Some finite double integral involving general class of polynomials, special functions , Aleph-function and multivariable Aleph-function

F.Y. AYANT[1]

1 Teacher in High School , France

ABSTRACT
The aim of the present document is to evaluate three triple double finite integrals involving general class of polynomials, special functions, Aleph-function and multivariable Aleph-function. Importance of our findings lies in the fact that they involve the multivariable Aleph-function, which are the sufficiently general in nature and are capable of yielding a large number of simpler and useful results merely by specializing the parameters in them. Further we establish some special cases.

KEYWORDS : Aleph-function of several variables, double integrals, special function, general class of polynomials, Aleph-function

2010 Mathematics Subject Classification. 33C99, 33C60, 44A20

1.Introduction and preliminaries.

The Aleph- function , introduced by Südland [10] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph_{P_i,Q_i,c_i;r}^{M,N}\left(\mathrm{z}\ \middle|\ \begin{matrix}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\(\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right)=\frac{1}{2\pi\omega}\int_L\Omega_{P_i,Q_i,c_i;r}^{M,N}(s)z^{-s}\mathrm{d}s \quad (1.1)$$

for all z different to 0 and

$$\Omega_{P_i,Q_i,c_i;r}^{M,N}(s)=\frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \quad (1.2)$$

With :$|argz|<\frac{1}{2}\pi\Omega$ Where $\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0\,;i=1,\cdots,r$

For convergence conditions and other details of Aleph-function , see Südland et al [10].the serie representation of Aleph-function is given by Chaurasia et al [2].

$$\aleph_{P_i,Q_i,c_i;r}^{M,N}(z)=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(s)}{B_Gg!}z^{-s} \quad (1.3)$$

With $s=\eta_{G,g}=\frac{b_G+g}{B_G},P_i<Q_i,|z|<1$ and $\Omega_{P_i,Q_i,c_i;r}^{M,N}(s)$ is given in (1.2) (1.4)

The generalized polynomials defined by Srivastava [8], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s]=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \quad (1.5)$$

Where $M_1, \cdots, M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s] \tag{1.6}$$

The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [6] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|c} z_1 & \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & \end{array}\right.$

$$[(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}] \ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$$
$$\cdots\cdots\cdots \ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$$

$$\left.\begin{array}{l}[(c_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(c_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(d_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(d_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.7}$$

with $\omega = \sqrt{-1}$

For more details, see Ayant [1].

The reals numbers τ_i are positives for $i = 1,\cdots,R$, $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1,\cdots,R^{(k)}$

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} - \tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)} - \tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1\cdots,r, i = 1,\cdots,R\ ,i^{(k)} = 1,\cdots,R^{(k)} \tag{1.8}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}\,), max(\,|z_1|\ldots|z_r|\,)\to 0$$

$$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\,), min(\,|z_1|\ldots|z_r|\,)\to\infty$$

where, with $k = 1,\cdots,r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1,\cdots,m_k$ and

$$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j = 1,\cdots,n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \;;\; V = m_1, n_1; \cdots ; m_r, n_r \tag{1.9}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots , p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.10}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots , \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots , \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.11}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots , \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.12}$$

$$C = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots ,\{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.13}$$

$$D = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots, \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.14}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots , z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\middle|\begin{array}{c} \mathrm{A} : \mathrm{C} \\ \cdots \\ \mathrm{B} : \mathrm{D} \end{array}\right) \tag{1.15}$$

2 . Required integrals

The following integral ([9],p.33;[3],p.172, eq(27);[4],p.46, eq(5) and [5],p.71] will be required to establish our main results :

a) $$\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a, b; a+b+1/2; x)\mathrm{d}x = \frac{\pi\Gamma(c)\Gamma(a+b+1/2)\Gamma(c-a-b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma(c-a+1/2)\Gamma(c-b+1/2)} \tag{2.1}$$

where Re(c) > 0, Re(2c-a-b) > -1

b) $$\int_0^\pi (sin\theta)^{\alpha-1} P_\upsilon^{-\mu}(cos\theta)\mathrm{d}\theta = \frac{2^{-\mu}\pi\Gamma\{(\alpha \pm \mu)/2\}}{\Gamma\{(\alpha+\upsilon+1)/2\}\Gamma\{(\alpha-\upsilon)/2\}\Gamma\{(\mu+\upsilon+2)/2\}\Gamma\{(\mu-\upsilon+1)/2\}} \tag{2.2}$$

