# On some multidimensional integral transforms of multivariable Aleph-function II

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ABSTRACT
The first integral evaluated here involve the exponential function, the product of two general polynomials . These integral is unified, useful and most general in nature. She is capable of yielding a large number of integrals and double Laplace transforms as their special cases. The second integral valuated here involve the exponential function, the product of two general polynomials and the Aleph-function of two variables. The third integral is a multivariable analogue of the second integral and is believed to be one of the most general integrals evaluated so far.

Keywords :Multivariable Aleph-function, multidimensional integral transforms ,General class of polynomials, Multivariable I-function, Aleph-function of two variables.

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## 1.Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [4] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c}\mathrm{y}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_r\end{array}\right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]$
$[(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\Bigg)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)y_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$a_j, j=1,\cdots,p; b_j, j=1,\cdots,q;$

$c_j^{(k)}, j=1,\cdots,n_k; c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R, i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers $\tau_i$ are positives for $i = 1$ to $R$ , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour $L_k$ is in the $s_k$-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where $\sigma$ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to $m_k$ are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to $n$ and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to $n_k$ to the left of the contour $L_k$ . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$ , where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1 \cdots, r, i = 1, \cdots, R, i^{(k)} = 1, \cdots, R^{(k)} \tag{1.5}$$

The complex numbers $z_i$ are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\alpha_1} \dots |y_r|^{\alpha_r}), max(|y_1| \dots |y_r|) \to 0$$

$$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\beta_1} \dots |y_r|^{\beta_r}), min(|y_1| \dots |y_r|) \to \infty$$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1) / \gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \; ; \; V = m_1, n_1; \cdots ; m_r, n_r \tag{1.6}$$

$$W = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.7}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.8}$$

$$B = \{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C_1 = \{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\{\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots, \tag{1.10}$$

$$C_r = \{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\{\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.11}$$

$$D = \{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.12}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r) = \aleph(z_1,\cdots,z_r) = \aleph_{U:W}^{0,\mathrm{n}:V}\left(\begin{array}{c|c} z_1 & \mathrm{A} : \mathrm{C}_1 : \cdots : C_r \\ \cdot & \\ \cdot & \cdots \\ \cdot & \\ z_r & \mathrm{B} : \mathrm{D} \end{array}\right) \tag{1.13}$$

If $r = 2$, we obtain the Aleph-function defined by K. Sharma[3].

The generalized polynomials defined by Srivastava [5], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.14}$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex.

In the present paper, we use the following notation

$$A = \frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s] \tag{1.15}$$

Srivastava [4] introduced the general class of polynomials :

$$S_N^M(x) = \sum_{k=0}^{[N/M]}\frac{(-N)_{Mk}}{k!}A_{N,k}x^k, N = 0,1,2,... \tag{1.16}$$

Where $M$ is an arbtrary positive integer and the coefficient $A_{N,k}$ are arbitrary constants, real or complex.By suitably specialized the coefficient $A_{N,k}$ the polynomials $S_N^M(x)$ can be reduced to the classical orthogonal polynomials such as Jacobi, Hermite, Legendre and Laguerre polynomials etc.

## 2. Main integrals

**First integral**

$$\int_0^\infty\int_0^\infty (a_1^1x_1+a_2^1x_2)^{\sigma_1-1}(a_1^2x_1+a_2^2x_2)^{\sigma_2-1}exp(-t_1(a_1^1x_1+a_2^1x_2)-t_2(a_1^2x_1+a_2^2x_2))$$

$$S_{N_1}^{M_1}(a_1^1x_1+a_2^1x_2)S_{N_2}^{M_2}(a_1^2x_1+a_2^2x_2)\mathrm{d}x_1\mathrm{d}x_2$$

$$=\frac{1}{K}\sum_{k_1=0}^{[N_1/M_1]}\sum_{k_2=0}^{[N_2/M_2]}\frac{(-N_1)_{M_1k_1}(-N_2)_{M_2k_2}A_{N_1,k_1}A_{N_2,k_2}\Gamma(\sigma_1+k_1)\Gamma(\sigma_2+k_2)}{k_1!k_2!t_1^{\sigma_1+k_1}t_2^{\sigma_2+k_2}} \tag{2.1}$$

where $Re(\sigma_i) > 0, Re(t_i) > 0, i = 1, 2$ and $K = \begin{vmatrix} a_1^1 & a_1^2 \\ a_2^1 & a_2^2 \end{vmatrix} \neq 0$

