Certain finite double integrals involving biorthogonal polynomial, a general class of polynomials and multivariable Aleph-function

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ABSTRACT
In this paper, we evaluate three finite double integrals involving varous products of biorthogonal polynomials, a general class of polynomial and multivariable Aleph-function with general arguments. The integrals evaluated are quite general in nature and yield a number of new integrals as special cases.

KEYWORDS : Aleph-function of several variables, finite double integral, multivariable I-function, Aleph-function of two variables , general class of polynomials.

2010 Mathematics Subject Classification. 33C99, 33C60, 44A20

1.Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [6] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c} z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r \end{array}\right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\cdots\cdots\cdots\cdots\cdots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$$\left.\begin{array}{c}[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{t_k}\,\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

For more details, see Ayant [1].
The reals numbers τ_i are positives for $i=1,\cdots,R$, $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1,\cdots,R^{(k)}$
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :
$|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0,\quad \text{with } k=1\cdots,r,i=1,\cdots,R\ ,i^{(k)}=1,\cdots,R^{(k)} \tag{1.2}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}\,)\,,max(\,|z_1|\ldots|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\,)\,,min(\,|z_1|\ldots|z_r|\,)\rightarrow \infty$$

where, with $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})],j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U=p_i,q_i,\tau_i;R\;;\;V=m_1,n_1;\cdots;m_r,n_r \tag{1.3}$$

$$W=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \tag{1.4}$$

$$A=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\}$$

$$B=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.5}$$

$$C=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.6}$$

$$D=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.7}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r)=\aleph_{U:W}^{0,n:V}\left(\begin{array}{c|c} z_1 & A:C \\ \cdot & \\ \cdot & \cdots \\ \cdot & \\ z_r & B:D \end{array}\right) \tag{1.8}$$

Chai and Carlitz [2] studied the following pair of biorthogonal polynomials.

$$J_n^{(\alpha,\beta)}(x;k)=\frac{(\alpha+1)_{kn}}{n!}\sum_{j=0}^{n}(-)^j\binom{n}{j}\frac{(\alpha+\beta+n+1)_{kj}}{(\alpha+1)_{kj}}\left(\frac{1-x}{2}\right)^{kj} \tag{1.9}$$

and $$K_n^{(\alpha,\beta)}(x;k)=\frac{1}{n!}\sum_{j=0}^{n}(-)^j\binom{\beta+n}{j}\left(\frac{x-1}{2}\right)^j\left(\frac{x+1}{2}\right)^{n-j}\sum_{l=0}^{j}(-)^l\binom{j}{l}\left(\frac{\alpha+l+1}{k}\right)_n \tag{1.10}$$

The generalized polynomials defined by Srivastava [8], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s]=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.11}$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex.

We shall require the following integrals for the evaluation of our main integrals :

$$\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}J_n^{(\alpha,\beta)}(1-2x;k)\mathrm{d}x = \frac{(\alpha+1)_{kn}}{n!}\sum_{m=0}^{n}\frac{(-n)_m}{m!}$$

$$\times\frac{(\alpha+\beta+n+1)_{km}\Gamma(\mu)\Gamma(\lambda+km)}{(\alpha+1)_{km}\Gamma(\lambda+\mu+km)} \tag{1.12}$$

$$\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}K_n^{(\alpha,\beta)}(1-2x;k)\mathrm{d}x= \frac{1}{n!}\sum_{m=0}^{n}(-)^m\binom{\beta+n}{m}\sum_{l=0}^{m}\frac{(-m)_l}{l!}(\frac{\alpha+l+1}{k})_m$$

$$\frac{\Gamma(\lambda+m)\Gamma(\mu+n-m)}{\Gamma(\lambda+\mu+n)} \text{ Where } Re(\lambda)>0, Re(\mu)>0, (\alpha)>-1, Re(\beta)>-1 \tag{1.13}$$

The above two integrals can be evaluated, if we make use of (1.9) and (1.10) respectively and the definition of Beta function.