provided that $Re(\alpha \pm \mu) > 0$

c) $$\int_0^\pi J_\mu(\theta)^{\mu+1}(cos\theta)^{2\rho+1}\mathrm{d}\theta = 2^\rho\Gamma(\rho+1)\alpha^{-\rho+1}J_{\rho+\mu+1}(\alpha) \tag{2.3}$$

where $Re(\rho) > -1; Re(\mu) > -1$

d) $$\int_0^{\pi/2} e^{i(\alpha+\beta)\theta}(sin\theta)^{\alpha-1}(cos\theta)^{\beta-1}{}_2F_1(\gamma, \delta; \beta; e^{i\theta}cos\theta)\mathrm{d}\theta = \frac{e^{i\pi\alpha/2}\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta-\delta-\gamma)}{\Gamma(\alpha+\beta-\gamma)\Gamma(\alpha+\beta-\delta)} \tag{2.4}$$

where $min\{Re(\alpha), Re(\beta), Re(\alpha+\beta-\gamma-\delta)\} > 0$

3. Main integrals

1) $$\int_0^1 \int_0^\pi x^{c-1}(1-x)^{-1/2}\ {}_2F_1(a, b; a+b+1/2; x)(sin\theta)^{\alpha-1}P_\upsilon^{-\mu}(cos\theta)\aleph_{P_i,Q_i,c_i;r'}^{M,N}(zx^{c'}(sin\theta)^d)$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} \mathrm{y}_1 x^{e_1}(sin\theta)^{f_1} \\ \cdots \\ \mathrm{y}_s x^{e_s}(sin\theta)^{f_s} \end{array}\right)\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 x^{c_1}(sin\theta)^{d_1} \\ \cdots \\ \mathrm{z}_r x^{c_r}(sin\theta)^{d_r} \end{array}\right)\mathrm{d}x\mathrm{d}\theta$$

$$= \frac{\pi^2 2^{-\mu}\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma\{(\mu+\upsilon+2)/2\}\Gamma\{(\mu-\upsilon+1)/2\}}$$

$$= \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a_1 \frac{(-)^g \Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!} z^{\eta_{G,g}} y_1^{K_1}\cdots y_s^{K_s}$$

$$\aleph_{U_{44}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c} z_1 \\ \cdots \\ z_r \end{array}\middle| \begin{array}{c} (1 - c - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1,\cdots,c_r), \\ \cdots \\ (\frac{1}{2} - c + a - c'\eta_{G,g} - \sum_{i=1}^{s} K e_i; c_1,\cdots,c_r), \end{array}\right.$$

$$\begin{array}{c} (\frac{1}{2} - c + a + b - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1,\cdots,c_r), (1 - \frac{\alpha+\mu+d\eta_{G,g}+\sum_{i=1}^{s} K_i f_i}{2}; d'_1,\cdots,d'_r), \\ \cdots \\ (\frac{1}{2} - c + b - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1,\cdots,c_r), (\frac{1}{2} - \frac{\alpha+\upsilon+d\eta_{G,g}+\sum_{i=1}^{s} K_i f_i}{2}; d'_1,\cdots,d'_r), \end{array}$$

$$\left.\begin{array}{c} (1 - \frac{\alpha-\mu+d\eta_{G,g}+\sum_{i=1}^{s} K_i f_i}{2}; d'_1,\cdots,d'_r), A : C \\ \cdots \\ (1 - \frac{\alpha-\upsilon+d\eta_{G,g}+\sum_{i=1}^{s} K_i f_i}{2}; d'_1,\cdots,d'_r), B : D \end{array}\right) \quad (3.1)$$

where $U_{44} = p_i + 4; q_i + 4; \tau_i; R$ and $d'_i = \frac{d_i}{2}, i = 1,\cdots,r$. Provided that

a) $Re[c + c' \min_{1\leqslant j\leqslant M} \frac{b_j}{B_j} + \sum_{i=1}^{s} K_i e_i + \sum_{i=1}^{r} c_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1$

b) $Re[c - a - b + c' \min_{1\leqslant j\leqslant M} \frac{b_j}{B_j} + \sum_{i=1}^{s} K_i e_i + \sum_{i=1}^{r} c_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 1/2$

c) $Re[c - a + c' \min_{1\leqslant j\leqslant M} \frac{b_j}{B_j} + \sum_{i=1}^{s} K_i e_i + \sum_{i=1}^{r} c_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1/2$

d) $Re[c - b + c' \min_{1\leqslant j\leqslant M} \frac{b_j}{B_j} + \sum_{i=1}^{s} K_i e_i + \sum_{i=1}^{r} c_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1/2$

