Finite integrals involving product of multivariable Jacobi polynomial and Aleph-function of several variables

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Abstract
In the present paper, few finite integrals involving products of multivariable Jacobi polynomial and Aleph-function of several variables of generalized arguments have been evaluated. These integrals have been utilized to established the expansion formula for multivariable Aleph-function in series involving product of multivariable Jacobi polynomials since multivariable Aleph-function quite general function in nature. On specializing the parameters of the function involving in results many news relations may be obtained as particular cases.

Keywords Multivariable Aleph-function, Multivariable I-function, Aleph-function of two variables, Multivariable Jacobi polynomial.

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1. Introduction and preliminaries.

The object of this document is to study a number of a general integrals involving the multivariables aleph-function. These function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [5]. The generalized multivariable I-function is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $$\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\right.$$

$$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}],[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$$
$$\cdots\cdots\cdots,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$$

$$\left.\begin{array}{c}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\phi_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

For more details, see Ayant [1].
The reals numbers τ_i are positives for $i=1,\cdots,R$, $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1,\cdots,R^{(k)}$
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)}=\sum_{j=1}^{n}\alpha_j^{(k)}-\tau_i\sum_{j=n+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0 \quad ,\text{with } k=1 \text{ to } r\ ,i=1 \text{ to } R\ ,i^{(k)}=1 \text{ to } R^{(k)} \tag{1.2}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \ ; \ V = m_1, n_1; \cdots ; m_r, n_r \tag{1.3}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots , p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.4}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots , \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots , \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.5}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots , \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.6}$$

$$C_1 = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots ,$$

$$C_r = \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.7}$$

$$D_1 = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots,$$

$$D_r = \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.8}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots , z_r) = \aleph_{U:W}^{0,\mathrm{n}:V} \left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A} \ ; \ \mathrm{C}_1; \cdots ; C_r \\ \cdot & \cdots \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B} \ ; \ \mathrm{D}_1; \cdots ; D_r \end{array} \right) \tag{1.9}$$

2. Multivariable Jacobi polynomial

The Jacobi polynomial for one variable is defined as [4; eq.(1), p.254].

$$P_n^{(a,b)}(x) = \frac{(1+a)_n}{n!} {}_2F_1 \left(\begin{array}{c} \text{-n,1+a+b+n} \\ \cdots \\ \text{1+a} \end{array} ; \frac{1-x}{2} \right) \tag{2.1}$$

The Jacobi polynomial of two variables and several variables is defined by Shrivastava [7 ,eq.(1.3), (1.7) p.159-161] in the following manner.

$$P_n^{(a_1,b_1;a_2,b_2)}(x_1, x_2) = \frac{(1+a_1)_n(1+a_2)_n}{(n!)^2} F_2 \left(\begin{array}{c} \text{-n;1+a}_2 + b_2 + n; 1 + a_1 + b_1 + n \\ \cdots \\ \text{1+a}_2; 1 + a_1 \end{array} ; \frac{1-x_2}{2}, ; \frac{1-x_1}{2} \right) \tag{2.2}$$

and $$P_n^{(a_1,b_1;\cdots;a_r,b_r)}(x_1, \cdots , x_r) = \prod_{i=1}^{r} \frac{(1+a_i)_n}{n!}$$

$$\times F_{0;1;\cdots;1}^{1;1;\cdots;1} \left(\begin{array}{c} \text{-n;1+a}_r + b_r + n; \cdots ; 1 + a_1 + b_1 + n \\ \cdots \\ \text{1+a}_r; \cdots ; 1 + a_1 \end{array} ; \frac{1-x_r}{2}, \cdots , \frac{1-x_1}{2} \right) \tag{2.3}$$

In this document, we will use the following result [1,p.58]

First integral

$$\int_{-1}^{1}\cdots\int_{-1}^{1}\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1-x_i)^{a_i}(1+x)^{\sigma_i}\right]P_n^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)\mathrm{d}x_1\cdots\mathrm{d}x_r$$

$$=\frac{1}{(n!)^r}\prod_{i=1}^{r}\frac{(-)^{ni}2^{\sigma_i+a_i+\frac{1}{r}}\Gamma(a_i+n+1)\Gamma(\sigma_i-b_i+1)\Gamma(\sigma_i+1)}{\Gamma(\sigma_i-b_i-n+1)\Gamma(\sigma_i+a_i+n+2)}$$

$$\times\prod_{i=1}^{r}{}_3F_2\left(\begin{array}{c}-\lambda_i,\sigma_i-b_i+1,\sigma_i+1\\ \cdots\\ \sigma_i-b_i-n+1,\sigma_i+a_i+n+2\end{array};2\right) \tag{2.4}$$