2. Main integrals

$$\text{Let } A=\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_t)_{M_tK_t}}{K_t!}A[N_1,K_1;\cdots;N_t,K_t] \tag{2.1}$$

m is a positive integer

First integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{\rho-1}z^{\rho+2\sigma}(1-x)^{\mu-1}(1-y^2)^{\sigma-1}J_n^{(\alpha,\beta)}(1-2x;k)$$

$$S^{M_1,\cdots,M_t}_{N_1,\cdots,N_t}\left(\begin{array}{c} \mathrm{a}_1 y^{u_1}(1-y^2)^{v_1}z^{u_1+2v_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{a}_t y^{u_t}(1-y^2)^{v_t}z^{u_t+2v_t} \end{array}\right) \aleph^{0,\mathrm{n}:V}_{U:W}\left(\begin{array}{c} \mathrm{z}_1 x^{\rho_1}(1-x)^{\sigma_1}y^{\mu_1}(1-y^2)^{\delta_1}z^{\mu_1+2\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\rho_r}(1-x)^{\sigma_r}y^{\mu_r}(1-y^2)^{\delta_r}z^{\mu_r+2\delta_r} \end{array}\right)\mathrm{d}x\mathrm{d}y$$

$$=\frac{(\alpha+1)_{kn}}{n!}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{j=0}^{n}\frac{(-n)_j(\alpha+\beta+n+1)_{kj}}{j!(\alpha+1)_{kj}}Aa_1^{K_1}\cdots a_t^{K_t}$$

$$e^{i\pi(\rho+\mu(K_1+\cdots+K_t)/2)}\aleph^{0,\mathrm{n}+4:V}_{U_{42}:W}\left(\begin{array}{c|c} \mathrm{z}_1 e^{i\pi\mu_1/2} & (1\text{-}\mu;\sigma_1,\cdots,\sigma_r), \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & \\ \mathrm{z}_r e^{i\pi\mu_r/2} & (1\text{-}\lambda-\mu-kj;\sigma_1+\rho_1,\cdots,\sigma_r+\rho_r), \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\lambda-kj;\rho_1,\cdots,\rho_r),(1-\rho-\sum_{l=1}^t K_l u_l;\mu_1,\cdots,\mu_r),(2\sigma-2\sum_{l=1}^t K_l v_l;2\delta_1,\cdots,2\delta_r),A:C \\ \cdots \\ \cdots \\ (1\text{-}\rho-2\sigma-\sum_{l=1}^t K_l(u_l+2v_l);\mu_1+2\delta_1,\cdots,\mu_r+2\delta_r),B:D \end{array}\right)$$

where $z=\sqrt{1-y^2}+iy$ and $\quad U_{42}=p_i+4,q_i+2,\tau_i;R$ (2.2)

Provided that

a) $\min_{1\leqslant i\leqslant r}\{\mu_i,\delta_j,\rho_i,\sigma_i\}>0, Re(a)>-1, Re(\beta)>-1$

b) $Re[\rho+\sum_{i=1}^{r}\mu_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0\,;Re[\lambda+\sum_{i=1}^{r}\mu_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

$Re[\sigma+\sum_{i=1}^{r}\mu_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$ and $Re[\mu+\sum_{i=1}^{r}\mu_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2)

Second integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{\rho-1}z^{\rho+\delta}(1-x)^{\mu-1}(1-y^2)^{\delta/2-1}J_n^{(\alpha,\beta)}(1-2x;k){}_2F_1[a,b;\delta;z\sqrt{1-y^2}]$$

$$S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\begin{pmatrix}\mathrm{a}_1y^{u_1}z^{u_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{a}_ty^{u_t}z^{u_t}\end{pmatrix}\aleph_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix}\mathrm{z}_1x^{\rho_1}(1-x)^{\sigma_1}(yz)^{\mu_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\sigma_r}(yz)^{\mu_r}\end{pmatrix}\mathrm{d}x\mathrm{d}y$$