e) $Re[\alpha \pm \mu + d \min_{1\leqslant j\leqslant M} \frac{b_j}{B_j} + \sum_{i=1}^{s} K_i f_i + \sum_{i=1}^{r} d_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

f) $Re[\alpha - \upsilon + d \min_{1\leqslant j\leqslant M} \frac{b_j}{B_j} + \sum_{i=1}^{s} K_i f_i + \sum_{i=1}^{r} d_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

g) $Re[\alpha \pm \upsilon + d \min_{1\leqslant j\leqslant M} \frac{b_j}{B_j} + \sum_{i=1}^{s} K_i f_i + \sum_{i=1}^{r} d_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1$

h) $|arg z_k| < \frac{1}{2} A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.8)

i) $|argz| < \frac{1}{2}\pi\Omega$ where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$

2) $$\int_0^1\int_0^\pi x^{c-1}(1-x)^{-1/2}\,{}_2F_1(a,b;a+b+1/2;x)(sin\theta)^{\mu+1}(cos\theta)^{2\rho+1}J_\mu(\alpha sin\theta)\aleph^{M,N}_{P_i,Q_i,c_i;r'}(zx^{c'}(sin\theta)^d)$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix} \mathrm{y}_1 x^{e_1}(cos\theta)^{2f_1} \\ \cdots \\ \mathrm{y}_s x^{e_s}(cos\theta)^{2f_s}\end{pmatrix}\aleph^{0,\mathfrak{n}:V}_{U:W}\begin{pmatrix} \mathrm{z}_1 x^{c_1}(cos\theta)^{2d_1} \\ \cdots \\ \mathrm{z}_r x^{c_r}(cos\theta)^{2d_r}\end{pmatrix}\mathrm{d}x\mathrm{d}\theta$$

$$= \frac{\pi 2^\rho \alpha^{-\rho}\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{g=0}^{\infty} a_1 \frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}\ \frac{(-)^g\alpha^{2g+\mu}}{2^{2g+\mu}g!}$$

$$z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}\ \aleph^{0,\mathfrak{n}+3:V}_{U_{33}:W}\left(\begin{array}{c|l} \mathrm{z}_1 & (1 - \mathrm{c}\text{-}\mathrm{c'}\eta_{G,g} - \sum_{i=1}^s K_i e_i; c_1,\cdots,c_r), \\ \cdots & \quad\cdots \\ \mathrm{z}_r & (\frac{1}{2} \text{ -c +a -c'}\eta_{G,g} - \sum_{i=1}^s K e_i; c_1,\cdots,c_r), \end{array}\right.$$

$$(\tfrac{1}{2}\text{ - c+a+b-c'}\eta_{G,g} - \sum_{i=1}^s K_i e_i; c_1,\cdots,c_r),$$
$$\cdots$$
$$(\tfrac{1}{2}\text{ -c +b -c'}\eta_{G,g} - \sum_{i=1}^s K_i e_i; c_1,\cdots,c_r),$$

$$\left.\begin{array}{c} (\text{ - }\rho - d\eta_{G,g} - \sum_{i=1}^s K_i f_i; d_1,\cdots,d_r), A : C \\ \cdots \\ (\text{-g-}\rho - \mu - 1 - d\eta_{G,g} - \sum_{i=1}^s K_i f_i; d_1,\cdots,d_r), B : D \end{array}\right) \tag{3.2}$$

where $U_{33} = p_i + 3; q_i + 3; \tau_i; R$. Provided that

a)$Re[c + c' \min_{1\leqslant j\leqslant M}\frac{b_j}{B_j} + \sum_{i=1}^s K_i e_i + \sum_{i=1}^r c_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1$

b)$Re[c - a - b + c' \min_{1\leqslant j\leqslant M}\frac{b_j}{B_j} + \sum_{i=1}^s K_i e_i + \sum_{i=1}^r c_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1/2$

c)$Re[c - a + c' \min_{1\leqslant j\leqslant M}\frac{b_j}{B_j} + \sum_{i=1}^s K_i e_i + \sum_{i=1}^r c_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1/2$

d)$Re[c - b + c' \min_{1\leqslant j\leqslant M}\frac{b_j}{B_j} + \sum_{i=1}^s K_i e_i + \sum_{i=1}^r c_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1/2$

e)$Re[\rho + d \min_{1\leqslant j\leqslant M}\frac{b_j}{B_j} + \sum_{i=1}^s K_i f_i + \sum_{i=1}^r d_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

f)$Re[g - \mu + \rho + d \min_{1\leqslant j\leqslant M}\frac{b_j}{B_j} + \sum_{i=1}^s K_i f_i + \sum_{i=1}^r d_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -2$

g) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.8)

h) $|argz| < \frac{1}{2}\pi\Omega$ where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$

