# Estimation of variance of time to recruitment for a two grade manpower system with two types of decisions when the wastages form a geometric process

Parameswari[#1], Srinivasan [#2]

[#1] *Assistant Professor, Department of Mathematics, St.Joseph's College of Engineering and Technology, Ellupatti, Thanjavur, India*

[#2] Professor Emeritus, PG & Research Department of Mathematics, Bishop Heber College, Trichy, India

***Abstract —*** *In this paper a two graded organization is considered in which depletion of manpower occurs due to its policy decisions. Assuming that the wastages (loss in man-hours) form a geometric process and the policy decisions are classified into two types interms of their intensity of attrition, three mathematical models are constructed according to different forms of the mandatory threshold for wastages in the organization. Explicit analytical expression for the system characteristics namely mean and variance of time to recruitment are obtained for all the models using an univariate CUM policy of recruitment based on shock model approach, when the inter-policy decision times form (i) a geometric process and (ii) an order statistics. The influence of the nodal parameters on these system characteristics are studied and relevant conclusions are presented.*

***Keywords —*** *Two types of policy decisions, Geometric Process, Order statistics, Univariate CUM policy of recruitment, Variance of time to recruitment.*

## I. Introduction

Exit of personnel is a common phenomenon in any marketing organization. In [1] and [3] several stochastic models for a manpower system with grades are discussed using Markovian and renewal theoretic approach. In [5] the authors have initiated the study on problem of time to recruitment for a single grade manpower system and obtained the variance of time to recruitment when the inter-decision times and the wastages form a sequence of indepenent and identically distributed exponential random variables, using shock model approach[2]. In [4] the authors have obtain the variance of time to recruitment for a two grade manpower system when the wastages are correlated and the inter-decision times which are classified into two types according to their intensity (high or low) of attrition, form a geometric process or an order statistics. The present paper studies the work in [4] when the wastages form a geometric process. In the present paper, three mathematical models are constructed which differ from each other in the context of permitting or not permitting transfer of personnel between two grades and providing a better allowable loss of manpower in the organization. More specifically, in Model-1, the breakdown threshold is minimum of the thresholds for the loss of manpower in the two grades. In Model-2, the breakdown threshold is the maximum of the thresholds for the grades. In Model-3, the breakdown threshold is the sum of the thresholds for the grades. This paper is organized as follows: In sections II, III and IV Models 1, 2 and 3 are respectively described and analytical expressions for mean and variance of time to recruitment are derived. The analytical results are numerically illustrated by assuming specific distributions and the influence of nodal parameters on the system characteristics are reported.

## II. Model description and analysis for Model - 1

Consider an organization having two grades in which decisions are taken at random epochs in $[0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated wastage in manpower (measured as loss in man-hours) to the organization, if a person quits and it is linear and cumulative. Let $X_i$ be the wastage due to the $i^{th}$ decision epoch, i=1, 2,3… … It is assumed that the wastages $X_i$ , i=1,2,3.. form a geometric process with rate 'b'(b >0) which are assumed to be an independent random variables. The distribution function G(x) of $X_1$ is assumed as $G(x)=1-e^{-cx}$ and g(.) be its probability density function. Let $S_k$ be the cumulative wastages in the organization in the first 'k' decisions with probability density function $g_k(.)$. Let $U_i, i=1,2,3...$ be the time between i-1$^{th}$ and $i^{th}$ decisions. The best distribution when the inter-decision times having high or low intensity of attrition is the hyper exponential distribution. Let $U_i, i=1,2,3...k$ are independent and identically distributed hyper exponential random variables with distribution (density) function F(.)(f(.)), and high(low) attrition rate $\lambda_h(\lambda_l)$ and p(q) be the proportion of decisions having high (low) attrition rate. Let $F_k(t)$ $(f_k(t))$ be the distribution(probability density) function of $\sum_{i=1}^{k} U_i$.