Second integral

$$\int_{-1}^{1}\cdots\int_{-1}^{1}\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1-x_i)^{\rho_i}(1+x)^{b_i}\right]P_n^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)\mathrm{d}x_1\cdots\mathrm{d}x_r$$

$$=\frac{1}{(n!)^r}\prod_{i=1}^{r}\frac{(-)^{ni}2^{\rho_i+b_i+\frac{1}{r}}\Gamma(b_i+n+1)\Gamma(\rho_i-b_i+1)\Gamma(\rho_i+1)}{\Gamma(\rho_i-a_i-n+1)\Gamma(\rho_i+b_i+n+2)}$$

$$\times\prod_{i=1}^{r}{}_3F_2\left(\begin{array}{c}-\lambda_i,\rho_i-b_i+1,\rho_i+1\\ \cdots\\ \rho_i-a_i-n+1,\rho_i+b_i+n+2\end{array};2\right) \tag{2.4}$$

3. Main integrals

In these section we evaluate four multiple integrals involving the Aleph-function of several variables and multivariable Jacobi polynomial. Throughout this paper, we note :

$$W_{33}=p_{i^{(1)}}+3,q_{i^{(1)}}+3,\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}}+3,q_{i^{(r)}}+3,\tau_{i^{(r)}};R^{(r)}$$

and $V+3=m_1,n_1+3;\cdots;m_r,n_r+3$

a) $$\int_{-1}^{1}\cdots\int_{-1}^{1}\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1-x_i)^{a_i}(1+x)^{\sigma_i}\right]P_n^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)$$

$$\aleph\left(z_1\left(\frac{1+x_1}{x_1}\right)^{\mu_1},\cdots,z_r\left(\frac{1+x_r}{x_r}\right)^{\mu_r}\right)\mathrm{d}x_1\cdots\mathrm{d}x_r=\frac{1}{(n!)^r}\prod_{i=1}^{r}\left[(-)^{ni}2^{\sigma_i+a_i+\frac{1}{r}}\Gamma(a_i+n+1)\right]$$

$$\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c}2^{\mu_1}z_1\\ \cdot\\ \cdot\\ 2^{\mu_r}z_r\end{array}\middle|\begin{array}{c}\mathrm{A}:\mathrm{C}_1,(\lambda_1+1-k_1;\mu_1),\ \ (\mathrm{b}_1-\sigma_1-k_1;\mu_1),\\ \cdots\qquad\cdots\\ \mathrm{B}:\mathrm{D}_1,(\lambda_1+1;\mu_1),\ \ (\mathrm{b}_1-\sigma_1+n-k_1;\mu_1),\end{array}\right.$$

$$\begin{array}{cc}(-\sigma_1-k_1;\mu_1);\cdots; & \mathrm{C}_r,(\lambda_r+1-k_r;\mu_r),\\ \cdots & \cdots\\ (-1-\sigma_1-a_1-n-k_1;\mu_1);\cdots; & \mathrm{D}_r,(\lambda_r+1;\mu_r),\end{array}$$

$$\left.\begin{matrix} (\mathrm{b}_r-\sigma_r-k_r;\mu_r),(-\sigma_r-k_r;\mu_r) \\ \cdots \\ (\mathrm{b}_r-\sigma_r+n-k_r;\mu_r),(-1-\sigma_r-a_r-n-k_r;\mu_r) \end{matrix}\right) \tag{3.1}$$

Provided that

a) $Re(\lambda_i)>-1, Re(a_i)>-1, \mu_i>0, i=1,\cdots,r$

b) $Re[\sigma_i+\mu_j \min\limits_{1\leqslant j\leqslant m_j}\frac{d_i^{(j)}}{\delta_i^{(j)}}]>0\,; Re[\lambda_i-\mu_j \min\limits_{1\leqslant j\leqslant m_j}\frac{d_i^{(j)}}{\delta_i^{(j)}}]>0\,, i,j=1,\cdots,r$

c) $|argz_i|<\frac{1}{2}A_i^{(k)}\pi\,,$ where $A_i^{(k)}$ is given in (1.2)

b) $\int_{-1}^{1}\cdots\int_{-1}^{1}\prod_{i=1}^{r}[x_i^{\lambda_i}(1-x_i)^{a_i}(1+x)^{\sigma_i}]P_n^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)$

$$\aleph(z_1x_1^{\mu_1}(1+x_1)^{\delta_1},\cdots,z_rx_r^{\mu_r}(1+x_r)^{\delta_r})\mathrm{d}x_1\cdots\mathrm{d}x_r=\frac{1}{(n!)^r}\prod_{i=1}^{r}[(-)^{ni}2^{\sigma_i+a_i+\frac{1}{r}}\Gamma(a_i+n+1)]$$

$$\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{matrix}2^{\delta_1}z_1\\ \cdot\\ \cdot\\ 2^{\delta_r}z_r\end{matrix}\left|\begin{matrix}\mathrm{A}:\mathrm{C}_1,(-\lambda_1+k_1;\mu_1), & (\mathrm{b}_1-\sigma_1-k_1;\delta_1),\\ \cdots & \cdots\\ \mathrm{B}:\mathrm{D}_1,(-\lambda_1;\mu_1), & (\mathrm{b}_1-\sigma_1+n-k_1;\delta_1),\end{matrix}\right.\right.$$

$$\begin{matrix}(\text{-}\sigma_1-k_1;\delta_1);\cdots; & \mathrm{C}_r,(-\lambda_r+k_r;\mu_r),\\ \cdots & \cdots\\ (\text{-}1\text{-}\sigma_1-a_1-n-k_1;\delta_1);\cdots; & \mathrm{D}_r,(-\lambda_r;\mu_r),\end{matrix}$$

$$\left.\begin{matrix} (\mathrm{b}_r-\sigma_r-k_r;\delta_r),(-\sigma_r-k_r;\delta_r) \\ \cdots \\ (\mathrm{b}_r-\sigma_r+n-k_r;\delta_r),(-1-\sigma_r-a_r-n-k_r;\delta_r) \end{matrix}\right) \tag{3.2}$$