$$=\frac{(\alpha+1)_{kn}\Gamma(\delta)}{n!}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{j=0}^{n}\frac{(-n)_j(\alpha+\beta+n+1)_{kj}}{j!(\alpha+1)_{kj}}Aa_1^{K_1}\cdots a_t^{K_t}$$

$$e^{i\pi(\rho+\mu(K_1+\cdots+K_t)/2)}\aleph_{U_{43}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c|c}\mathrm{z}_1e^{i\pi\mu_1/2} & (1\text{-}\mu;\sigma_1,\cdots,\sigma_r),(1-\lambda-kj;\rho_1,\cdots,\rho_r)\\ \cdot & \cdots\\ \cdot & \cdots\\ \mathrm{z}_re^{i\pi\mu_r/2} & (1\text{-}\lambda-\mu-kj;\sigma_1+\rho_1,\cdots,\sigma_r+\rho_r),\end{array}\right.$$

$$\left.\begin{array}{c},(1\text{-}\rho-\sum_{l=1}^{t}K_lu_l;\mu_1,\cdots,\mu_r),(1-\rho-\delta+a+b-\sum_{l=1}^{t}K_lu_l;\mu_1,\cdots,\mu_r),A:C\\ \cdots\\ \cdots\\ (1\text{-}\rho-\delta-\sum_{l=1}^{t}K_lu_l+a;\mu_1,\cdots,\mu_r),(1-\rho-\delta-\sum_{l=1}^{t}K_lu_l+b;\mu_1,\cdots,\mu_r),B:D\end{array}\right)\quad(2.3)$$

where $z=\sqrt{1-y^2}+iy$ and $U_{42}=p_i+4,q_i+3,\tau_i;R$,provided that

a) $\min_{1\leqslant i\leqslant r}\{\mu_i,\delta_j,\rho_i,\sigma_i\}>0, Re(a)>-1, Re(\beta)>-1$

b) $Re[\rho+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0\,; Re[\lambda+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

$Re[\mu+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$ and $Re(\delta)>0, Re(\rho+\delta-a-b)>0$

c) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2)

Third integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{2\rho-1}\zeta^{-\rho-\sigma}(1-x)^{\mu-1}(1-y^2)^{\sigma-1}J_n^{(\alpha,\beta)}(1-2x;k)$$

$$S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\begin{pmatrix} \mathrm{a}_1 y^{2u_1}(1-y^2)^{v_1}\zeta^{-u_1-v_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{a}_t y^{2u_t}(1-y^2)^{v_t}\zeta^{-u_t-v_t} \end{pmatrix} \aleph_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix} \mathrm{z}_1 x^{\rho_1}(1-x)^{\sigma_1}y^{2\mu_1}(1-y^2)^{\delta_1}\zeta^{-\mu_1-\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\rho_r}(1-x)^{\sigma_r}y^{2\mu_r}(1-y^2)^{\delta_r}\zeta^{-\mu_r-\delta_r} \end{pmatrix}\mathrm{d}x\mathrm{d}y$$

$$=\frac{1}{2}\frac{(\alpha+1)_{kn}}{n!}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{j=0}^{n}\frac{(-n)_j(\alpha+\beta+n+1)_{kj}}{j!(\alpha+1)_{kj}}Aa_1^{K_1}\cdots a_t^{K_t}$$

$$(1+b)^{-(\rho+\mu(K_1+\cdots+K_t))}\aleph_{U_{42}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c|c} \mathrm{z}_1(1+b)^{-\mu_1} & (1\text{-}\mu;\sigma_1,\cdots,\sigma_r),(1-\lambda-kj;\rho_1,\cdots,\rho_r) \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & \\ \mathrm{z}_r(1+b)^{-\mu_r} & (1\text{-}\lambda-\mu-kj;\rho_1,\cdots,\rho_r), \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\rho-\sum_{l=1}^t K_l u_l;\mu_1,\cdots,\mu_r),(1-\sigma-\sum_{l=1}^t K_l v_l;\delta_1,\cdots,\delta_r),A:C \\ \cdots \\ \cdots \\ (1\text{-}\rho-\sigma-\sum_{l=1}^t K_l(u_l+v_l);\mu_1+\delta_1,\cdots,\mu_r+\delta_r),B:D \end{array}\right) \qquad (2.4)$$