The time to recruitment is denoted by T and its cumulative distribution function, probability density function, mean and variance are denoted by $L(.), \ell(.), E[T]$ *and* $V[T]$ respectively. Let $\sigma^*(.)$ be the Laplace transform of $\sigma(.)$. Let Y be the breakdown threshold for the cumulative wastage in the organization. Let $Y_A(Y_B)$ be the threshold level for the cumulative wastage in grade A(B) is a exponential random variable with mean $1/\alpha_A$ $(1/\alpha_B)$. The loss of man-hours process and the inter-decision time process are statistically independent. The univariate policy of recruitment is ***Recruitment is done as and when the total loss of man-hours in the organization exceeds the breakdown threshold Y.*** Let $V_k(t)$ be the probability that there are exactly k-decision epochs in (0,t]. Since the number of decisions made in (0,t] form a renewal process, we note that $V_k(t)=F_k(t)-F_{k+1}(t)$, where $F_0(t)=1$.

**Main Results**

By definition, $S_{N(t)}$ is the total loss of man-hours in the N(t) decisions taken in (0,t]. Therefore

$$P(T>t)=P(S_{N(t)}<Y). \tag{1}$$

Since Y= min $(Y_A,Y_B)$ and by using laws of probability and on simplification we get

$$P(T>t)=\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\ g_k^*(\alpha_A+\alpha_B). \tag{2}$$

Since $\{X_i\}$ is a geometric process it is known that

$$g_k^*(\alpha_A+\alpha_B)=\prod_{i=1}^{k} f^*\left(\frac{\alpha_A+\alpha_B}{i^{n-1}}\right) \tag{3}$$

From (2) and (3) we get

$$P(T>t)=\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\left[V_{(\alpha_A+\alpha_B,k)}\right] \tag{4}$$

where

$$V_{(\tau,k)}=\prod_{i=1}^{k}\frac{c\,b^{i-1}}{\left(c\,b^{i-1}+\tau\right)}. \tag{5}$$

Since $L(t)=1-P(T>t)$

From (4) and by taking Laplace transforms on both sidesweget

$$\ell^*(s)=-\sum_{k=0}^{\infty}\left[f_k^*(s)-f_{k+1}^*(s)\right]\left[V_{(\alpha_A+\alpha_B,k)}\right]. \tag{6}$$

It is known that

$$E[T]=-\left.\frac{d(\ell^*(s))}{ds}\right|_{s=0},\ E[T^2]=\left.\frac{d^2(\ell^*(s))}{ds^2}\right|_{s=0}\ \text{and}\ V[T]=E[T^2]-(E[T])^2. \tag{7}$$

**Case(i)**

Assume that the inter-decision times $U_i$, i=1,2,3..form a geometric process with parameter'a'.

Since $\{U_i\}$ is a geometric process it is known that

$$f_k^*(s)=\prod_{n=1}^{k} f^*\left(\frac{s}{a^{n-1}}\right). \tag{8}$$

From (6), (7) and (8) we get

$$E[T]=E[U]\sum_{k=0}^{\infty}\left(\frac{1}{a}\right)^k V_{(\alpha_A+\alpha_B,k)} \tag{9}$$

and

$$E[T^2]=V[U]\sum_{k=0}^{\infty}\left(\frac{1}{a^2}\right)^k V_{(\alpha_A+\alpha_B,k)}-(E[U])^2$$
$$\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)V_{(\alpha_A+\alpha_B,k)} \tag{10}$$

In (9) and (10), by hypothesis , $U=U_i$ ,i=1,2,3…

$$\text{where}\ E[U]=\frac{p\lambda_l+q\lambda_h}{\lambda_h\lambda_l}\ \text{and}\ E[U^2]=2\left(\frac{p\lambda_l^2+q\lambda_h^2}{\lambda_h^2\lambda_l^2}\right). \tag{11}$$

**Case(ii)**

If $U_{(1)},U_{(2)},...U_{(m)}$ be the order statistics selected from the sample $U_1,U_2,...U_m$ with respective density function $f_{u(1)}, f_{u(2)},...f_{u(m)}$.

From (6) and (7) we get

$$E[T]=E[U]\sum_{k=0}^{\infty}V_{(\alpha_A+\alpha_B,k)} \tag{12}$$

and

$$E[T^2]=\sum_{k=0}^{\infty}\left(2k(E[U])^2+E[U^2]\right)V_{(\alpha_A+\alpha_B,k)} \tag{13}$$