Provided that

a) $Re(a_i)>1, \mu_i>0, \delta>0, i=1,\cdots,r$

b) $Re[\sigma_i+\delta_j \min\limits_{1\leqslant j\leqslant m_j}\frac{d_i^{(j)}}{\delta_i^{(j)}}]>0\,; Re[\lambda_i+\mu_j \min\limits_{1\leqslant j\leqslant m_j}\frac{d_i^{(j)}}{\delta_i^{(j)}}]>0\,, i,j=1,\cdots,r$

c) $|argz_i|<\frac{1}{2}A_i^{(k)}\pi\,,$ where $A_i^{(k)}$ is given in (1.2)

c) $\int_{-1}^{1}\cdots\int_{-1}^{1}\prod_{i=1}^{r}[x_i^{\lambda_i}(1-x_i)^{\rho_i}(1+x)^{b_i}]P_n^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)$

$$\aleph(z_1x_1^{\mu_1}(1-x_1)^{\delta_1},\cdots,z_rx_r^{\mu_r}(1-x_r)^{\delta_r})\mathrm{d}x_1\cdots\mathrm{d}x_r=\frac{1}{(n!)^r}\prod_{i=1}^{r}[(-)^{ni}2^{\sigma_i+a_i+\frac{1}{r}}\Gamma(b_i+n+1)]$$

$$\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c}2^{\delta_1}z_1\\ \cdot\\ \cdot\\ 2^{\delta_r}z_r\end{array}\middle|\begin{array}{l}\mathrm{A}:\mathrm{C}_1,(-\lambda_1+k_1;\mu_1),\quad (\mathrm{b}_1-\rho_1-k_1;\delta_1),\\ \quad\cdots\qquad\qquad\qquad\cdots\\ \mathrm{B}:\mathrm{D}_1,(-\lambda_1;\mu_1),\quad (\mathrm{a}_1-\rho_1+n-k_1;\delta_1),\end{array}\right.$$

$$\begin{array}{l}(\text{-}\rho_1-k_1;\delta_1);\cdots;\qquad \mathrm{C}_r,(-\lambda_r+k_r;\mu_r),\\ \qquad\cdots\qquad\qquad\qquad\cdots\\ (\text{-}1\text{-}\rho_1-b_1-n-k_1;\delta_1);\cdots;\quad \mathrm{D}_r,(-\lambda_r;\mu_r),\end{array}$$

$$\left.\begin{array}{c}(\mathrm{b}_r-\rho_r-k_r;\delta_r),(-\rho_r-k_r;\delta_r)\\ \cdots\\ (\mathrm{a}_r-\rho_r+n-k_r;\delta_r),(-1-\rho_r-b_r-n-k_r;\delta_r)\end{array}\right) \tag{3.2}$$

Provided that

a)$Re(\lambda_i)>-1, Re(b_i)>-1, \mu_i>0, i=1,\cdots,r$

b) $Re[\rho_i+\delta_j\min\limits_{1\leqslant j\leqslant m_j}\frac{d_i^{(j)}}{\delta_i^{(j)}}]>0\,; Re[\lambda_i+\mu_j\min\limits_{1\leqslant j\leqslant m_j}\frac{d_i^{(j)}}{\delta_i^{(j)}}]>0\,, i,j=1,\cdots,r$

c)$|argz_i|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.2)

d) $\int_{-1}^{1}\cdots\int_{-1}^{1}\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1-x_i)^{\rho_i}(1+x)^{b_i}\right]P_n^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)$

$$\aleph\big(z_1\big(\frac{1-x_1}{x_1}\big)^{\mu_1},\cdots,z_r\big(\frac{1-x_r}{x_r}\big)^{\mu_r}\big)\mathrm{d}x_1\cdots\mathrm{d}x_r=\frac{1}{(n!)^r}\prod_{i=1}^{r}\left[(-)^{ni}2^{\sigma_i+a_i+\frac{1}{r}}\Gamma(b_i+n+1)\right]$$

$$\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c}2^{\mu_1}z_1\\ \cdot\\ \cdot\\ 2^{\mu_r}z_r\end{array}\middle|\begin{array}{l}\mathrm{A}:\mathrm{C}_1,(\lambda_1-k_1;\mu_1),\quad (\mathrm{b}_1-\rho_1-k_1;\mu_1),\\ \quad\cdots\qquad\qquad\qquad\cdots\\ \mathrm{B}:\mathrm{D}_1,(-\lambda_1-1;\mu_1),(\mathrm{a}_1-\rho_1+n-k_1;\mu_1),\end{array}\right.$$