where $\zeta=(1+by^2), b>-1$ and $U_{42}=p_i+4,q_i+2,\tau_i;R$, provided that

a) $\min_{1\leqslant i\leqslant r}\{\mu_i,\delta_j,\rho_i,\sigma_i\}>0, Re(a)>-1, Re(\beta)>-1$

b) $Re[\rho+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0\,; Re[\lambda+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

$Re[\sigma+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$ and $Re[\mu+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2) remark : Similarly integrals involving $K_n^{(\alpha,\beta)}(1-2x;k)$ can also be evaluating.

Proof of the first integral

To establish the integral (2.2), we first use the definition of $S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}$ and the Aleph-function of r variables in Mellin-Barnes contour integral, changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). Evaluating the resulting integral with the help of [4, p.450, eq.(4)]. Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

The proof of the other integral formulas are similar to that of the first integral with the only difference that here we use other known integral [5,p.71, eq.(3.1.8)] and [3,p.10, eq.20]

3. Multivariable I-function

If $\tau_i = \tau_{i(1)} = \cdots = \tau_{i(r)} = 1$, the Aleph-function of several variables degenere to the I-function of several variables. The finite double integrals have been derived in this section for multivariable I-functions defined by Sharma et al [6]. In these section, we note

$$B_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_{i(k)}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_{i(k)}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1\cdots,r, i = 1,\cdots,R\ , i^{(k)} = 1,\cdots,R^{(k)} \tag{3.1}$$

First integral

$$\int_0^1 \int_0^1 x^{\lambda-1} y^{\rho-1} z^{\rho+2\sigma} (1-x)^{\mu-1} (1-y^2)^{\sigma-1} J_n^{(\alpha,\beta)}(1-2x;k)$$

$$S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\left(\begin{array}{c} \mathrm{a}_1 y^{u_1}(1-y^2)^{v_1} z^{u_1+2v_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{a}_t y^{u_t}(1-y^2)^{v_t} z^{u_t+2v_t} \end{array}\right) I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 x^{\rho_1}(1-x)^{\sigma_1} y^{\mu_1}(1-y^2)^{\delta_1} z^{\mu_1+2\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\rho_r}(1-x)^{\sigma_r} y^{\mu_r}(1-y^2)^{\delta_r} z^{\mu_r+2\delta_r} \end{array}\right) \mathrm{d}x\mathrm{d}y$$

$$= \frac{(\alpha+1)_{kn}}{n!} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \sum_{j=0}^{n} \frac{(-n)_j(\alpha+\beta+n+1)_{kj}}{j!(\alpha+1)_{kj}} A a_1^{K_1}\cdots a_t^{K_t}$$

$$e^{i\pi(\rho+\mu(K_1+\cdots+K_t)/2)} I_{U_{42}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c|c} \mathrm{z}_1 e^{i\pi\mu_1/2} & (1\text{-}\mu;\sigma_1,\cdots,\sigma_r), \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & \\ \mathrm{z}_r e^{i\pi\mu_r/2} & (1\text{-}\lambda-\mu-kj;\sigma_1+\rho_1,\cdots,\sigma_r+\rho_r), \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\lambda-kj;\rho_1,\cdots,\rho_r),(1-\rho-\sum_{l=1}^t K_l u_l;\mu_1,\cdots,\mu_r),(2\sigma-2\sum_{l=1}^t K_l v_l;2\delta_1,\cdots,2\delta_r),A:C \\ \cdots \\ \cdots \\ (1\text{-}\rho-2\sigma-\sum_{l=1}^t K_l(u_l+2v_l);\mu_1+2\delta_1,\cdots,\mu_r+2\delta_r),B:D \end{array}\right)$$

where $z=\sqrt{1-y^2}+iy$ and $\qquad U_{42}=p_i+4,q_i+2,\tau_i;R$ (3.2)