**Proof of (2.1)**

we have [widder [6],1989 p.241, eq;(7)]

$$\int_0^\infty \int_0^\infty F(a_1^1 x_1 + a_2^1 x_2, a_1^2 x_1 + a_2^2 x_2) \mathrm{d}x_1 \mathrm{d}x_2 = \frac{1}{K} \int_0^\infty \int_0^\infty F(u_1, u_2) \, \mathrm{d}u_1 \mathrm{d}u_2 \tag{2.2}$$

where $K$ stands for the expression mentioned in (2.1)

If we take $F(a_1^1 x_1 + a_2^1 x_2, a_1^2 x_1 + a_2^2 x_2) = f_1(a_1^1 x_1 + a_2^1 x_2) \, f_2(a_1^2 x_1 + a_2^2 x_2)$, then we have

$$\int_0^\infty \int_0^\infty f_1(a_1^1 x_1 + a_2^1 x_2) f_2(a_1^2 x_1 + a_2^2 x_2) \mathrm{d}x_1 \mathrm{d}x_2 = \frac{1}{K} \int_0^\infty f_1(u_1) \mathrm{d}u_1 \int_0^\infty f_2(u_2) \mathrm{d}u_2 \tag{2.3}$$

consider $f_1(a_1^1 x_1 + a_2^1 x_2) = (a_1^1 x_1 + a_2^1 x_2)^{\sigma_1 - 1} exp(-t_1(a_1^1 x_1 + a_2^1 x_2)) S_{N_1}^{M_1}(a_1^1 x_1 + a_2^1 x_2)$

and $f_2(a_1^2 x_1 + a_2^2 x_2) = (a_1^2 x_1 + a_2^2 x_2)^{\sigma_2 - 1} exp(-t_2(a_1^2 x_1 + a_2^2 x_2)) S_{N_2}^{M_2}(a_1^2 x_1 + a_2^2 x_2)$

then from (2.3), we get

$$\int_0^\infty \int_0^\infty (a_1^1 x_1 + a_2^1 x_2)^{\sigma_1 - 1} (a_1^2 x_1 + a_2^2 x_2)^{\sigma_2 - 1} exp(-t_1(a_1^1 x_1 + a_2^1 x_2) - t_2(a_1^2 x_1 + a_2^2 x_2))$$

$$S_{N_1}^{M_1}(a_1^1 x_1 + a_2^1 x_2) S_{N_2}^{M_2}(a_1^2 x_1 + a_2^2 x_2) \mathrm{d}x_1 \mathrm{d}x_2$$

$$= \frac{1}{K} \int_0^\infty u_1^{\sigma_1 - 1} exp(-t_1 u_1) S_{N_1}^{M_1}(u_1) \mathrm{d}u_1 \int_0^\infty u_2^{\sigma_2 - 1} exp(-t_2 u_2) S_{N_2}^{M_2}(u_2) \mathrm{d}u_2 \tag{2.4}$$

On expressing the general class of polynomials occuring on the right hand side of (2.4) in terms of series with the help of (1.16) interchanging the order of integrals and summation in the result thus obtained and integrating the $u_1$and $u_2$ integrals, we arrive with the help of a known formula (Gradshteyn and Ryzhik [1] 1980, p.317, eq.(3.381(14)), we get the desired result.

**Second integral**

$$\int_0^\infty \int_0^\infty (a_1^1 x_1 + a_2^1 x_2)^{\sigma_1 - 1} (a_1^2 x_1 + a_2^2 x_2)^{\sigma_2 - 1} exp(-t_1(a_1^1 x_1 + a_2^1 x_2) - t_2(a_1^2 x_1 + a_2^2 x_2))$$

$$S_{N_1}^{M_1}(a_1^1 x_1 + a_2^1 x_2) S_{N_2}^{M_2}(a_1^2 x_1 + a_2^2 x_2) \; \aleph\big(z_1(a_1^1 x_1 + a_2^1 x_2)^{\rho_1}, z_2(a_1^2 x_1 + a_2^2 x_2)^{\rho_2}\big) \; \mathrm{d}x_1 \mathrm{d}x_2$$