Using the theory of order statistics it can be shown that

$$E[U]=\begin{cases}\displaystyle\sum_{r_1=0}^{m}\frac{mc_{r_1}p^{r_1}q^{m-r_1}}{(\lambda_h-\lambda_l)r_1+\lambda_l m}, & \text{if}\ f(t)=f_{u(1)}(t)\\ \displaystyle\sum_{r_1=0}^{m}\sum_{r_2=0}^{m-r_1}\frac{(-1)^{m-r_1}mc_{r_1}(m-r_1)c_{r2}p^{r2}q^{m-r_1-r2}}{(\lambda_h+\lambda_l)r_2+\lambda_l r_1-m\lambda_l}, & \text{if}\ f(t)=f_{u(m)}(t)\end{cases} \tag{14}$$

and

$$E[U^2]=\begin{cases}\displaystyle 2\sum_{r_1=0}^{m}\frac{mc_{r_1}p^{r_1}q^{mk-r_1}}{((\lambda_h-\lambda_l)r_1+\lambda_l m)^2}, & \text{if}\ f(t)=f_{u(1)}(t)\\ \displaystyle 2\sum_{r_1=0}^{k}\sum_{r_2=0}^{k-r_1}\frac{(-1)^{m-r_1}mc_{r_1}(m-r_1)c_{r2}p^{r2}q^{m-r_1-r2}}{((\lambda_h+\lambda_l)r_2+\lambda_l r_1-m\lambda_l)^2}, & \text{if}\ f(t)=f_{u(m)}(t)\end{cases} \tag{15}$$

## III. MODEL DESCRIPTION AND ANALYSIS FOR MODEL – 2

For this model $Y=\max(Y_A,Y_B)$. All the other assumptions and notations are as in Model- 1. In this model it can be shown that

$$P(S_k > Y) = V_{(\alpha_A,k)} + V_{(\alpha_B,k)} - V_{(\alpha_B+\alpha_B,k)}. \quad (16)$$

Proceeding as in Model- 1 we get

$$\ell^*(s) = -\sum_{k=0}^{\infty}\left[f_k^*(s) - f_{k+1}^*(s)\right]\left(V_{(\alpha_B,k)} + V_{(\alpha_A,k)} - V_{(\alpha_B+\alpha_B,k)}\right). \quad (17)$$

**Case(i) :** $\{U_i\}$ form a geometric process

Proceeding as in Model- 1 it is found that

$$E[T] = E[U]\sum_{k=0}^{\infty}\left(\frac{1}{a}\right)^k\left[V_{(\alpha_B,k)} + V_{(\alpha_A,k)} - V_{(\alpha_B+\alpha_B,k)}\right] \quad (18)$$

and

$$E[T^2] = V[U]\sum_{k=0}^{\infty}\left(\frac{1}{a^2}\right)^k\left[V_{(\alpha_B,k)} + V_{(\alpha_A,k)} - V_{(\alpha_B+\alpha_B,k)}\right] - (E[U])^2$$
$$\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2 - \left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\left[V_{(\alpha_B,k)} + V_{(\alpha_A,k)} - V_{(\alpha_B+\alpha_B,k)}\right] \quad (19)$$

where E[U] and V[U] are given by (11).

**Case (ii) :** $\{U_i\}$ form an order statistics.

Proceeding as in Model-1, we get

$$E[T] = E[U]\sum_{k=0}^{\infty}\left[V_{(\alpha_B,k)} + V_{(\alpha_A,k)} - V_{(\alpha_B+\alpha_B,k)}\right] \quad (20)$$

and

$$E[T^2] = \sum_{k=0}^{\infty}\left(2k(E[U])^2 + E[U^2]\right)\left[V_{(\alpha_B,k)} + V_{(\alpha_A,k)} - V_{(\alpha_B+\alpha_B,k)}\right] \quad (21)$$

where E[U] and E[U$^2$] are given by (14) and (15).

## IV. MODEL DESCRIPTION AND ANALYSIS FOR MODEL – 3

For this model $Y = Y_A + Y_B$ . All the other assumptions and notations are as in Model-1.