Provided that

a) $\min\limits_{1\leqslant i\leqslant r}\{\mu_i,\delta_j,\rho_i,\sigma_i\}>0, Re(a)>-1, Re(\beta)>-1$

b) $Re[\rho+\sum_{i=1}^{r}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0\,; Re[\lambda+\sum_{i=1}^{r}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

$Re[\sigma+\sum_{i=1}^{r}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$ and $Re[\mu+\sum_{i=1}^{r}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c) $|argz_k|<\frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (3.1)

Second integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{\rho-1}z^{\rho+\delta}(1-x)^{\mu-1}(1-y^2)^{\delta/2-1}J_n^{(\alpha,\beta)}(1-2x;k)\,{}_2F_1[a,b;\delta;z\sqrt{1-y^2}]$$

$$S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\begin{pmatrix}\mathrm{a}_1y^{u_1}z^{u_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{a}_ty^{u_t}z^{u_t}\end{pmatrix} I_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix}\mathrm{z}_1x^{\rho_1}(1-x)^{\sigma_1}(yz)^{\mu_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\sigma_r}(yz)^{\mu_r}\end{pmatrix}\mathrm{d}x\mathrm{d}y$$

$$=\frac{(\alpha+1)_{kn}\Gamma(\delta)}{n!}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{j=0}^{n}\frac{(-n)_j(\alpha+\beta+n+1)_{kj}}{j!(\alpha+1)_{kj}}Aa_1^{K_1}\cdots a_t^{K_t}$$

$$e^{i\pi(\rho+\mu(K_1+\cdots+K_t)/2)}I_{U_{43}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c|c}\mathrm{z}_1e^{i\pi\mu_1/2} & (1\text{-}\mu;\sigma_1,\cdots,\sigma_r),(1-\lambda-kj;\rho_1,\cdots,\rho_r)\\ \cdot & \cdots\\ \cdot & \cdots\\ \mathrm{z}_re^{i\pi\mu_r/2} & (1\text{-}\lambda-\mu-kj;\sigma_1+\rho_1,\cdots,\sigma_r+\rho_r),\end{array}\right.$$

$$\left.\begin{array}{c},(1\text{-}\rho-\sum_{l=1}^tK_lu_l;\mu_1,\cdots,\mu_r),(1-\rho-\delta+a+b-\sum_{l=1}^tK_lu_l;\mu_1,\cdots,\mu_r),A:C\\ \cdots\\ \cdots\\ (1\text{-}\rho-\delta-\sum_{l=1}^tK_lu_l+a;\mu_1,\cdots,\mu_r),(1-\rho-\delta-\sum_{l=1}^tK_lu_l+b;\mu_1,\cdots,\mu_r),B:D\end{array}\right) \quad (3.3)$$

where $z=\sqrt{1-y^2}+iy$ and $U_{42}=p_i+4,q_i+3,\tau_i;R$, provided that

a) $\min\limits_{1\leqslant i\leqslant r}\{\mu_i,\delta_j,\rho_i,\sigma_i\}>0, Re(a)>-1, Re(\beta)>-1$

b) $Re[\rho+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0\,;Re[\lambda+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

$Re[\mu+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$ and $Re(\delta)>0, Re(\rho+\delta-a-b)>0$

c) $|argz_k|<\frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (3.1)