$$\begin{array}{l}\mathrm{C}_r,(\lambda_r-k_1;\mu_r),\qquad (\text{-}\rho_1-k_1;\mu_1);\cdots;\\ \qquad\cdots\qquad\qquad\qquad\cdots\\ \mathrm{D}_r,(-\lambda_r-1;\mu_r),(\text{-}1\text{-}\rho_1-b_1-n-k_1;\mu_1);\cdots;\end{array}$$

$$\left.\begin{array}{c}(\mathrm{b}_r-\rho_r-k_1;\mu_r),(-\rho_r-k_r;\mu_r)\\ \cdots\\ (\mathrm{a}_r-\rho_r+n-k_r;\mu_r),(-1-\rho_r-b_r-n-k_r;\mu_r)\end{array}\right) \tag{3.1}$$

Provided that

a)$Re(\lambda_i)>-1, Re(b_i)>-1, \mu_i>0, i=1,\cdots,r$

b) $Re[\rho_i+\mu_j\min\limits_{1\leqslant j\leqslant m_j}\frac{d_i^{(j)}}{\delta_i^{(j)}}]>0\,; Re[\lambda_i-\mu_j\min\limits_{1\leqslant j\leqslant m_j}\frac{d_i^{(j)}}{\delta_i^{(j)}}]>0\,, i,j=1,\cdots,r$

c) $|argz_i| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.2)

Proof of (3.1) : Let $M = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\phi_k(s_k)$

To establish (3.1), expressing the multivariable Aleph-function on L.H.S. as Mellin-Barnes contour integral (1.1) and then change the order of integration (which is permissible under the stated conditions). We get the following result.

$$M\{\frac{z_1^{s_1}\cdots z_r^{s_r}}{(n!)^r}\int_{-1}^{1}\cdots\int_{-1}^{1}\prod_{i=1}^{r}[x_i^{\lambda_i-\mu_i s_i}(1-x_i)^{a_i}(1+x)^{\sigma_i+\mu_i s_i}]P_n^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_r\}\mathrm{d}s_1\cdots\mathrm{d}s_r$$

Evaluate the inner multiple integral with the help (2.4), we get the following result.

$$M\{\frac{z_1^{s_1}\cdots z_r^{s_r}}{(n!)^r}\prod_{i=1}^{r}\frac{(-)^{ni}2^{\sigma_i+a_i+\mu_i s_i+\frac{1}{r}}\Gamma(a_i+n+1)\Gamma(\sigma_i+\mu_i s_i-b_i+1)\Gamma(\sigma_i+\mu_i s_i+1)}{\Gamma(\sigma_i+\mu_i s_i-b_i-n+1)\Gamma(\sigma_i+\mu_i s_i+a_i+n+2)}$$

$$\times\prod_{i=1}^{r}{}_3F_2\left(\begin{array}{c}\text{-}\lambda_i+\mu_i s_i,\sigma_i+\mu_i s_i-b_i+1,\sigma_i+\mu_i s_i+1\\ \cdots\\ \sigma_i+\mu_i s_i-b_i-n+1,\sigma_i+a_i+\mu_i s_i+n+2\end{array};2\right)\Bigg\}\mathrm{d}s_1\cdots\mathrm{d}s_r$$

Now we represent the hypergeometric function into series and addition and integration [2,p.176 (75)] which is valid under the stated conditions.and on reinterpreting the Mellin-Barnes contour integral in the R.H.S. of (3.1) in term of the multivariable Aleph-function given by (1.1), we arrive at the desired result. To prove (3.2) to (3.4), we use the similar methods.

4. Expansion

The objective of these section is to apply the result of (3.1) to (3.4) and established four expansion formulas.

a) $$\prod_{i=1}^{r}[x_i^{\lambda_i}(1+x_i)^{\sigma_i}]\aleph(z_1(\frac{1+x_1}{x_1})^{\mu_1},\cdots,z_r(\frac{1+x_r}{x_r})^{\mu_r})=\prod_{i=1}^{r}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c}2^{\mu_1}z_1\\ \cdot\\ \cdot\\ 2^{\mu_r}z_r\end{array}\right|$$

$$\begin{array}{l}\mathrm{A}:\mathrm{C}_1,(\lambda_1+1-k_1;\mu_1),\quad(\text{-a}_1-k_1;\mu_1),(\sigma_1+b_1-k_1;\mu_1);\cdots;\\ \quad\cdots\qquad\qquad\qquad\qquad\cdots\\ \mathrm{B}:\mathrm{D}_1,(\lambda_1+1;\mu_1),\quad(1\text{-}\sigma_1-k_1;\mu_1),(1-l-\sigma_1-k_1;\mu_1);\cdots;\end{array}$$

$$\left.\begin{array}{l}\mathrm{C}_r,(\lambda_r+1-k_r;\mu_r),\quad(\text{-a}_r-k_r;\mu_r),(\sigma_r+b_r-k_r;\mu_r)\\ \quad\cdots\qquad\qquad\qquad\cdots\\ \mathrm{D}_r,(\lambda_r+1;\mu_r),\quad(1\text{-}\sigma_r-k_r;\mu_r),(1-\sigma_r-k_r-l;\mu_r)\end{array}\right) \quad (4.1)$$

the equation (4.1) is valid under the same conditions mentioned in (3.1).