$$= \frac{1}{K} \sum_{k_1=0}^{[N_1/M_1]} \sum_{k_2=0}^{[N_2/M_2]} \frac{(-N_1)_{M_1 k_1} (-N_2)_{M_2 k_2} A_{N_1,k_1} A_{N_2,k_2} \Gamma(\sigma_1 + k_1) \Gamma(\sigma_2 + k_2)}{k_1! k_2! t_1^{\sigma_1 + k_1} t_2^{\sigma_2 + k_2}}$$

$$\aleph_{U:W+1}^{0,\mathfrak{n}:V+1} \left( \begin{array}{c} \mathrm{z}_1 t_1^{-\rho_1} \\ \cdot \\ \cdot \\ \mathrm{z}_2 t_2^{-\rho_2} \end{array} \middle| \begin{array}{lll} , \mathrm{A} : & (1\text{-}\sigma_1 - k_1; \rho_1), C_1 : & (1\text{-}\sigma_2 - k_2; \rho_2), C_2 \\ \cdots & \cdots & \cdots \\ \mathrm{B} : & \cdots\cdots & \mathrm{D} \end{array} \right) \tag{2.5}$$

where $V + 1 = m_1, n_1 + 1; m_2, n_2 + 1$ and $W + 1 = p_{i^{(1)}} + 1, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, p_{i^{(2)}} + 1, q_{i^{(2)}}, \tau_{i^{(2)}}; R^{(2)}$

Provided that

$$\rho_i > 0, Re(t_i) > 0, i = 1,2\,; Re[\sigma_i + \rho_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0\,, i = 1,2; \text{ and } K = \begin{vmatrix} a_1^1 & a_1^2 \\ a_2^1 & a_2^2 \end{vmatrix} \neq 0$$

**Proof of (2.5)**

If we first express the Aleph-function of two variables occuring in the left hand side of (2.5) in terms of mellin-barnes type contour integral, it reduces of interchanging the $(s_1, s_2)$double integral and $(x_1, x_2)$ double integral to the following result after a slight simplification

$$\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\psi(s_1,s_2)\,\theta_1(s_1)\theta_2(s_2)\,z_1^{s_1}z_2^{s_2}\int_0^\infty\int_0^\infty exp(-t_1(a_1^1x_1+a_2^1x_2)-t_2(a_1^2x_1+a_2^2x_2))$$

$$(a_1^1x_1+a_2^1x_2)^{\rho_1 s_1+\sigma_1-1}(a_1^2x_1+a_2^2x_2)^{\rho_2 s_2+\sigma_2-1}S_{N_1}^{M_1}(a_1^1x_1+a_2^1x_2)S_{N_2}^{M_2}(a_1^2x_1+a_2^2x_2)\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}s_1\mathrm{d}s_2$$

Now evaluating the inner $(x_1, x_2)$ double integral with the help of (2 .1) and reinterpreting the result so obtained in terms of the Aleph-function of two variables, we get the desired result after a little simplification.

**Third integral**

Let $X_1 = \sum_{i=1}^{n} \lambda_i^1 x_i\,, \cdots, X_r = \sum_{i=1}^{r}\lambda_i^r x_i$ , we have the following result

$$\int_0^\infty\cdots\int_0^\infty X_1^{\sigma_1-1}\cdots X_r^{\sigma_r-1}exp(-\sum_{i=1}^r t_iX_i)\; S_{N_1,\cdots,N_r}^{M_1,\cdots,M_r}[e_1X_1^{v_1},\cdots,e_rX_r^{v_r}]$$

$$\aleph(z_1X_1^{\rho_1},\cdots,z_rX_r^{\rho_r})\}\mathrm{d}x_1\cdots\mathrm{d}x_r = \frac{1}{K}\sum_{k_1=0}^{[N_1/M_1]}\cdots\sum_{k_r=0}^{[N_r/M_r]} A\prod_{i=1}^r t_i^{-(\sigma_i+v_ik_i)}e_1^{k_1}\cdots e_r^{k_r}$$