Third integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{2\rho-1}\zeta^{-\rho-\sigma}(1-x)^{\mu-1}(1-y^2)^{\sigma-1}J_n^{(\alpha,\beta)}(1-2x;k)$$

$$S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\begin{pmatrix} \mathrm{a}_1y^{2u_1}(1-y^2)^{v_1}\zeta^{-u_1-v_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{a}_ty^{2u_t}(1-y^2)^{v_t}\zeta^{-u_t-v_t}\end{pmatrix} I_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix} \mathrm{z}_1x^{\rho_1}(1-x)^{\sigma_1}y^{2\mu_1}(1-y^2)^{\delta_1}\zeta^{-\mu_1-\delta_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\sigma_r}y^{2\mu_r}(1-y^2)^{\delta_r}\zeta^{-\mu_r-\delta_r}\end{pmatrix}\mathrm{d}x\mathrm{d}y$$

$$=\frac{1}{2}\frac{(\alpha+1)_{kn}}{n!}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{j=0}^{n}\frac{(-n)_j(\alpha+\beta+n+1)_{kj}}{j!(\alpha+1)_{kj}}Aa_1^{K_1}\cdots a_t^{K_t}$$

$$(1+b)^{-(\rho+\mu(K_1+\cdots+K_t))}I_{U_{42}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c|c} \mathrm{z}_1(1+b)^{-\mu_1} & (1\text{-}\mu;\sigma_1,\cdots,\sigma_r),(1-\lambda-kj;\rho_1,\cdots,\rho_r)\\ \cdot & \cdots\\ \cdot & \cdots\\ \cdot & (1\text{-}\lambda-\mu-kj;\rho_1,\cdots,\rho_r),\\ \mathrm{z}_r(1+b)^{-\mu_r} & \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\rho-\sum_{l=1}^t K_lu_l;\mu_1,\cdots,\mu_r),(1-\sigma-\sum_{l=1}^t K_lv_l;\delta_1,\cdots,\delta_r),A:C\\ \cdots\\ \cdots\\ (1\text{-}\rho-\sigma-\sum_{l=1}^t K_l(u_l+v_l);\mu_1+\delta_1,\cdots,\mu_r+\delta_r),B:D\end{array}\right) \tag{3.4}$$

where $\zeta=(1+by^2), b>-1$ and $U_{42}=p_i+4,q_i+2,\tau_i;R,$ provided that

a) $\min_{1\leqslant i\leqslant r}\{\mu_i,\delta_j,\rho_i,\sigma_i\}>0, Re(a)>-1, Re(\beta)>-1$

b) $Re[\rho+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0\,;Re[\lambda+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

$Re[\sigma+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$ and $Re[\mu+\sum_{i=1}^{r}\mu_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c) $|argz_k| < \frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (3.1)

4. Aleph-function of two variables

If $r=2$, we obtain the Aleph-function of two variables defined by K.Sharma [7], and we have the following relations.

First integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{\rho-1}z^{\rho+2\sigma}(1-x)^{\mu-1}(1-y^2)^{\sigma-1}J_n^{(\alpha,\beta)}(1-2x;k)$$

$$S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\begin{pmatrix} \mathrm{a}_1 y^{u_1}(1-y^2)^{v_1}z^{u_1+2v_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{a}_t y^{u_t}(1-y^2)^{v_t}z^{u_t+2v_t}\end{pmatrix}\aleph_{U:W}^{0,\mathrm{n}:V}\begin{pmatrix} \mathrm{z}_1 x^{\rho_1}(1-x)^{\sigma_1}y^{\mu_1}(1-y^2)^{\delta_1}z^{\mu_1+2\delta_1}\\ \cdot\\ \cdot\\ \mathrm{z}_2 x^{\rho_2}(1-x)^{\sigma_2}y^{\mu_2}(1-y^2)^{\delta_2}z^{\mu_2+2\delta_2}\end{pmatrix}\mathrm{d}x\mathrm{d}y$$

$$=\frac{(\alpha+1)_{kn}}{n!}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{j=0}^{n}\frac{(-n)_j(\alpha+\beta+n+1)_{kj}}{j!(\alpha+1)_{kj}}Aa_1^{K_1}\cdots a_t^{K_t}$$