Proceeding as in model-I it can be shown that

$$\ell^*(s) = \left(\frac{\alpha_B}{\alpha_B - \alpha_A}\right)\sum_{k=0}^{\infty}\left[f_k^*(s) - f_{k+1}^*(s)\right]V_{(\alpha_A,k)} -$$
$$\left(\frac{\alpha_A}{\alpha_B - \alpha_A}\right)\sum_{k=0}^{\infty}\left[f_k^*(s) - f_{k+1}^*(s)\right]V_{(\alpha_B,k)}. \quad (22)$$

**Case(i) :** $\{U_i\}$ form a geometric process

Proceeding as in Model-I it is found that

$$E[T] = E[U]\sum_{k=0}^{\infty}\left(\frac{1}{a}\right)^k\left[\left(\frac{\alpha_B}{\alpha_B-\alpha_A}\right)V_{(\alpha_A,k)} - \left(\frac{\alpha_A}{\alpha_B-\alpha_A}\right)V_{(\alpha_B,k)}\right] \quad (23)$$

and

$$E[T^2] = V[U]\sum_{k=0}^{\infty}\left(\frac{1}{a^2}\right)^k\left[\left(\frac{\alpha_B}{\alpha_B-\alpha_A}\right)V_{(\alpha_A,k)} - \left(\frac{\alpha_A}{\alpha_B-\alpha_A}\right)V_{(\alpha_B,k)}\right] - (E[U])^2$$
$$\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2 - \left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\left[\left(\frac{\alpha_B}{\alpha_B-\alpha_A}\right)V_{(\alpha_A,k)} - \left(\frac{\alpha_A}{\alpha_B-\alpha_A}\right)V_{(\alpha_B,k)}\right] \quad (24)$$

where E[U] and V[U] are given by (11).

**Case (ii) :** $\{U_i\}$ form an order statistics.

Proceeding as in Model-1, we get

$$E[T] = E[U]\sum_{k=0}^{\infty}\left[\left(\frac{\alpha_B}{\alpha_B-\alpha_A}\right)V_{(\alpha_A,k)} - \left(\frac{\alpha_A}{\alpha_B-\alpha_A}\right)V_{(\alpha_B,k)}\right] \quad (25)$$

and

$$E[T^2] = \sum_{k=0}^{\infty}\left(2k(E[U])^2 + E[U^2]\right)\left[\left(\frac{\alpha_B}{\alpha_B-\alpha_A}\right)V_{(\alpha_A,k)} - \left(\frac{\alpha_A}{\alpha_B-\alpha_A}\right)V_{(\alpha_B,k)}\right] \quad (26)$$

where E[U] and E[U$^2$] are given by (14) and (15).

## Note :

Let $Y_{A1}$ be the normal exponential threshold for the cumulative wastage in grade A and $1/\alpha_{A1}$ ($\alpha_{A1}>0$) be its mean. Let $Y_{A2}$ be the exponential threshold for frequent breaks taken by the existing workers in grade A and $1/\alpha_{A2}$ ($\alpha_{A2}>0$) be its mean. Similarly, let $Y_{B1}$ and $Y_{B2}$ be the normal exponential threshold for wastages and exponential threshold for frequent breaks of existing workers in grade B with means $1/\alpha_{B1}, 1/\alpha_{B2}$($\alpha_{B1}$ , $\alpha_{B2}$ >0) respectively. Let $H_A$, and $H_B$ be the cumulative distribution function of $Y_A$, and $Y_B$ respectively. It is assumed that $Y_{A1}$, $Y_{A2}$, $Y_{B1}$ and $Y_{B2}$ are statistically independent. Here the thresholds $Y_A$ and $Y_B$ for the grades A and B respectively are taken as $Y_A = Y_{A1} + Y_{A2}$ and $Y_B = Y_{B1} + Y_{B2}$.

**Model- 1**

Since $Y = \min(Y_A, Y_B)$ . From the above assumptions it is shown that

$$\ell^*(s) = -\sum_{k=0}^{\infty}\left[f_k^*(s) - f_{k+1}^*(s)\right]\begin{pmatrix}\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)} + \gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)} - \\ \gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)} - \gamma_4 V_{(\alpha_{A2}+\alpha_{B1},k)}\end{pmatrix}. \quad (27)$$

where

$$\gamma_1 = \frac{\alpha_{A2}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}, \gamma_2 = \frac{\alpha_{A1}\alpha_{B1}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})},$$
$$\gamma_3 = \frac{\alpha_{A2}\alpha_{B1}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})} \text{ and } \gamma_4 = \frac{\alpha_{A1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}. \quad (28)$$