b) $$\prod_{i=1}^{r}[x_i^{\lambda_i}(1+x_i)^{\sigma_i}]\aleph(z_1x_1^{\mu_1}(1+x_1)^{\delta_1},\cdots,z_rx_r^{\mu_r}(1+x_r)^{\delta_r})=\prod_{i=1}^{r}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c} 2^{\delta_1}z_1 \\ \cdot \\ \cdot \\ 2^{\delta_r}z_r \end{array}\middle| \begin{array}{l} \mathrm{A}:\mathrm{C}_1,(-\lambda_1;\mu_1),\ (\text{-}\sigma_1-k_1;\delta_1),(-\sigma_1-b_1-k_1;\delta_1);\cdots; \\ \cdots \\ \mathrm{B}:\mathrm{D}_1,(k_1-\lambda_1;\mu_1),\ (1\text{-}\sigma_1-k_1;\delta_1),(-1-l-\sigma_1;\delta_1);\cdots; \end{array}\right.$$

$$\left.\begin{array}{l} \mathrm{C}_r,(-\lambda_r;\mu_r),\ (\text{-}\sigma_r-k_r;\delta_r),(-\sigma_r-b_r-k_r;\delta_r) \\ \cdots \\ \mathrm{D}_r,(k_r-\lambda_r;\mu_r),\ (1\text{-}\sigma_r-k_r;\delta_r),(-1-l-\sigma_r;\delta_r) \end{array}\right) \tag{4.2}$$

the equation (4.2) is valid under the same conditions mentioned in (3.2).

c) $$\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1-x_i)^{\sigma_i}\right]\aleph\left(z_1x_1^{\mu_1}(1-x_1)^{\delta_1},\cdots,z_rx_r^{\mu_r}(1-x_r)^{\delta_r}\right)=\prod_{i=1}^{r}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c} 2^{\delta_1}z_1 \\ \cdot \\ \cdot \\ 2^{\delta_r}z_r \end{array}\middle| \begin{array}{l} \mathrm{A}:\mathrm{C}_1,(-\lambda_1;\mu_1),\ (\text{-}\rho_1+k_1;\delta_1),(-\rho_1-l-k_1;\delta_1);\cdots; \\ \cdots \\ \mathrm{B}:\mathrm{D}_1,(k_1-\lambda_1;\mu_1),(1\text{-}\rho_1-k_1;\delta_1),(1-l-a_1-b_1-\rho_1-k_1;\delta_1);\cdots; \end{array}\right.$$

$$\left.\begin{array}{l} \mathrm{C}_r,(-\lambda_r;\mu_r),\ (\text{-}\rho_r+k_r;\delta_r),(-\rho_r-l-k_r;\delta_r) \\ \cdots \\ \mathrm{D}_r,(k_r-\lambda_r;\mu_r),(1\text{-}\rho_r-k_r;\delta_r),(1-l-a_r-b_r-\rho_r-k_r;\delta_r) \end{array}\right) \tag{4.3}$$

the equation (4.3) is valid under the same conditions mentioned in (3.3).

d) $$\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1-x_i)^{\sigma_i}\right]\aleph\left(z_1\left(\frac{1-x_1}{x_1}\right)^{\mu_1},\cdots,z_r\left(\frac{1-x_r}{x_r}\right)^{\mu_r}\right)=\prod_{i=1}^{r}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c} 2^{\mu_1}z_1 \\ \cdot \\ \cdot \\ 2^{\mu_r}z_r \end{array}\middle| \begin{array}{l} \mathrm{A}:\mathrm{C}_1,(\lambda_1+1-k_1;\mu_1),\ (\rho_1-k_1;\mu_1),(\rho_1+a_1-k_1;\mu_1);\cdots; \\ \cdots \\ \mathrm{B}:\mathrm{D}_1,(\lambda_1+1;\mu_1),\ (1\text{-}\rho_1-k_1;\mu_1),(1-\rho_1-k_1-b-l;\mu_1);\cdots; \end{array}\right.$$

$$\left.\begin{array}{l} \mathrm{C}_r,(\lambda_r+1-k_r;\mu_r),\ (\rho_r-k_r;\mu_r),(\rho_r+a_r-k_r;\mu_r) \\ \cdots \\ \mathrm{D}_r,(\lambda_r+1;\mu_r),\ (1\text{-}\rho_r-k_r;\mu_r),(1-\rho_r-k_r-b_r-l;\mu_r) \end{array}\right) \tag{4.4}$$

the equation (4.4) is valid under the same conditions mentioned in (3.4).

