Difference Speed sequence graph

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Abstract: *A (p.q) graph G(V,E) is said to be a difference speed sequence graceful graph if there exists a bijection f: V(G) → { 0, 1, 2, …q² } such that the induced mapping f: E(G) →{$\Delta_i(x)$}/ i=1, 2, 3, …n}defined by f(uv) = |f(u) – f(v) |is a bijection. Here $\Delta_m(x) = (\Delta_m x_k) = |(x_k - x_{k+m})|$ and (x) is the Fibonacci sequence . The function f is called a difference speed sequence graceful graph. In this paper we prove that the star $K_{1,n}$,the path graph, the comb graph, bistar $B_{m,n}$, crown graph $C_3 \odot k_{1,2}$, and various types of graph are difference speed sequence graph.*

Keywords: *graceful labeling, speed sequence labeling, speed sequence graphs*

AMS Subject Classification: 05C78, 94C15

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected graph G(V, E) with 'p 'vertices and 'q' edges. A detailed survey of graph labeling can be found in the dynamic survey of labeling by J.A. Gallian. In this paper we introduce a new labeling called speed sequence labeling. We use the following definitions in the following sections.

Definition: 1.1. A (p.q) graph G(V,E) is said to be a difference speed sequence graceful graph if there exists a bijection f: V(G) → { 0, 1, 2, …q^2 } such that the induced mapping f: E(G) →{$\Delta_i(x)$}/ i=1, 2, 3, …n} defined by f(uv) = |f(u) – f(v) |is a bijection. Here $\Delta_m(x) = (\Delta_m x_k) = |(x_k - x_{k+m})|$ and (x) is the Fibonacci sequence. The function f is called a difference speed sequence graceful graph.

Definition: 1.2. A complete bipartite graph $K_{1,n}$ is called a star and it has n + 1 vertices and n edges.

Definition: 1.3. The bistar graph $B_{m,n}$ is the graph obtained from a copy of star $K_{1,m}$ and a copy of start $K_{1,n}$ by joining the vertices of maximum degree by an edge.

Definition:1.4. A star S_n is the complete bipartite graph $K_{1,n}$ is a tree with one internal node and n leaves.

Definition: 1.5. The graph obtained by joining a pendent edge at each vertex of a path P_n is called a comb and is denoted by $Pn \odot k_1$ or P_n^+

Definition: 1.6. The corona $G_1 \odot G_2$ is defined as the graph obtained by taking one copy of G_1 to every point in the i^{th} copy of G_2.

Definition: 1.7. The graph (P_m, S_n) is obtained from m copies of the star graph S_n and the path Pm: {u_1, u_2, …u_m } by joining u_i with the centre of the j^{th} copy of S_n by means of an edge $1 \le j \le m$.

Definition: 1.8. A subdivision of a graph G is a graph that can be obtained from G by a sequence of edge subdivisions.

Definition: 1.9. An (n,t) kite graph consists of a cycle of length n with a t edge path attached to one vertex.

2. Main Results:

In this paper, we prove that path Pn, $K_{1,n}$, bistar $B_{m,n}$, the graph by subdividing the edges of the star $K_{1,n}$, the generalized crown $C_3 \odot K_{1,n}$, the comb $Pn \odot k_1$, the graph (P_m, S_n), (3,t) kite graph are the difference speed sequence graphs.

Theorem:2.1

The Path P_n is a difference speed sequence graceful for all $n \geq 2$.

Proof:

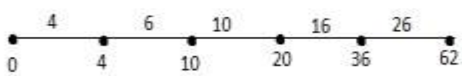
Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n.

Define f : V(G) → { 0, 1, 2, ….q^2 } and E(G) →{$\Delta_i(x)$/ i=1, 2, 3, …n} and (x) is the fibonacci sequence. Then f induces a bijection f : E(G) →{$\Delta_i(x)$/ i=1, 2, 3, …n}

Example:

$\Delta_2(x)$

Figure 1



$\Delta_3(x)$

Figure 2

Theorem: 2.2

Every comb graph Pn ⊙ k_1 is a difference speed sequence graceful graph.

Proof:

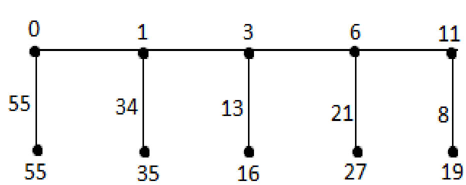
Let u_1, u_2, …, u_n be the vertices of the path P_n.

