighbourhood $N(e_i)$ of a line e_i in $E(G)$ is the set of lines adjacent to e_i. $N(e_i) = (e_j/e_i$, e_j are adjacent in $G)$. For each line e_i of G, we take a new point e'_i and the resulting set of points is denoted by $E_1(G)$.

Definition 1. The line splitting graph $L_S(G)$ of a graph G is defined as the graph having point set $E(G) \cup E_1(G)$ with two points are adjacent if they correspond to adjacent lines of G or one corresponds to a point e'_i of $E_1(G)$ and the other to a point e_j of $E(G)$ and e_j is $N(e'_i)$. This concept was introduced by Kulli and Biradar [4]. In Figure.1, a graph G, its line splitting graph $L_s(G)$ are shown.

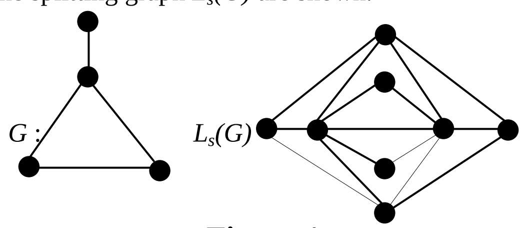

Figure 1.

In the following theorem, we present a characterization of graphs whose line splitting graphs are eulerian.

Theorem 1. *The line splitting graph $L_S(G)$ of a graph G is eulerian if and only if the degrees of all the points of G are of the same parity and G is not K_2.*

Proof. Suppose $L_S(G)$ is eulerian. Without loss of generality, we assume that G is other than K_2. Since if $G = K_2$, then by definition $L_S(G)$ has two isolated points.

Suppose G has a point v_1 with odd degree and a point v_2 with even degree. Since G is connected, v_1 and v_2 are joined by a path on which exist two adjacent points v_3 and v_4 of opposite parity.

If $e = v_3v_4$ and e and e' are the corresponding points of $L_S(G)$, then

$$\deg_{L_s(G)} e = (2\deg_G v_3 - 2) + (2\deg_G v_4 - 2)$$

= even and

$$\deg_{L_s(G)} e' = (\deg_G v_3 - 1) + (\deg_G v_4 - 1)$$

= odd, since v_3 and v_4 are of opposite parity and G is other than K_2. Thus the degree of e' in $L_S(G)$ is odd, a contradiction. Therefore, this proves that the degrees of all points of G are of the same parity.

Conversely, suppose the degrees of all points of G are of the same parity. We consider the following two cases.

Case 1. Assume the degrees of all points of G are odd. If $v_1v_2 = e \in E(G)$ and e and e' are the corresponding points in $L_S(G)$. Then $\deg_{L_s(G)} e = (2\deg_G v_1 - 2) + (2\deg_G v_2 - 2)$ = *even* and also $\deg_{L_s(G)} e' = (\deg_G v_1 - 1) + (\deg_G v_2 - 1)$ = *even*, since v_1 and v_2 are of odd degree and G is other than K_2. Therefore, $\deg_{L_s(G)} e$ and $\deg_{L_s(G)} e'$ are even. Therefore $L_S(G)$ is eulerian.

Case 2. Assume the degrees of all points of G are even. If $v_1v_2 = e \in E(G)$ and e and e' are the corresponding points in $L_S(G)$. Then $\deg_{L_s(G)} e = (2\deg_G v_1 - 2) + (2\deg_G v_2 - 2)$ = *even* and also $\deg_{L_s(G)} e' = (\deg_G v_1 - 1) + (\deg_G v_2 - 1)$ = *even*, since v_1 and v_2 are of even degree and G is other than K_2. Therefore, $\deg_{L_S(G)} e$ and $\deg_{L_S(G)} e'$ are even. Therefore $L_S(G)$ is eulerian in both the cases. This completes the proof of the theorem.

3. Eulerianity of total blitact graph of a graph

The points, lines and blocks of a graph are called its members.

Definition 2. The total blitact graph $T_n(G)$ of a graph G is the graph whose set of points is the union of the set of points, lines and blocks of G and in which two points are adjacent if the corresponding members of G are adjacent or one corresponds to a point and the other to a line incident with it or one corresponds to block B of G and other to a point v of G and v is in B. This concept was introduced by M.S.Biradar and S.S.Hiremath in [14, 15]. In Figure 2, the graph G and its total blitact graph $T_n(G)$ are shown.