3) $$\int_0^1\int_0^{\pi/2} x^{c-1}(1-x)^{-1/2}\ {}_2F_1(a,b;a+b+1/2;x)(sin\theta)^{\alpha-1}(cos\theta)^{\beta-1}\ {}_2F_1\left(\gamma,\delta;\beta;e^{i\theta}cos\theta\right)$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(zx^{c'}(sin\theta)^d)S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} y_1x^{e_1}e^{if_1\theta}(cos\theta)^{f_1} \\ \cdots \\ y_sx^{e_s}e^{if_s\theta}(cos\theta)^{f_s}\end{array}\right)\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} z_1x^{c_1}e^{i\theta d_1}(cos\theta)^{d_1} \\ \cdots \\ z_rx^{c_r}e^{i\theta d_r}(cos\theta)^{d_r}\end{array}\right)dxd\theta$$

$$= \frac{\pi e^{i\pi\alpha/2}\Gamma(\beta)\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_Gg!}$$

$$z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}\,e^{i\pi(d\eta_{G,g}+\sum_{j=1}^s K_jf_j)}\aleph_{U_{44}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c|c} z_1e^{i\pi\rho_1/2} & (1 - c - c'\eta_{G,g}-\sum_{j=1}^s K_je_j;c_1,\cdots,c_r), \cdots \\ \cdots & \\ z_re^{i\pi\rho_r/2} & (\frac{1}{2} - c + a - c'\eta_{G,g}-\sum_{j=1}^s Ke_i;c_1,\cdots,c_r),\end{array}\right.$$

$$(1-\alpha-d\eta_{G,g}-\sum_{j=1}^s K_jf_j;d_1,\cdots,d_r),\quad (\tfrac{1}{2} - c+a+b-c'\eta_{G,g}-\sum_{j=1}^s K_je_j;c_1,\cdots,c_r),$$
$$\cdots \qquad \cdots$$
$$(1-\alpha-\beta+\gamma-d\eta_{G,g}-\sum_{j=1}^s K_jf_j;d_1,\cdots,d_r),\ (\tfrac{1}{2} - c + b - c'\eta_{G,g}-\sum_{j=1}^s K_je_j;c_1,\cdots,c_r),$$

$$\left.\begin{array}{c}(1-\alpha-\beta+\gamma+\delta-d\eta_{G,g}-\sum_{j=1}^s K_jf_j;d_1,\cdots,d_r),A:C \\ \cdots \\ (1-\alpha-\beta+\delta-d\eta_{G,g}-\sum_{j=1}^s K_jf_j;d_1,\cdots,d_r),B:D\end{array}\right) \tag{3.3}$$

where $U_{44} = p_i+4;q_i+4;\tau_i;R$. Provided that

a) $Re[c+c'\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{s}K_ie_i+\sum_{i=1}^{r}c_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1$

b) $Re[c-a-b+c'\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{s}K_ie_i+\sum_{i=1}^{r}c_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1/2$

c) $Re[c-a+c'\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{s}K_ie_i+\sum_{i=1}^{r}c_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1/2$

d) $Re[c-b+c'\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{s}K_ie_i+\sum_{i=1}^{r}c_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1/2$

e) $Re[\alpha+\beta-\delta-\gamma+d\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{s}K_if_i+\sum_{i=1}^{r}d_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

f) $Re[\alpha+\beta-\delta+d\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{s}K_if_i+\sum_{i=1}^{r}d_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

g) $Re[\alpha+\beta-\gamma+d\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{s}K_if_i+\sum_{i=1}^{r}d_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

h) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.8)

i) $|argz|<\frac{1}{2}\pi\Omega$ where $\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0$

Proof of (3.1)

We first express the general class of polynomial occuring on the L.H.S of (3.1) in series form with the help of (1.5), the Aleph-function in series form with the help of (1.3) and replace the multivariable Aleph-function by its Mellin-Barnes contour integral with the help of (1.7). Now we interchange the order of summation and integrations, we obtain

L.H.S of (3.1) = $\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{\tilde{(-)}^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}$

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\left(\int_0^1 x^{c+c'\eta_{G,g}+\sum_{i=1}^{s}e_iK_i+\sum_{i=1}^{r}c_is_i-1}(1-x)^{-1/2}\right.$$

$$\left.{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x\right)\left(\int_0^{\pi}(sin\theta)^{\alpha+d\eta_{G,g}+\sum_{i=1}^{s}f_iK_i+\sum_{i=1}^{r}d_is_i-1}P_{\upsilon}^{-\mu}(cos\theta)\mathrm{d}\theta\right)\mathrm{d}s_1\cdots\mathrm{d}s_r$$

Now, using the result (2.1) and (2.2) to evaluate the x-integral and θ-integral and reinterpreting the multiple contour integral so obtained in the form of multivariable Aleph-function withe help of (1.7), we obtain the desired result.
The result (3.2) and (3.1) can be proved by similar prrofs with the help of integrals given by (2.2), (2.3) and (2.4).