$$e^{i\pi(\rho+\mu(K_1+\cdots+K_t)/2)}\aleph_{U_{42}:W}^{0,\mathrm{n}+4:V}\left(\begin{array}{c|c} \mathrm{z}_1 e^{i\pi\mu_1/2} & (1\text{-}\mu;\sigma_1,\sigma_2),\\ \cdot & \cdots\\ \cdot & \cdots\\ \mathrm{z}_2 e^{i\pi\mu_2/2} & (1\text{-}\lambda-\mu-kj;\sigma_1+\rho_1,\sigma_2+\rho_2),\end{array}\right.$$

$$\left.\begin{array}{c}(1\text{-}\lambda-kj;\rho_1,\rho_2),(1-\rho-\sum_{l=1}^t K_l u_l;\mu_1,\mu_2),(2\sigma-2\sum_{l=1}^t K_l v_l;2\delta_1,2\delta_2),A:C\\ \cdots\\ \cdots\\ (1\text{-}\rho-2\sigma-\sum_{l=1}^t K_l(u_l+2v_l);\mu_1+2\delta_1,\mu_2+2\delta_2),B:D\end{array}\right) \tag{4.1}$$

where $z=\sqrt{1-y^2}+iy$ and $U_{42}=p_i+4,q_i+2,\tau_i;R$, provided that

a) $\min\limits_{1\leqslant i\leqslant 2}\{\mu_i,\delta_j,\rho_i,\sigma_i\}>0, Re(a)>-1, Re(\beta)>-1$

b) $Re[\rho+\sum_{i=1}^2\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$; $Re[\lambda+\sum_{i=1}^2\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

$Re[\sigma+\sum_{i=1}^2\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$ and $Re[\mu+\sum_{i=1}^2\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.5) with $r=2$

Second integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{\rho-1}z^{\rho+\delta}(1-x)^{\mu-1}(1-y^2)^{\delta/2-1}J_n^{(\alpha,\beta)}(1-2x;k)\,{}_2F_1[a,b;\delta;z\sqrt{1-y^2}]$$

$$S^{M_1,\cdots,M_t}_{N_1,\cdots,N_t}\begin{pmatrix} \mathrm{a}_1 y^{u_1}z^{u_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{a}_t y^{u_t}z^{u_t}\end{pmatrix}\aleph^{0,\mathfrak{n}:V}_{U:W}\begin{pmatrix} \mathrm{z}_1 x^{\rho_1}(1-x)^{\sigma_1}(yz)^{\mu_1} \\ \cdot \\ \cdot \\ \mathrm{z}_2 x^{\rho_2}(1-x)^{\sigma_2}(yz)^{\mu_2}\end{pmatrix}\mathrm{d}x\mathrm{d}y$$

$$=\frac{(\alpha+1)_{kn}\Gamma(\delta)}{n!}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{j=0}^{n}\frac{(-n)_j(\alpha+\beta+n+1)_{kj}}{j!(\alpha+1)_{kj}}Aa_1^{K_1}\cdots a_t^{K_t}$$

$$e^{i\pi(\rho+\mu(K_1+\cdots+K_t)/2)}\aleph^{0,\mathfrak{n}+4:V}_{U_{43}:W}\left(\begin{array}{c|c} \mathrm{z}_1 e^{i\pi\mu_1/2} & (1\text{-}\mu;\sigma_1,\sigma_2),(1-\lambda-kj;\rho_1,\cdots,\rho_r) \\ \cdot & \cdots \\ \cdot & \cdots \\ \mathrm{z}_2 e^{i\pi\mu_2/2} & (1\text{-}\lambda-\mu-kj;\sigma_1+\rho_1,\sigma_2+\rho_2), \end{array}\right.$$

$$\left.\begin{array}{c} ,(1\text{-}\rho-\sum_{l=1}^t K_l u_l;\mu_1,\mu_2),(1-\rho-\delta+a+b-\sum_{l=1}^t K_l u_l;\mu_1,\mu_2),A:C \\ \cdots \\ \cdots \\ (1\text{-}\rho-\delta-\sum_{l=1}^t K_l u_l+a;\mu_1,\mu_2),(1-\rho-\delta-\sum_{l=1}^t K_l u_l+b;\mu_1,\mu_2),B:D \end{array}\right) \quad (4.2)$$