Let v_1, v_2, …, v_n be the vertices adjacent to each vertex of P_n.

Define f : V(Pn ⊙ k_1) →{$\Delta_i(x)$/ i=1, 2, 3, …n}

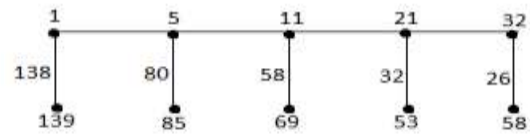
Then f induces a bijection f : E(Pn ⊙ k_1) →{$\Delta_i(x)$/ i=1, 2, 3, …n}

Example:



$\Delta_2(x)$

Figure 3



$\Delta_3(x)$

Figure 4

Theorem :2.3

The star $K_{1,n}$ is a difference speed sequence graph.

Proof:

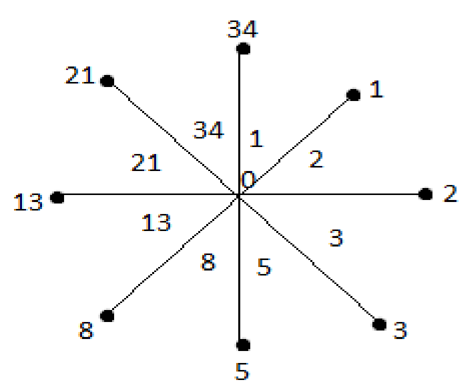
Let V ($K_{1,n}$) = { u_i / $1 \leq i \leq n + 1$ }

Let E ($K_{1,n}$) = {$\Delta_i(x)$/ i=1, 2, 3, …n}

Define an injection f : V ($K_{1,n}$) → { 0, 1, 2, ….$2q^2$ } by f(u_i) = $\Delta_i(x)$ if / $1 \leq i \leq n$

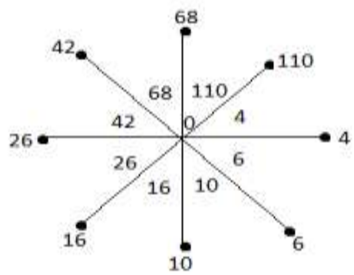
Then f induces a bijection f: E ($K_{1,n}$) → {$\Delta_i(x)$/ i=1, 2, 3, …n}

Example:



$\Delta_2(x)$

Figure 5



$\Delta_3(x)$

Figure 6

Theorem: 2.4

The graph obtained by the subdivision of the edges of the star $K_{1,n}$ is a difference speed sequence graph.

Proof:

Let G be the graph obtained by the subdivision of the edges of the star $K_{1,n}$.

Let $V(G) = \{v, u_i, w_i / 1 \leq i \leq n\}$ and $E(G) = \{vu_i, u_iw_i / 1 \leq i \leq n\}$

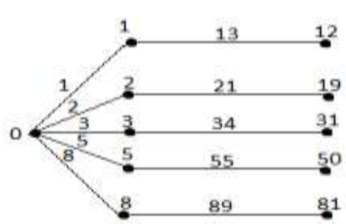
Define an injection $f : V(G) \rightarrow \{0, 1, 2, \ldots 3q^2\}$ and $E(G) \rightarrow \{\Delta_i(x) / i=1, 2, 3, \ldots n\}$

and $f(v) = 0$.

Then f induces a bijection $f: E(G) \rightarrow \{\Delta_i(x) / i=1, 2, 3, \ldots n\}$

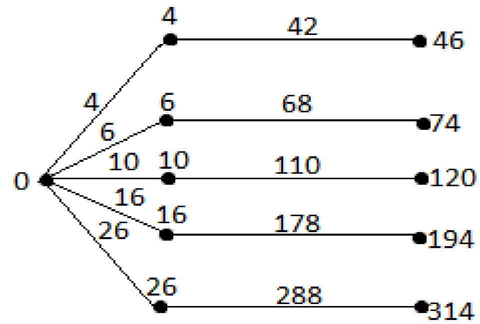
Therefore the subdivision of the edges of the star $K_{1,n}$ is a difference speed sequence graph.

Example:



$\Delta_2(x)$

Figure 7



$\Delta_3(x)$

Figure 8

Theorem:2.5

Every bistar $B_{m,n}$ is a difference speed sequence graceful.