Proof

To prove (4.1), Let

$$\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1+x_i)^{\sigma_i}\right]\aleph\left(z_1\left(\frac{1+x_1}{x_1}\right)^{\mu_1},\cdots,z_r\left(\frac{1+x_r}{x_r}\right)^{\mu_r}\right)=\sum_{i=1}^{\infty}a_i P_i^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r) \quad (4.5)$$

where a_i is a constant. For finding this we multiply $\prod_{i=1}^{r}\left[(1-x_i)^{a_i}(1+x_i)^{b_i}\right]$ on both sides of the equation (4.5) and then integrate in the limit -1 and $+1$. Lastly in the left side of equation (3.1) is recalculated and on right side , use the orthogonal property of Jacobi polynomial [3,p.285 (5) and (9)] , we obtain the desired result.

To prove (4.2) to (4.4) , we use the similar methods.

5. Multivariable I-function

If $\tau_i=\tau_{i^{(1)}}=\tau_{i^{(r)}}=1$, the multivariable Aleph-function degenere to the multivariable I-function defined by Sharma et al [5]. In this section , we establish four expansion formulas concerning the multivariable I-function.

a) $$\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1+x_i)^{\sigma_i}\right]I\left(z_1\left(\frac{1+x_1}{x_1}\right)^{\mu_1},\cdots,z_r\left(\frac{1+x_r}{x_r}\right)^{\mu_r}\right)=\prod_{i=1}^{r}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)I_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c}2^{\mu_1}z_1\\ \cdot\\ \cdot\\ 2^{\mu_r}z_r\end{array}\middle|\right.$$

$$\mathrm{A}:\mathrm{C}_1,(\lambda_1+1-k_1;\mu_1),\ (\text{-a}_1-k_1;\mu_1),(\sigma_1+b_1-k_1;\mu_1);\cdots;$$
$$\cdots \qquad \cdots$$
$$\mathrm{B}:\mathrm{D}_1,(\lambda_1+1;\mu_1),\ (1\text{-}\sigma_1-k_1;\mu_1),(1-l-\sigma_1-k_1;\mu_1);\cdots;$$

$$\left.\begin{array}{l}\mathrm{C}_r,(\lambda_r+1-k_r;\mu_r),\ (\text{-a}_r-k_r;\mu_r),(\sigma_r+b_r-k_r;\mu_r)\\ \cdots \qquad \cdots\\ \mathrm{D}_r,(\lambda_r+1;\mu_r),\ (1\text{-}\sigma_r-k_r;\mu_r),(1-\sigma_r-k_r-l;\mu_r)\end{array}\right) \quad (5.1)$$

the equation (4.1) is valid under the same conditions mentioned in (3.1).

b) $$\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1+x_i)^{\sigma_i}\right]I\left(z_1x_1^{\mu_1}(1+x_1)^{\delta_1},\cdots,z_rx_r^{\mu_r}(1+x_r)^{\delta_r}\right)=\prod_{i=1}^{r}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)I_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c}2^{\delta_1}z_1\\ \cdot\\ \cdot\\ 2^{\delta_r}z_r\end{array}\middle|\right.$$

$$\mathrm{A}:\mathrm{C}_1,(-\lambda_1;\mu_1),\ (\text{-}\sigma_1-k_1;\delta_1),(-\sigma_1-b_1-k_1;\delta_1);\cdots;$$
$$\cdots \qquad \cdots$$
$$\mathrm{B}:\mathrm{D}_1,(k_1-\lambda_1;\mu_1),(1\text{-}\sigma_1-k_1;\delta_1),(-1-l-\sigma_1;\delta_1);\cdots;$$

$$\left.\begin{matrix} \mathrm{C}_r,(-\lambda_r;\mu_r), & (\text{-}\sigma_r-k_r;\delta_r),(-\sigma_r-b_r-k_r;\delta_r) \\ \cdots & \cdots \\ \mathrm{D}_r,(k_r-\lambda_r;\mu_r), & (1\text{-}\sigma_r-k_r;\delta_r),(-1-l-\sigma_r;\delta_r) \end{matrix}\right) \tag{5.2}$$

the equation (4.2) is valid under the same conditions mentioned in (3.2).

c) $$\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1-x_i)^{\sigma_i}\right]I\left(z_1x_1^{\mu_1}(1-x_1)^{\delta_1},\cdots,z_rx_r^{\mu_r}(1-x_r)^{\delta_r}\right)=\prod_{i=1}^{r}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)I_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{matrix}2^{\delta_1}z_1\\ \cdot\\ \cdot\\ 2^{\delta_r}z_r\end{matrix}\right|$$

$$\begin{matrix} \mathrm{A}:\mathrm{C}_1,(-\lambda_1;\mu_1), & (\text{-}\rho_1+k_1;\delta_1),(-\rho_1-l-k_1;\delta_1);\cdots; \\ \cdots & \cdots \\ \mathrm{B}:\mathrm{D}_1,(k_1-\lambda_1;\mu_1), & (1\text{-}\rho_1-k_1;\delta_1),(1-l-a_1-b_1-\rho_1-k_1;\delta_1);\cdots; \end{matrix}$$

$$\left.\begin{matrix} \mathrm{C}_r,(-\lambda_r;\mu_r), & (\text{-}\rho_r+k_r;\delta_r),(-\rho_r-l-k_r;\delta_r) \\ \cdots & \cdots \\ \mathrm{D}_r,(k_r-\lambda_r;\mu_r), & (1\text{-}\rho_r-k_r;\delta_r),(1-l-a_r-b_r-\rho_r-k_r;\delta_r) \end{matrix}\right) \tag{5.3}$$

the equation (4.3) is valid under the same conditions mentioned in (3.3).