4. Multivariable I-function

If $\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$ the Aleph-function of several variables degenere to the I-function of several variables. The Four doubles integrals have been derived in this section for multivariable I-functions defined by Sharma et al [6]. In these section, we note

$$B_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}-\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0,\text{ with } k=1\cdots,r,i=1,\cdots,R\ ,i^{(k)}=1,\cdots,R^{(k)} \tag{4.1}$$

1) $\int_0^1\int_0^{\pi}x^{c-1}(1-x)^{-1/2}\,{}_2F_1(a,b;a+b+1/2;x)(sin\theta)^{\alpha-1}P_{\upsilon}^{-\mu}(cos\theta)\aleph_{P_i,Q_i,c_i;r'}^{M,N}(zx^{c'}(sin\theta)^d)$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{y}_1x^{e_1}(sin\theta)^{f_1}\\ \cdot\cdot\cdot\\ \mathrm{y}_sx^{e_s}(sin\theta)^{f_s}\end{pmatrix}I_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix}\mathrm{z}_1x^{c_1}(sin\theta)^{d_1}\\ \cdot\cdot\cdot\\ \mathrm{z}_rx^{c_r}(sin\theta)^{d_r}\end{pmatrix}\mathrm{d}x\mathrm{d}\theta$$

$$= \frac{\pi^2 2^{-\mu}\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma\{(\mu+\upsilon+2)/2\}\Gamma\{(\mu-\upsilon+1)/2\}}$$

$$= \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a_1 \frac{(-)^g \Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!} z^{\eta_{G,g}} y_1^{K_1}\cdots y_s^{K_s}$$

$$I^{0,\mathfrak{n}+4:V}_{U_{44}:W}\left(\begin{array}{c} z_1 \\ \cdots \\ z_r \end{array}\middle| \begin{array}{c} (1 - c\text{-}c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1,\cdots,c_r), \\ \cdots \\ (\frac{1}{2} - c + a - c'\eta_{G,g} - \sum_{i=1}^{s} K e_i; c_1,\cdots,c_r), \end{array}\right.$$

$$\begin{array}{l} (\frac{1}{2} - c+a+b-c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1,\cdots,c_r), (1 - \frac{\alpha+\mu+d\eta_{G,g}+\sum_{i=1}^{s}K_i f_i}{2}; d'_1,\cdots,d'_r), \\ \cdots \\ (\frac{1}{2} - c + b - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1,\cdots,c_r),\ (\frac{1}{2} - \frac{\alpha+\upsilon+d\eta_{G,g}+\sum_{i=1}^{s}K_i f_i}{2}; d'_1,\cdots,d'_r), \end{array}$$

$$\left.\begin{array}{l} (1 - \frac{\alpha-\mu+d\eta_{G,g}+\sum_{i=1}^{s}K_i f_i}{2}; d'_1,\cdots,d'_r), A : C \\ \cdots \\ (1 - \frac{\alpha-\upsilon+d\eta_{G,g}+\sum_{i=1}^{s}K_i f_i}{2}; d'_1,\cdots,d'_r), B : D \end{array}\right) \tag{4.1}$$

where the same notations and validity conditions that (3.1)

2) $$\int_0^1\int_0^\pi x^{c-1}(1-x)^{-1/2}\,{}_2F_1(a,b;a+b+1/2;x)(sin\theta)^{\mu+1}(cos\theta)^{2\rho+1}J_\mu(\alpha sin\theta)\aleph^{M,N}_{P_i,Q_i,c_i;r'}(zx^{c'}(sin\theta)^d)$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c} y_1 x^{e_1}(cos\theta)^{2f_1} \\ \cdots \\ y_s x^{e_s}(cos\theta)^{2f_s} \end{array}\right) I^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c} z_1 x^{c_1}(cos\theta)^{2d_1} \\ \cdots \\ z_r x^{c_r}(cos\theta)^{2d_r} \end{array}\right) dx d\theta$$