Proof:

Let $V(B_{m,n}) = \{0,1,2,\ldots 2(m+n+1)^2\}$ and $E(B_{m,n}) \rightarrow \{\Delta_i(x) / i=1, 2, 3, \ldots n\}$

Then f induces a bijection $f: E(B_{m,n}) \rightarrow \{\Delta_i(x) / i=1, 2, 3, \ldots n\}$

Case(i) : m > n

Define an injection $f: V(B_{m,n}) \rightarrow \{0,1,2,\ldots 2(m+n+1)^2\}$ by

$f(u_i) = |m+n-6i|$ for i=1

$f(u_i) = |m+n-(2i+1)|$ for i=2

$f(u_i) = |m+n-5i|$ for i=3

$f(u_i) = |m+n-4i|$ for i=4

and $f(u) = 0$; $f(v) = m+n+1$

$f(v_i) = |m+n-4i(m+n)|$ for i=1

$f(v_i) = |m+n-(5i/2)(m+n) + 1|$ for i=2

$f(v_i) = |m+n-mn|$ for i=3

Case(ii) : m < n

Define an injection f: V $(B_{m,n}) \rightarrow \{0,1,2,\ldots 2(m+n+1)^2\}$by

$f(u_i) = |m+n-4i(m+n)|$ for i=1

$f(u_i) = |m+n-(5i/2)(m+n) + 1|$ for i=2

$f(u_i) = |m+n- mn|$ for i=3

and f(u) = m+n+1 ; f(v) = 0

$f(v_i) = |m+n-6i|$ for i=1

$f(v_i) = |m+n-(2i+1)|$ for i=2

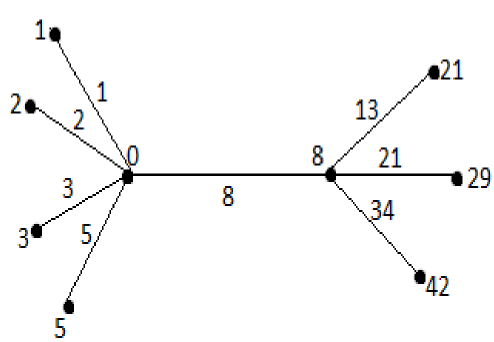
$f(v_i) = |m+n- 5i|$ for i=3

$f(v_i) = |m+n-4i|$ for i=4

Then f induces a bijection f: $E(G) \rightarrow \{\Delta_i(x)/ i=1, 2, 3, \ldots n\}$

Therefore the bistar $B_{m,n}$ is a difference speed sequence graph.

Example:



$\Delta_2(x)$

Figure 9

Theorem:2.6

The crown graph $C_3 \odot k_{1,2}$ is a difference speed sequence graceful

Proof:

Let $\{v_1, v_2, v_3, u_1, u_2, u_3, u_4, u_5, u_6\}$ be the vertices of $C_3 \odot k_{1,2}$

Here $\{v_1, v_2, v_3\}$ are the vertices of the cycle C_3 and $\{u_1, u_2, u_3, u_4, u_5, u_6\}$are the vertices of the copies of $k_{1,2}$ adjacent to v_i for i=1,2,3

The size of the graph is q = 3n+3

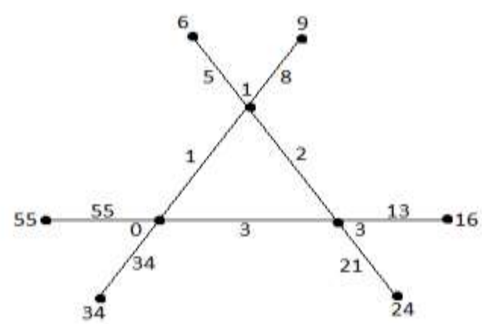
Define an injection f: V $(C_3 \odot k_{1,2}) \rightarrow \{0,1,2,\ldots(3n+3)^2\}$ by $f(v_1) = 0$, $f(v_2) = 1$, $f(v_3) = 3$,

and $E(G) \rightarrow \{\Delta_i(x)/ i=1, 2, 3, \ldots n\}$

Then f induces a bijection f: $E(G) \rightarrow \{\Delta_i(x)/ i=1, 2, 3, \ldots n\}$