d) $$\prod_{i=1}^{r}\left[x_i^{\lambda_i}(1-x_i)^{\sigma_i}\right]I\left(z_1\left(\frac{1-x_1}{x_1}\right)^{\mu_1},\cdots,z_r\left(\frac{1-x_r}{x_r}\right)^{\mu_r}\right)=\prod_{i=1}^{r}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\frac{2^{\sum_{i=1}^{r}k_i}}{k_1!\cdots k_r!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;\cdots;a_r,b_r)}(x_1,\cdots,x_r)I_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{matrix}2^{\mu_1}z_1\\ \cdot\\ \cdot\\ 2^{\mu_r}z_r\end{matrix}\right|$$

$$\begin{matrix} \mathrm{A}:\mathrm{C}_1,(\lambda_1+1-k_1;\mu_1), & (\rho_1-k_1;\mu_1),(\rho_1+a_1-k_1;\mu_1);\cdots; \\ \cdots & \cdots \\ \mathrm{B}:\mathrm{D}_1,(\lambda_1+1;\mu_1), & (1\text{-}\rho_1-k_1;\mu_1),(1-\rho_1-k_1-b-l;\mu_1);\cdots; \end{matrix}$$

$$\left.\begin{matrix} \mathrm{C}_r,(\lambda_r+1-k_r;\mu_r), & (\rho_r-k_r;\mu_r),(\rho_r+a_r-k_r;\mu_r) \\ \cdots & \cdots \\ \mathrm{D}_r,(\lambda_r+1;\mu_r), & (1\text{-}\rho_r-k_r;\mu_r),(1-\rho_r-k_r-b_r-l;\mu_r) \end{matrix}\right) \tag{5.4}$$

the equation (4.4) is valid under the same conditions mentioned in (3.4).

6. Aleph-function of two variables

If $r=2$, we obtain the Aleph-function of two variables defined by K.Sharma [6]. In this section , we establish four expansion formulas concerning the Aleph-function of two variables.

$$\text{a)}\ \prod_{i=1}^{2}[x_i^{\lambda_i}(1+x_i)^{\sigma_i}]\aleph\Big(z_1\big(\frac{1+x_1}{x_1}\big)^{\mu_1},z_2\big(\frac{1+x_2}{x_2}\big)^{\mu_r}\Big)=\prod_{i=1}^{2}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{2^{\sum_{i=1}^{2}k_i}}{k_1!k_2!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;a_2,b_2)}(x_1,x_2)\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c} 2^{\mu_1}z_1 \\ \cdot \\ \cdot \\ 2^{\mu_2}z_2 \end{array}\right|$$

$$\begin{array}{l} \mathrm{A}:\mathrm{C}_1,(\lambda_1+1-k_1;\mu_1),\quad (\text{-a}_1-k_1;\mu_1),(\sigma_1+b_1-k_1;\mu_1); \\ \quad\cdots \qquad\qquad\qquad \cdots \\ \mathrm{B}:\mathrm{D}_1,(\lambda_1+1;\mu_1),\quad (1\text{-}\sigma_1-k_1;\mu_1),(1-l-\sigma_1-k_1;\mu_1); \end{array}$$

$$\left.\begin{array}{l} \mathrm{C}_2,(\lambda_2+1-k_2;\mu_2),\quad (\text{-a}_2-k_2;\mu_r),(\sigma_2+b_2-k_2;\mu_2) \\ \quad\cdots \qquad\qquad\qquad \cdots \\ \mathrm{D}_2,(\lambda_2+1;\mu_2),\quad (1\text{-}\sigma_2-k_2;\mu_2),(1-\sigma_2-k_2-l;\mu_2) \end{array}\right) \tag{4.1}$$

the equation (4.1) is valid under the same conditions mentioned in (3.1).