$$= \frac{\pi 2^\rho \alpha^{-\rho}\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{g=0}^{\infty} a_1 \frac{(-)^g \Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!} \frac{(-)^g \alpha^{2g+\mu}}{2^{2g+\mu} g!}$$

$$z^{\eta_{G,g}} y_1^{K_1}\cdots y_s^{K_s}\ I^{0,\mathfrak{n}+3:V}_{U_{33}:W}\left(\begin{array}{c} z_1 \\ \cdots \\ z_r \end{array}\middle| \begin{array}{c} (1 - c\text{-}c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1,\cdots,c_r), \\ \cdots \\ (\frac{1}{2} - c + a - c'\eta_{G,g} - \sum_{i=1}^{s} K e_i; c_1,\cdots,c_r), \end{array}\right.$$

$$\begin{array}{l} (\frac{1}{2} - c+a+b-c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1,\cdots,c_r), \\ \cdots \\ (\frac{1}{2} - c + b - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1,\cdots,c_r), \end{array}$$

$$\left.\begin{array}{c} (- \rho - d\eta_{G,g} - \sum_{i=1}^{s} K_i f_i; d_1,\cdots,d_r), A : C \\ \cdots \\ (\text{-}g\text{-}\rho - \mu - 1 - d\eta_{G,g} - \sum_{i=1}^{s} K_i f_i; d_1,\cdots,d_r), B : D \end{array}\right) \tag{4.2}$$

where the same notations and validity conditions that (3.2)

3) $\int_0^1\int_0^{\pi/2} x^{c-1}(1-x)^{-1/2}\ {}_2F_1(a,b;a+b+1/2;x)(sin\theta)^{\alpha-1}(cos\theta)^{\beta-1}\ {}_2F_1\left(\gamma,\delta;\beta;e^{i\theta}cos\theta\right)$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(zx^{c'}(sin\theta)^d)S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} \mathrm{y}_1x^{e_1}e^{if_1\theta}(cos\theta)^{f_1} \\ \cdots \\ \mathrm{y}_sx^{e_s}e^{if_s\theta}(cos\theta)^{f_s}\end{array}\right)I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1x^{c_1}e^{i\theta d_1}(cos\theta)^{d_1} \\ \cdots \\ \mathrm{z}_rx^{c_r}e^{i\theta d_r}(cos\theta)^{d_r}\end{array}\right)\mathrm{d}x\mathrm{d}\theta$$

$$=\frac{\pi e^{i\pi\alpha/2}\Gamma(\beta)\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_Gg!}$$

$$z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}\,e^{i\pi(d\eta_{G,g}+\sum_{j=1}^sK_jf_j)}I_{U_{44}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c} \mathrm{z}_1e^{i\pi\rho_1/2} \\ \cdots \\ \mathrm{z}_re^{i\pi\rho_r/2}\end{array}\middle|\begin{array}{c}(1\text{ - c-c'}\eta_{G,g}-\sum_{j=1}^sK_je_j;c_1,\cdots,c_r), \\ \cdots \\ (\tfrac{1}{2}\text{ -c +a -c'}\eta_{G,g}-\sum_{j=1}^sKe_i;c_1,\cdots,c_r),\end{array}\right.$$

$$\begin{array}{cc}(1\text{ -}\alpha-d\eta_{G,g}-\sum_{j=1}^sK_jf_j;d_1,\cdots,d_r), & (\tfrac{1}{2}\text{ - c+a+b-c'}\eta_{G,g}-\sum_{j=1}^sK_je_j;c_1,\cdots,c_r), \\ \cdots & \cdots \\ (1\text{-}\alpha-\beta+\gamma-d\eta_{G,g}-\sum_{j=1}^sK_jf_j;d_1,\cdots,d_r), & (\tfrac{1}{2}\text{ -c +b -c'}\eta_{G,g}-\sum_{j=1}^sK_je_j;c_1,\cdots,c_r),\end{array}$$

$$\left.\begin{array}{c}(1\text{ -}\alpha-\beta+\gamma+\delta-d\eta_{G,g}-\sum_{j=1}^sK_jf_j;d_1,\cdots,d_r),A:C \\ \cdots \\ (1\text{-}\alpha-\beta+\delta-d\eta_{G,g}-\sum_{j=1}^sK_jf_j;d_1,\cdots,d_r),B:D\end{array}\right) \tag{4.3}$$

where the same notations and validity conditions that (3.3)

5. Aleph-function of two variables

If $r=2$, we obtain the Aleph-function of two variables defined by K.Sharma [7], and we have the following results.