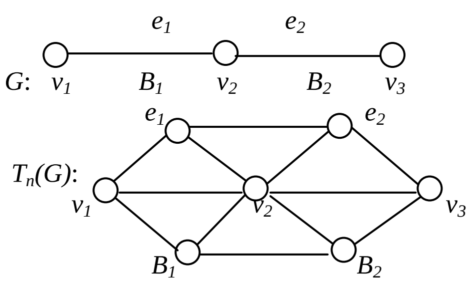

Figure 2.

Theorem 2. *If v is a non cutpoint of a graph G, then $\deg_{T_n(G)} v$ is always odd.*

Proof. Let v be a noncutpoint of G, then v is on some block of G. Suppose w be the point corresponding to v in $T_n(G)$. By definition, $degw=2degv+1$, which is an odd number.

Theorem 3. *For any nontrivial connected graph G, the total blitact graph $T_n(G)$ is not eulerian.*

Proof. Since G is a nontrivial connected graph, by Theorem A, it has at least two points which are not cutpoints. By Theorem 2, the points corresponding to these in $T_n(G)$ are odd degree points. Thus $T_n(G)$ is not eulerian.

4. Eulerianity of full graph of a graph

The points, lines and blocks of a graph are called its members.

Definition 3. The full graph $F(G)$ of a graph G is the graph whose set of points is the union of the set of points, lines and blocks of G in which two points are adjacent if the corresponding members of G are adjacent or incident. In [6], Kulli introduced this concept. In Figure 3, the graph G and its Full graph $F(G)$ are shown.

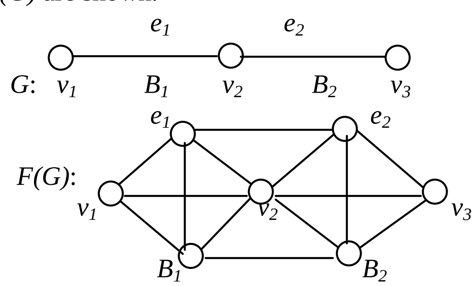

Figure 3.

Remark 4. *The graph $T_n(G)$ of G is a spanning subgraph of $F(G)$.*

Theorem 5. For any nontrivial connected graph G, the full graph $F(G)$ is not eulerian.

Proof. By definitions of $T_n(G)$ and $F(G)$, the degree of a point corresponding to a non cutpoint of G is same in both $T_n(G)$ and $F(G)$. Thus by Remark 4 and by Theorem 3, $F(G)$ also not eulerian.

5. Some results on middle blict graph of a graph

The points, lines and blocks of a graph are called its members.

Definition 4. The middle blict graph $M_n(G)$ of a graph G as the graph whose set of points is the union of the set of blocks, points and lines of G and in which two points are adjacent if and only if the corresponding blocks and lines of G are adjacent or the corresponding members are incident. In [5], Kulli and Biradar introduced this concept. In Figure 4, the graph G and its middle blict graph $M_n(G)$ are shown.

Let $\Delta(G)$ denote the maximum degree of a point in G and let b_i is the number of blocks to which point v_i belongs in G. For a point v_i of G we define the α-degree denoted by α-$deg\, v_i$ as, α-$deg\, v_i = deg\, v_i + b_i$. For a line $x = uv$, deg $x = degu + deg\, v$ and for a block $B= \{v_1, v_2, \ldots, v_n, n \geq 2\}$, $degB = \sum_{i=1}^{n} v_i$.

Let $\Delta_1(G)$, $\Delta_2(G)$ and $\Delta_3(G)$ denote the maximum α-degree of a point, degree of a line and degree of a block in G respectively.

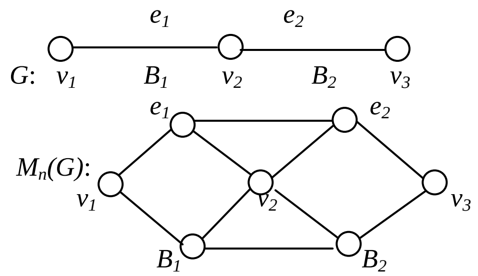

Figure 4

It is easy to prove that.

Theorem 6. *For a graph G,* $\Delta(M_n(G)) = \max\{\Delta_1(G), \Delta_2(G), \Delta_3(G)\}$.

Proof. It follows from the definition of $M_n(G)$.

Theorem 7. *For a graph G,* $\Delta(M_n(G)) \cap \{\Delta_1(G), \Delta_2(G), \Delta_3(G)\} \neq \phi$.

Theorem 8. *For a tree T,* $\Delta(M_n(T)) = 2\Delta(T)$.

Theorem 9. If $M_n(G)$ *is eulerian, then G has even number of points.*

Proof. Suppose $M_n(G)$ is eulerian, then by Theorem *C*, degree of every point of *G* is odd and hence by Theorem *B*, *G* has even number of points.

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