**Case(i) :** $\{U_i\}$ form a geometric process

In this case it is shown that

$$E[T]=E[U]\sum_{k=0}^{\infty}\left(\frac{1}{a}\right)^{k}\begin{bmatrix}\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)}+\gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)}-\\ \gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)}-\gamma_4 V_{(\alpha_{A2}+\alpha_{B1},k)}\end{bmatrix} \qquad (29)$$

and

$$E[T^2]=V[U]\sum_{k=0}^{\infty}\left(\frac{1}{a^2}\right)^{k}\begin{bmatrix}\gamma_1 W_{(\alpha_{A1}+\alpha_{B1},k)}+\gamma_2 W_{(\alpha_{A2}+\alpha_{B2},k)}-\\ \gamma_3 W_{(\alpha_{A1}+\alpha_{B2},k)}-\gamma_4 W_{(\alpha_{A2}+\alpha_{B1},k)}\end{bmatrix}-\left(E[U]\right)^2$$

$$\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\begin{bmatrix}\gamma_1 W_{(\alpha_{A1}+\alpha_{B1},k)}+\gamma_2 W_{(\alpha_{A2}+\alpha_{B2},k)}-\\ \gamma_3 W_{(\alpha_{A1}+\alpha_{B2},k)}-\gamma_4 W_{(\alpha_{A2}+\alpha_{B1},k)}\end{bmatrix} \qquad (30)$$

where E[U] and V[U] are given by (11).

**Case (ii) :** $\{U_i\}$ form an order statistics.

In this case it is shown that

$$E[T]=E[U]\sum_{k=0}^{\infty}\begin{bmatrix}\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)}+\gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)}-\\ \gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)}-\gamma_4 V_{(\alpha_{A2}+\alpha_{B},k)}\end{bmatrix} \qquad (31)$$

and

$$E[T^2]=\sum_{k=0}^{\infty}\left(2k\left(E[U]\right)^2+E[U^2]\right)\begin{bmatrix}\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)}+\gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)}-\\ \gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)}-\gamma_4 V_{(\alpha_{A2}+\alpha_{B1},k)}\end{bmatrix} \qquad (32)$$

where E[U] and E[U$^2$] are given by (14) and (15).

**Model- 2**

Since $Y=\max\,(Y_A,Y_B)$ . From the above assumptions it is shown that

$$\ell^*(s)=-\sum_{k=0}^{\infty}\left[f_k^*(s)-f_{k+1}^*(s)\right]\begin{bmatrix}\gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)}+\gamma_4 V_{(\alpha_{A2}+\alpha_{B1},k)}+\\ \gamma_5 V_{(\alpha_{A1},k)}+\gamma_6 V_{(\alpha_{B1},k)}-\gamma_7 V_{(\alpha_{A2},k)}\\ -\gamma_8 V_{(\alpha_{B2},k)}-\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)}-\\ \gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)}\end{bmatrix}. \qquad (33)$$

where

$$\gamma_5=\frac{\alpha_{A2}}{(\alpha_{A2}-\alpha_{A1})},\gamma_6=\frac{\alpha_{A1}}{(\alpha_{A2}-\alpha_{A1})},\gamma_7=\frac{\alpha_{B2}}{(\alpha_{B2}-\alpha_{B1})}\ and$$

$$\gamma_8=\frac{\alpha_{B1}}{(\alpha_{B2}-\alpha_{B1})}\ . \qquad (34)$$

**Case(i) :** $\{U_i\}$ form a geometric process

In this case it is shown that

$$E[T]=E[U]\sum_{k=0}^{\infty}\left(\frac{1}{a}\right)^{k}\begin{bmatrix}\gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)}+\gamma_4 V_{(\alpha_{A2}+\alpha_{B1},k)}+\\ \gamma_5 V_{(\alpha_{A1},k)}+\gamma_6 V_{(\alpha_{B1},k)}-\gamma_7 V_{(\alpha_{A2},k)}\\ -\gamma_8 V_{(\alpha_{B2},k)}-\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)}-\\ \gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)}\end{bmatrix} \qquad (35)$$

and

$$E[T^2]=V[U]\sum_{k=0}^{\infty}\left(\frac{1}{a^2}\right)^{k}\begin{bmatrix}\gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)}+\gamma_4 V_{(\alpha_{A2}+\alpha_{B1},k)}+\\ \gamma_5 V_{(\alpha_{A1},k)}+\gamma_6 V_{(\alpha_{B1},k)}-\gamma_7 V_{(\alpha_{A2},k)}\\ -\gamma_8 V_{(\alpha_{B2},k)}-\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)}-\\ \gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)}\end{bmatrix}-\left(E[U]\right)^2$$

$$\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\begin{bmatrix}\gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)}+\gamma_4 V_{(\alpha_{A2}+\alpha_{B1},k)}+\\ \gamma_5 V_{(\alpha_{A1},k)}+\gamma_6 V_{(\alpha_{B1},k)}-\gamma_7 V_{(\alpha_{A2},k)}\\ -\gamma_8 V_{(\alpha_{B2},k)}-\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)}-\\ \gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)}\end{bmatrix} \qquad (36)$$

where E[U] and V[U] are given by (11).