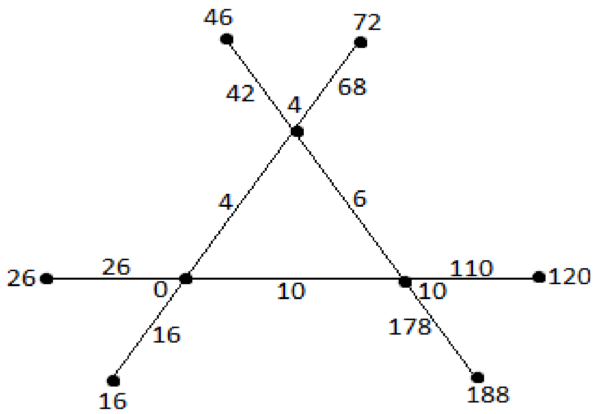
Therefore the crown graph $C_3 \odot k_{1,2}$ is a difference speed sequence graph.

Example:



$\Delta_2(x)$

Figure 10



$\Delta_3(x)$

Figure 11

Theorem:2.7

The kite graph (3,t) is difference speed graceful for t ≥ 1

Proof:

Let $\{v_1, v_2, v_3\}$be the vertices of a cycle C_3 and $\{u_1, u_2, \ldots, u_t\}$be the t vertices of the tail with v_1 adjacent to u_1.

The size of G is q = 3 + t

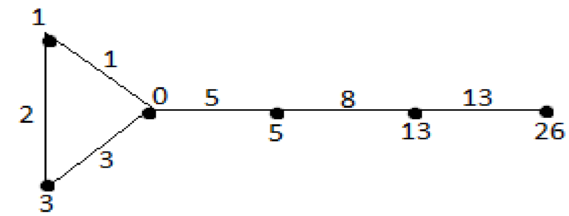
Define a bijection f: V(G) $\rightarrow$ $\{0,1,2,\ldots(3+2t)^2\}$ for all t = 1…n and

E(G) $\rightarrow \{\Delta_i(x)/\ i=1, 2, 3, \ldots n\}$

Therefore f induces a bijection.

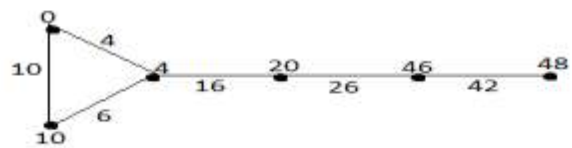
Hence (3,t) is a kite difference speed sequence graceful graph.

Example:



$\Delta_2(x)$

Figure 12



$\Delta_3(x)$

Figure 13

Theorem: 2.8

The graph $P_m \odot nk_1$ $(n \geq 2)$ is a difference speed sequence graceful graph.

Proof:

Let $\{u_1, u_2, \ldots, u_m\}$be the vertices of the path P_m and $\{v_{1j}, v_{2j}, \ldots, v_{nj}\}$be the i^{th} copy of the null graph nK_1.

Then $\{v_{1j}, v_{2j}, \ldots, v_{nj}\}$are the n pendent vertices adjacent to the vertex u_j of P_m for $1 \leq j \leq m$.

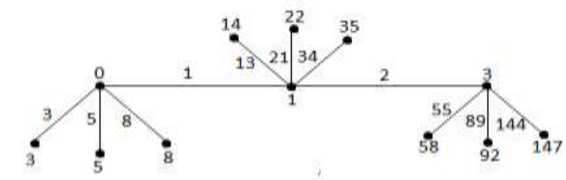
Define an injection f: $V(P_m \odot nk_1) \rightarrow \{0,1,2,\ldots 3(mn+m-1)^2\}$ and

E(G) $\rightarrow \{\Delta_i(x)/\ i=1, 2, 3, \ldots n\}$

Then f induces a bijection f: E $(P_m \odot nk_1) \rightarrow \{\Delta_i(x)/\ i=1, 2, 3, \ldots n\}$

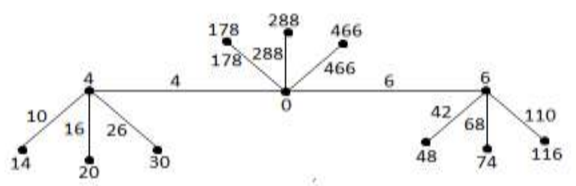
Therefore $P_m \odot nk_1$ is a difference speed sequence graceful graph.

Example:



$\Delta_2(x)$

Figure 14



$\Delta_3(x)$

Figure 15

Conclusion:

Here we have introduced a new labeling called difference speed sequence labeling. We have also proved it for various types of graphs.

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