1) $\int_0^1\int_0^{\pi} x^{c-1}(1-x)^{-1/2}\ {}_2F_1(a,b;a+b+1/2;x)(sin\theta)^{\alpha-1}P_\upsilon^{-\mu}(cos\theta)\aleph_{P_i,Q_i,c_i;r'}^{M,N}(zx^{c'}(sin\theta)^d)$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} \mathrm{y}_1x^{e_1}(sin\theta)^{f_1} \\ \cdots \\ \mathrm{y}_sx^{e_s}(sin\theta)^{f_s}\end{array}\right)\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1x^{c_1}(sin\theta)^{d_1} \\ \cdots \\ \mathrm{z}_2x^{c_2}(sin\theta)^{d_2}\end{array}\right)\mathrm{d}x\mathrm{d}\theta$$

$$=\frac{\pi^22^{-\mu}\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma\{(\mu+\upsilon+2)/2\}\Gamma\{(\mu-\upsilon+1)/2\}}$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_Gg!}z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}$$

$$\aleph_{U_{44}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c} z_1 \\ \cdots \\ z_2 \end{array}\middle| \begin{array}{c} (1 - c - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1, c_2), \quad (1 - \frac{\alpha+\mu+d\eta_{G,g}+\sum_{i=1}^{s} K_i f_i}{2}; d'_1, d'_2), \\ \cdots \\ (\frac{1}{2} - c + a - c'\eta_{G,g} - \sum_{i=1}^{s} K e_i; c_1, c_2), \ (\frac{1}{2} - \frac{\alpha+\upsilon+d\eta_{G,g}+\sum_{i=1}^{s} K_i f_i}{2}; d'_1, d'_2), \end{array}\right.$$

$$\left.\begin{array}{c} (\frac{1}{2} - c + a + b - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1, c_2), (1 - \frac{\alpha-\mu+d\eta_{G,g}+\sum_{i=1}^{s} K_i f_i}{2}; d'_1, d'2), A : C \\ \cdots \\ (\frac{1}{2} - c + b - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1, c_2), \ (1 - \frac{\alpha-\upsilon+d\eta_{G,g}+\sum_{i=1}^{s} K_i f_i}{2}; d'_1, d'_2), B : D \end{array}\right) \quad (5.1)$$

where the same notations and validity conditions that (3.1)

2) $$\int_0^1 \int_0^\pi x^{c-1}(1-x)^{-1/2} \, {}_2F_1(a,b;a+b+1/2;x)(sin\theta)^{\mu+1}(cos\theta)^{2\rho+1} J_\mu(\alpha sin\theta) \aleph_{P_i,Q_i,c_i;r'}^{M,N}(zx^{c'}(sin\theta)^d)$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} y_1 x^{e_1}(cos\theta)^{2f_1} \\ \cdots \\ y_s x^{e_s}(cos\theta)^{2f_s} \end{array}\right) \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} z_1 x^{c_1}(cos\theta)^{2d_1} \\ \cdots \\ z_2 x^{c_2}(cos\theta)^{2d_2} \end{array}\right) dx d\theta$$

$$= \frac{\pi 2^\rho \alpha^{-\rho}\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{g=0}^{\infty} a_1 \frac{(-)^g \Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!} \ \frac{(-)^g \alpha^{2g+\mu}}{2^{2g+\mu} g!}$$

$$z^{\eta_{G,g}} y_1^{K_1} \cdots y_s^{K_s} \ \aleph_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{array}{c} z_1 \\ \cdots \\ z_2 \end{array}\middle| \begin{array}{c} (1 - c - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1, c_2), \\ \cdots \\ (\frac{1}{2} - c + a - c'\eta_{G,g} - \sum_{i=1}^{s} K e_i; c_1, c_2), \end{array}\right.$$

$$\left.\begin{array}{c} (\frac{1}{2} - c + a + b - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1, c_2), \quad (-\rho - d\eta_{G,g} - \sum_{i=1}^{s} K_i f_i; d_1, d_2), A : C \\ \cdots \\ (\frac{1}{2} - c + b - c'\eta_{G,g} - \sum_{i=1}^{s} K_i e_i; c_1, c_2), \ (-g-\rho - \mu - 1 - d\eta_{G,g} - \sum_{i=1}^{s} K_i f_i; d_1, d_2), B : D \end{array}\right) \quad (5.2)$$

where the same notations and validity conditions that (3.2)