**Case (ii) :** $\{U_i\}$ form an order statistics.

In this case it is shown that

$$E[T]=E[U]\sum_{k=0}^{\infty}\begin{bmatrix}\gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)}+\gamma_4 V_{(\alpha_{A2}+\alpha_{B1},k)}+\\ \gamma_5 V_{(\alpha_{A1},k)}+\gamma_6 V_{(\alpha_{B1},k)}-\gamma_7 V_{(\alpha_{A2},k)}\\ -\gamma_8 V_{(\alpha_{B2},k)}-\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)}-\\ \gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)}\end{bmatrix} \qquad (37)$$

and

$$E[T^2]=\sum_{k=0}^{\infty}\left(2k\left(E[U]\right)^2+E[U^2]\right)\begin{bmatrix}\gamma_3 V_{(\alpha_{A1}+\alpha_{B2},k)}+\gamma_4 V_{(\alpha_{A2}+\alpha_{B1},k)}+\\ \gamma_5 V_{(\alpha_{A1},k)}+\gamma_6 V_{(\alpha_{B1},k)}-\gamma_7 V_{(\alpha_{A2},k)}\\ -\gamma_8 V_{(\alpha_{B2},k)}-\gamma_1 V_{(\alpha_{A1}+\alpha_{B1},k)}-\\ \gamma_2 V_{(\alpha_{A2}+\alpha_{B2},k)}\end{bmatrix} \qquad (38)$$

where E[U] and E[U$^2$] are given by (14) and (15).

**Model- 3**

Since $Y=Y_A+Y_B$ . From the above assumptions it is shown that

$$\ell^*(s)=-\sum_{k=0}^{\infty}\left[f_k^*(s)-f_{k+1}^*(s)\right]\begin{bmatrix}\gamma_9 V_{(\alpha_{A1},k)}+\gamma_{10} V_{(\alpha_{B1},k)}-\\ \gamma_{11} V_{(\alpha_{A2},k)}-\gamma_{12} V_{(\alpha_{B2},k)}\end{bmatrix}. \qquad (39)$$

where

$$\gamma_9=\frac{\alpha_{A2}\alpha_{B1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B1}-\alpha_{A1})(\alpha_{B2}-\alpha_{A1})},\gamma_{10}=\frac{\alpha_{A1}\alpha_{A2}\alpha_{B2}}{(\alpha_{B2}-\alpha_{B1})(\alpha_{B1}-\alpha_{A1})(\alpha_{B1}-\alpha_{A2})},$$

$$\gamma_{11}=\frac{\alpha_{A1}\alpha_{B1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B1}-\alpha_{A2})(\alpha_{B2}-\alpha_{A2})}\ and\ \gamma_{12}=\frac{\alpha_{A1}\alpha_{A2}\alpha_{B1}}{(\alpha_{B2}-\alpha_{B1})(\alpha_{B2}-\alpha_{A1})(\alpha_{B2}-\alpha_{A2})}. \qquad (40)$$

**Case(i) :** $\{U_i\}$ form a geometric process

In this case it is shown that

$$E[T]=E[U]\sum_{k=0}^{\infty}\left(\frac{1}{a}\right)^{k}\begin{bmatrix}\gamma_9 V_{(\alpha_{A1},k)}+\gamma_{10} V_{(\alpha_{B1},k)}-\\ \gamma_{11} V_{(\alpha_{A2},k)}-\gamma_{12} V_{(\alpha_{B2},k)}\end{bmatrix} \qquad (41)$$

and

$$E[T^2]=V[U]\sum_{k=0}^{\infty}\left(\frac{1}{a^2}\right)^{k}\begin{bmatrix}\gamma_9 V_{(\alpha_{A1},k)}+\gamma_{10} V_{(\alpha_{B1},k)}-\\ \gamma_{11} V_{(\alpha_{A2},k)}-\gamma_{12} V_{(\alpha_{B2},k)}\end{bmatrix}-$$

$$\left(E[U]\right)^2\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\begin{bmatrix}\gamma_9 V_{(\alpha_{A1},k)}+\gamma_{10} V_{(\alpha_{B1},k)}-\\ \gamma_{11} V_{(\alpha_{A2},k)}-\gamma_{12} V_{(\alpha_{B2},k)}\end{bmatrix} \qquad (42)$$

where E[U] and V[U] are given by (11).