$$\text{b)}\ \prod_{i=1}^{2}[x_i^{\lambda_i}(1+x_i)^{\sigma_i}]\ \aleph\big(z_1x_1^{\mu_1}(1+x_1)^{\delta_1},z_2x_2^{\mu_2}(1+x_2)^{\delta_2}\big)=\prod_{i=1}^{2}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{2^{\sum_{i=1}^{2}k_i}}{k_1!k_2!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;a_2,b_2)}(x_1,x_2)\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c} 2^{\delta_1}z_1 \\ \cdot \\ \cdot \\ 2^{\delta_2}z_2 \end{array}\right|$$

$$\begin{array}{l} \mathrm{A}:\mathrm{C}_1,(-\lambda_1;\mu_1),\quad (\text{-}\sigma_1-k_1;\delta_1),(-\sigma_1-b_1-k_1;\delta_1); \\ \quad\cdots \qquad\qquad\qquad \cdots \\ \mathrm{B}:\mathrm{D}_1,(k_1-\lambda_1;\mu_1),\ (1\text{-}\sigma_1-k_1;\delta_1),(-1-l-\sigma_1;\delta_1); \end{array}$$

$$\left.\begin{array}{l} \mathrm{C}_2,(-\lambda_2;\mu_2),\quad (\text{-}\sigma_2-k_2;\delta_2),(-\sigma_2-b_2-k_2;\delta_2) \\ \quad\cdots \qquad\qquad\qquad \cdots \\ \mathrm{D}_2,(k_2-\lambda_2;\mu_2),\ (1\text{-}\sigma_2-k_2;\delta_2),(-1-l-\sigma_2;\delta_2) \end{array}\right) \tag{4.2}$$

the equation (4.2) is valid under the same conditions mentioned in (3.2).

$$\text{c)}\ \prod_{i=1}^{2}[x_i^{\lambda_i}(1-x_i)^{\sigma_i}]\aleph\big(z_1x_1^{\mu_1}(1-x_1)^{\delta_1},z_2x_r^{\mu_2}(1-x_2)^{\delta_2}\big)=\prod_{i=1}^{2}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{2^{\sum_{i=1}^{2}k_i}}{k_1!k_2!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;a_2,b_2)}(x_1,x_2)\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{array}{c} 2^{\delta_1}z_1 \\ \cdot \\ \cdot \\ 2^{\delta_2}z_2 \end{array}\right|$$

$$\begin{array}{l} \mathrm{A}:\mathrm{C}_1,(-\lambda_1;\mu_1),\qquad (\text{-}\rho_1+k_1;\delta_1),(-\rho_1-l-k_1;\delta_1); \\ \quad\cdots \qquad\qquad\qquad \cdots \\ \mathrm{B}:\mathrm{D}_1,(k_1-\lambda_1;\mu_1),(1\text{-}\rho_1-k_1;\delta_1),(1-l-a_1-b_1-\rho_1-k_1;\delta_1); \end{array}$$

$$\left.\begin{matrix} \mathrm{C}_2,(-\lambda_2;\mu_2), & (\text{-}\rho_2+k_2;\delta_2),(-\rho_2-l-k_2;\delta_2) \\ \cdots & \cdots \\ \mathrm{D}_2,(k_2-\lambda_2;\mu_2), & (1\text{-}\rho_2-k_2;\delta_2),(1-l-a_2-b_2-\rho_2-k_2;\delta_2) \end{matrix}\right) \tag{4.3}$$

the equation (4.3) is valid under the same conditions mentioned in (3.3).

d) $$\prod_{i=1}^{2}\left[x_i^{\lambda_i}(1-x_i)^{\sigma_i}\right]\aleph\left(z_1\left(\frac{1-x_1}{x_1}\right)^{\mu_1},z_2\left(\frac{1-x_2}{x_2}\right)^{\mu_2}\right)=\prod_{i=1}^{2}2^{\sigma_i}(-1)^{l_i}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{2^{\sum_{i=1}^{2}k_i}}{k_1!k_2!}$$

$$\sum_{l=0}^{\infty}(1+a_i+b_i+l)(1+a_i+b_i+2l)P_l^{(a_1,b_1;a_2,b_2)}(x_1,x_2)\aleph_{U:W_{33}}^{0,\mathfrak{n}:V+3}\left(\begin{matrix} 2^{\mu_1}z_1 \\ \cdot \\ \cdot \\ 2^{\mu_2}z_2 \end{matrix}\ \middle|\right.$$

$$\begin{matrix} \mathrm{A}:\mathrm{C}_1,(\lambda_1+1-k_1;\mu_1), & (\rho_1-k_1;\mu_1),(\rho_1+a_1-k_1;\mu_1); \\ \cdots & \cdots \\ \mathrm{B}:\mathrm{D}_1,(\lambda_1+1;\mu_1), & (1\text{-}\rho_1-k_1;\mu_1),(1-\rho_1-k_1-b-l;\mu_1); \end{matrix}$$

$$\left.\begin{matrix} \mathrm{C}_2,(\lambda_2+1-k_2;\mu_2), & (\rho_2-k_2;\mu_2),(\rho_2+a_2-k_2;\mu_2) \\ \cdots & \cdots \\ \mathrm{D}_2,(\lambda_2+1;\mu_2), & (1\text{-}\rho_2-k_2;\mu_2),(1-\rho_2-k_2-b_2-l;\mu_2) \end{matrix}\right) \tag{4.4}$$

6. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as , multivariable I-function , defined by Sharma and Ahmad et al [5] , the Aleph-function of two variables defined by K.sharma [6].

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