where $z=\sqrt{1-y^2}+iy$ and $U_{42}=p_i+4,q_i+3,\tau_i;R$, provided that

a) $\min\limits_{1\leqslant i\leqslant 2}\{\mu_i,\delta_j,\rho_i,\sigma_i\}>0, Re(a)>-1, Re(\beta)>-1$

b) $Re[\rho+\sum_{i=1}^{2}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0\,;Re[\lambda+\sum_{i=1}^{2}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

$Re[\mu+\sum_{i=1}^{2}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$ and $Re(\delta)>0, Re(\rho+\delta-a-b)>0$

c) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.5) with $r=2$

Third integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{2\rho-1}\zeta^{-\rho-\sigma}(1-x)^{\mu-1}(1-y^2)^{\sigma-1}J_n^{(\alpha,\beta)}(1-2x;k)$$

$$S^{M_1,\cdots,M_t}_{N_1,\cdots,N_t}\begin{pmatrix} \mathrm{a}_1 y^{2u_1}(1-y^2)^{v_1}\zeta^{-u_1-v_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{a}_t y^{2u_t}(1-y^2)^{v_t}\zeta^{-u_t-v_t}\end{pmatrix}\aleph^{0,\mathfrak{n}:V}_{U:W}\begin{pmatrix} \mathrm{z}_1 x^{\rho_1}(1-x)^{\sigma_1}y^{2\mu_1}(1-y^2)^{\delta_1}\zeta^{-\mu_1-\delta_1} \\ \cdot \\ \cdot \\ \mathrm{z}_2 x^{\rho_2}(1-x)^{\sigma_2}y^{2\mu_2}(1-y^2)^{\delta_2}\zeta^{-\mu_2-\delta_2}\end{pmatrix}\mathrm{d}x\mathrm{d}y$$

$$=\frac{1}{2}\frac{(\alpha+1)_{kn}}{n!}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{j=0}^{n}\frac{(-n)_j(\alpha+\beta+n+1)_{kj}}{j!(\alpha+1)_{kj}}Aa_1^{K_1}\cdots a_t^{K_t}$$

$$(1+b)^{-(\rho+\mu(K_1+\cdots+K_t))}\aleph_{U_{42}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c|c} z_1(1+b)^{-\mu_1} & (1\text{-}\mu;\sigma_1,\sigma_2),(1-\lambda-kj;\rho_1,\rho_2) \\ \cdot & \cdots \\ \cdot & \cdots \\ z_2(1+b)^{-\mu_r} & (1\text{-}\lambda-\mu-kj;\rho_1,\rho_2), \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\rho-\sum_{l=1}^t K_l u_l;\mu_1,\mu_2),(1-\sigma-\sum_{l=1}^t K_l v_l;\delta_1,\delta_2),A:C \\ \cdots \\ \cdots \\ (1\text{-}\rho-\sigma-\sum_{l=1}^t K_l(u_l+v_l);\mu_1+\delta_1,\mu_2+\delta_2),B:D \end{array}\right) \quad (4.3)$$

where $\zeta=(1+by^2), b>-1$ and $U_{42}=p_i+4,q_i+2,\tau_i;R$, provided that

a) $\min\limits_{1\leqslant i\leqslant 2}\{\mu_i,\delta_j,\rho_i,\sigma_i\}>0, Re(a)>-1, Re(\beta)>-1$

b) $Re[\rho+\sum_{i=1}^{2}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0\,; Re[\lambda+\sum_{i=1}^{2}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

$Re[\sigma+\sum_{i=1}^{2}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$ and $Re[\mu+\sum_{i=1}^{2}\mu_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.5) with $r=2$

5. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions o several variables such as multivariable I-function ,multivariable Fox's H-function, Fox's H-function , Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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