3) $$\int_0^1 \int_0^\pi x^{c-1}(1-x)^{-1/2} \ {}_2F_1(a,b;a+b+1/2;x)(sin\theta)^{\alpha-1}(cos\theta)^{\beta-1} \ {}_2F_1(\gamma,\delta;\beta;e^{i\theta}cos\theta)$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(zx^{c'}(sin\theta)^d) S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} y_1 x^{e_1} e^{if_1\theta}(cos\theta)^{f_1} \\ \cdots \\ y_s x^{e_s} e^{if_s\theta}(cos\theta)^{f_s} \end{array}\right) \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} z_1 x^{c_1} e^{i\theta d_1}(cos\theta)^{d_1} \\ \cdots \\ z_2 x^{c_2} e^{i\theta d_2}(cos\theta)^{d_2} \end{array}\right) dx d\theta$$

$$= \frac{\pi e^{i\pi\alpha/2}\Gamma(\beta)\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} a_1 \frac{(-)^g \Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}$$

$$z^{\eta_{G,g}} y_1^{K_1} \cdots y_s^{K_s} e^{i\pi(d\eta_{G,g}+\sum_{j=1}^{s} K_j f_j)} \aleph_{U_{44}:W}^{0,\mathfrak{n}+4:V}\left(\begin{array}{c} z_1 e^{i\pi\rho_1/2} \\ \cdots \\ z_2 e^{i\pi\rho_2/2} \end{array}\middle| \begin{array}{c} (1 - c - c'\eta_{G,g} - \sum_{j=1}^{s} K_j e_j; c_1, c_2), \\ \cdots \\ (\frac{1}{2} - c + a - c'\eta_{G,g} - \sum_{j=1}^{s} K e_i; c_1, c_2), \end{array}\right.$$

$$(1 \text{-}\alpha - d\eta_{G,g} - \textstyle\sum_{j=1}^{s} K_j f_j; d_1, d_2), \quad (\tfrac{1}{2} \text{ - c+a+b-c'}\eta_{G,g} - \textstyle\sum_{j=1}^{s} K_j e_j; c_1, c_2),$$

$$\cdots \qquad \cdots$$

$$(1\text{-}\alpha - \beta + \gamma - d\eta_{G,g} - \textstyle\sum_{j=1}^{s} K_j f_j; d_1, d_2), \; (\tfrac{1}{2} \text{ -c +b -c'}\eta_{G,g} - \textstyle\sum_{j=1}^{s} K_j e_j; c_1, c_2),$$

$$\left.\begin{matrix} (1 \text{-}\alpha - \beta + \gamma + \delta - d\eta_{G,g} - \sum_{j=1}^{s} K_j f_j; d_1, d_2), \\ \cdots \\ (1\text{-}\alpha - \beta + \delta - d\eta_{G,g} - \sum_{j=1}^{s} K_j f_j; d_1, d_2), \end{matrix}\right) \tag{5.3}$$

6. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions o several variables such as multivariable I-function ,multivariable Fox's H-function, Fox's H-function , Meijer's G-function, Wright's generalized Bessel function,

Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

REFERENCES

[1] Ayant F.Y. An integral associated with the Aleph-functions of several variables. *International Journal of Mathematics Trends and Technology (IJMTT)*. 2016 Vol 31 (3), page 142-154.

[2] Chaurasia V.B.L and Singh Y. New generalization of integral equations of fredholm type using the Aleph-function Int. J. of Modern Math. Sci. 9(3), 2014, p 208-220

[3] Erdelyi A. et al. Higher transcendental functions. , Vol 1, McGraw-Hill, New-York (1953).

[4] Erdelyi A. et al. Higher transcendental functions. , Vol 2, McGraw-Hill, New-York (1953).

[5] Mathai A.M. And Saxena R.K. (1973) : Generalized hypergeometric functions with applications in statistics and physical sciences, springer-Verlag, Lecture Notes No348, Heidelberg.

[6] Sharma C.K.and Ahmad S.S.: On the multivariable I-function. Acta ciencia Indica Math , 1994 vol 20,no2, p 113-116.

[7] Sharma K. On the integral representation and applications of the generalized function of two variables , International Journal of Mathematical Engineering and Sciences , Vol 3 , issue1 (2014) , page1-13.

[8] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. Vol 77(1985), page183-191

[9] Südland, N.; Baumann, B. and Nonnenmacher,T.F., Open problem : who knows about the Aleph-functions? Fract. Calc. Appl. Anal., 1(4) (1998): 401-402.

[10] Vyas V.M. and Rathie K., An integral involving hypergeometric function. The mathematics education 31(1997) page33

Personal adress : 411 Avenue Joseph Raynaud
Le parc Fleuri , Bat B
83140 , Six-Fours les plages
Tel : 06-83-12-49-68
Department : VAR
Country : FRANCE