$$\aleph_{U:W+1}^{0,\mathfrak{n}:V+1}\left(\begin{array}{c} \mathrm{z}_1 t_1^{-\rho_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r t_r^{-\rho_r}\end{array}\middle| \begin{array}{l} \mathrm{A}:\ (1\text{-}\sigma_1 - v_1k_1;\rho_1), C_1 : \cdots :(1\text{-}\sigma_r - v_rk_r;\rho_r), C_r \\ \cdot\ \cdot\ \cdot \qquad\qquad \cdot\ \cdot\ \cdot \qquad\qquad \cdot\ \cdot\ \cdot \\ \mathrm{B}: \qquad\qquad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \qquad\qquad \mathrm{D} \end{array}\right) \qquad (2.6)$$

where $A$ is defined by (1.15) ; $K = \begin{vmatrix} \lambda_1^1 & \cdots & \lambda_r^1 \\ \cdots & & \\ \lambda_1^r & \cdots & \lambda_r^r \end{vmatrix} \neq 0\,; V+1 = m_1, n_1+1;\cdots; m_r, n_r+1$

and $W+1 = p_{i^{(1)}}+1, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}+1, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)}$

Provided that $\rho_i > 0, Re(t_i) > 0, i = 1,2\,; Re[\sigma_i + \rho_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0\,, i = 1,\cdots,r$

**Proof of (2.6)**

On making use the result given below (which is a $r$-variable analogue of (2.2))

$$\int_0^\infty\cdots\int_0^\infty F(X_1,\cdots,X_r)\mathrm{d}x_1\cdots\mathrm{d}x_r = \frac{1}{K}\int_0^\infty\cdots\int_0^\infty F(u_1,\cdots,,u_r)\,\mathrm{d}u_1\cdots\mathrm{d}u_r$$

Taking the definition of generalized polynomial given by (1.14) into consideration and proceeding in a manner indicated early in the proof of (2.5), we arrive at the desired result after simplications.

On account of the general nature of multivariable Aleph-function and the generalized class of polynomial $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1,\cdots,x_s]$, the result given by (2.6) is capable of yielding numerous integrals involving products of several special functions and simple polynomials.

**Special case**

Let $.\tau_i = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = 1$ The Multivariable Aleph-function degenere to the multivariable I-function defined by Sharma et al [2], and we obtain

$$\int_0^\infty \cdots \int_0^\infty X_1^{\sigma_1 - 1} \cdots X_r^{\sigma_r - 1} exp(-\sum_{i=1}^r t_i X_i)\, S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[e_1 X_1^{v_1}, \cdots, e_r X_r^{v_r}]$$

$$I(z_1 X_1^{\rho_1}, \cdots, z_r X_r^{\rho_r})\}\mathrm{d}x_1 \cdots \mathrm{d}x_r = \frac{1}{K} \sum_{k_1=0}^{[N_1/M_1]} \cdots \sum_{k_r=0}^{[N_r/M_r]} A \prod_{i=1}^r t_i^{-(\sigma_i + v_i k_i)} e_1^{k_1} \cdots e_r^{k_r}$$

$$I_{U:W+1}^{0,\mathfrak{n}:V+1}\left(\begin{array}{c} \mathrm{z}_1 t_1^{-\rho_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r t_r^{-\rho_r} \end{array} \middle| \begin{array}{c} \mathrm{A} : (1\text{-}\sigma_1 - v_1 k_1; \rho_1), C_1 : \cdots : (1\text{-}\sigma_r - v_r k_r; \rho_r), C_r \\ \cdots \qquad \cdots \qquad \cdots \\ \mathrm{B} : \qquad \cdots\cdots \qquad \mathrm{D} \end{array}\right) \tag{2.7}$$

where $A$ is defined by (1.15) ; $K = \begin{vmatrix} \lambda_1^1 & \cdots & \lambda_r^1 \\ \cdots & & \\ \lambda_1^r & \cdots & \lambda_r^r \end{vmatrix} \neq 0$ ; $V + 1 = m_1, n_1 + 1; \cdots; m_r, n_r + 1$

and $W + 1 = p_{i^{(1)}} + 1, q_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}} + 1, q_{i^{(r)}}; R^{(r)}$

Provided that $\rho_i > 0, Re(t_i) > 0$, $i = 1, 2$ ; $Re[\sigma_i + \rho_i \min_{1 \leqslant j \leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$ , $i = 1, \cdots, r$

## 3. Conclusion

Due to the nature of the multivariable Aleph-function and the general class of polynomials $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}$ , we can get general product of Laguerre, Legendre, Jacobi and other polynomials, the special functions of one and several variables.

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