**Case (ii) :** $\{U_i\}$ form an order statistics.

In this case it is shown that

$$E[T]=E[U]\sum_{k=0}^{\infty}\begin{bmatrix}\gamma_9 V_{(\alpha_{A1},k)}+\gamma_{10}V_{(\alpha_{B1},k)}- \\ \gamma_{11}V_{(\alpha_{A2},k)}-\gamma_{12}V_{(\alpha_{B2},k)}\end{bmatrix} \quad (43)$$

and

$$E[T^2]=\sum_{k=0}^{\infty}\left(2k(E[U])^2+E[U^2]\right)\begin{bmatrix}\gamma_9 V_{(\alpha_{A1},k)}+\gamma_{10}V_{(\alpha_{B1},k)}- \\ \gamma_{11}V_{(\alpha_{A2},k)}-\gamma_{12}V_{(\alpha_{B2},k)}\end{bmatrix} \quad (44)$$

where E[U] and $E[U^2]$ are given by (14) and (15).

## V. NUMERICAL ILLUSTRATION

The influence of parameters on the performance measures namely the mean and variance of the time for recruitment is studied numerically. In the following table these performance measures are calculated by varying the parameter 'c' and the other parameters $\alpha_{A1}$=0.1, $\alpha_{A2}$=0.2, $\alpha_{B1}$=0.3 , $\alpha_{B2}$=0.4 , p=0.4, $\lambda_h$=0.3and $\lambda_l$=0.2.

Table : Effect of 'c' on the performance measures E[T] and V[T]

| *c* | *Case(i)* | | | | *Case(ii)* | |
|---|---|---|---|---|---|---|
| | *f(t) = $f_{U(1)}(t)$* | | *f(t) = $f_{U(m)}(t)$* | | | |
| | *E[T]* | *V[T]* | *E[T]* | *V[T]* | *E[T]* | *V[T]* |
| ***MODEL - 1*** | | | | | | |
| ***1*** | *0.1930* | *0.1794* | *4.2421* | *45.688* | *0.926* | *9.6733* |
| ***1.5*** | *0.2579* | *19.424* | *5.6684* | *58.096* | *1.178* | *12.294* |
| ***2*** | *0.3078* | *23.707* | *6.7646* | *65.961* | *1.359* | *14.156* |
| ***2.5*** | *0.3470* | *27.14* | *7.6267* | *71.008* | *1.496* | *15.542* |
| ***MODEL - 2*** | | | | | | |
| ***1*** | *0.3767* | *0.3076* | *8.2794* | *68.666* | *1.659* | *16.638* |
| ***1.5*** | *0.4541* | *0.3538* | *9.9805* | *74.556* | *1.899* | *19.076* |
| ***2*** | *0.5021* | *0.3782* | *11.035* | *76.181* | *2.037* | *20.493* |
| ***2.5*** | *0.5338* | *0.3925* | *11.733* | *76.361* | *2.125* | *21.398* |
| ***MODEL - 3*** | | | | | | |
| ***1*** | *0.4594* | *0.3559* | *10.098* | *74.414* | *1.918* | *19.362* |
| ***1.5*** | *0.5310* | *0.3904* | *11.671* | *75.918* | *2.122* | *21.435* |
| ***2*** | *0.5703* | *0.4062* | *12.535* | *75.226* | *2.227* | *22.511* |
| ***2.5*** | *0.5940* | *0.4147* | *13.056* | *74.284* | *2.288* | *23.140* |

From the above table it is found that

- E(T) and V(T) increases in all the three models as 'c' increases.

## Conclusion

Since the time to recruitment is more elongated in Model- 3 than the first two Models, Model- 3 is preferable from the organization point of view.

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