Some finite integrals and Fourier serie involving general class of polynomials, biorthogonal polynomials, Aleph-function and the multivariable Aleph-function

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ABSTRACT
In this document, we obtain four integrals and four Fourier series expansions involving the multivariable Aleph-function, the general class of polynomials, biorthogonal polynomials and Aleph-function which are the sufficiently general in nature and are capable of yielding a large number of simpler and useful results merely by specializing the parameters in them. Further we establish some special cases.

KEYWORDS : Aleph-function of several variables, integrals, Fourier serie, general class of polynomials,biorthogonal polynomial,Aleph-function

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1.Introduction and preliminaries.

The Aleph- function , introduced by Südland [7] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z) = \aleph^{M,N}_{P_i,Q_i,c_i;r}\left(z \;\middle|\; \begin{matrix} (\mathrm{a}_j, A_j)_{1,\mathfrak{n}}, [c_i(a_{ji}, A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ (\mathrm{b}_j, B_j)_{1,m}, [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix} \right) = \frac{1}{2\pi\omega}\int_L \Omega^{M,N}_{P_i,Q_i,c_i;r}(s) z^{-s} \mathrm{d}s \quad (1.1)$$

for all z different to 0 and

$$\Omega^{M,N}_{P_i,Q_i,c_i;r}(s) = \frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r} c_i \prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \quad (1.2)$$

With $|argz| < \frac{1}{2}\pi\Omega$ where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0, i = 1, \cdots, r$

For convergence conditions and other details of Aleph-function , see Südland et al [7]. The serie representation of Aleph-function is given by Chaurasia et al [2].

$$\aleph^{M,N}_{P_i,Q_i,c_i;r}(z) = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r}(s)}{B_G g!} z^{-s} \quad (1.3)$$

With $s = \eta_{G,g} = \dfrac{b_G+g}{B_G}, P_i < Q_i, |z| < 1$ and $\Omega^{M,N}_{P_i,Q_i,c_i;r}(s)$ is given in (1.2) (1.4)

The generalized polynomials defined by Srivastava [6], is given in the following manner :

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}[y_1,\cdots,y_s] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \quad (1.5)$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1=\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s] \tag{1.6}$$

The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [4] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c|c} z_1 & \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & \end{array}\right.$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.7}$$

with $\omega=\sqrt{-1}$

For more details, see Ayant [1].
The reals numbers τ_i are positives for $i=1,\cdots,R$, $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1,\cdots,R^{(k)}$
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0,\ \text{with}\ k=1\cdots,r,i=1,\cdots,R\ ,i^{(k)}=1,\cdots,R^{(k)} \tag{1.8}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r)=0(\ |z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}\ ),max(\ |z_1|\ldots|z_r|\ )\rightarrow 0$

$\aleph(z_1,\cdots,z_r)=0(\ |z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\ ),min(\ |z_1|\ldots|z_r|\ )\rightarrow\infty$

where, with $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})],j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U=p_i,q_i,\tau_i;R\ ;\ V=m_1,n_1;\cdots;m_r,n_r \tag{1.9}$$

$$\mathrm{W}=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \tag{1.10}$$

$$A = \{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.11}$$

$$B = \{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.12}$$

$$C = \{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.13}$$

$$D = \{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.14}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} z_1 & \mathrm{A : C} \\ \cdot & \cdot\ \cdot\ \cdot \\ \cdot & \\ \cdot & \\ z_r & \mathrm{B : D}\end{array}\right) \tag{1.15}$$

Prabhakar and Tomar [3] have given a biorthogonal pair of polynomial sets.

$$U_n(x;k) = \sum_{j=0}^{n}(-)^j\binom{n}{j}\frac{(\frac{j+1}{k})_n}{(1/k)_n}\left(\frac{1-x}{2}\right)^j \tag{1.16}$$

and $$V_n(x;k) = \sum_{j=0}^{n}(-)^j\binom{n}{j}\frac{(1+n)_{kj}}{(1)_{kj}}\left(\frac{1-x}{2}\right)^{kj} \tag{1.17}$$

We have the following integrals

2; Mains integrals

Integral 1

$$\int_0^{\pi/2} cos2\rho\theta(sin\theta)^{\mu}U_n(1-2xsin^{2h}\theta;k)\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(sin\theta)^{2d})S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdot\ \cdot\ \cdot\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_r(sin\theta)^{2d_r}\end{array}\right)\mathrm{d}\theta = \sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{(\frac{j+1}{k})_n(-x)^j}{(1/k)_n}$$

$$\binom{n}{j}\frac{\Gamma(1/2+\rho)\Gamma(1/2-\rho)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i+1}\qquad B_Gg!}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c|c}2^{-2d_1}z_1 & (\text{-}\mu-2hj-2d\eta_{G,g}-2\sum_{i=1}^{s}K_if_i;2d_1,\cdots,2d_r),A:C\\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot & (\text{-}\frac{\mu\pm\rho}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}Kf_i;d_1,\cdots,d_r),B:D\\ 2^{-2d_r}z_r & \end{array}\right) \tag{2.1}$$

where $U_{12}=p_i+1;q_i+2;\tau_i;R$,provided that

a) $\rho,d>0,f_i>0,i=1,\cdots,s;d_i>0;i=1,\cdots,r$

b)$Re[\mu+2d\min\limits_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+2\sum_{i=1}^{r}d_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c)$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.8)

d) $|argz| < \frac{1}{2}\pi\Omega$ where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$

Integral 2

$$\int_0^{\pi/2} cos\rho\theta(cos\theta)^{\mu}U_n(1-2xsin^{2h}\theta;k)\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(sin\theta)^{2d})S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} \mathrm{y}_1(sin\theta)^{2f_1} \\ \cdots \\ \mathrm{y}_s(sin\theta)^{2f_s} \end{array}\right)$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(sin\theta)^{2d_1} \\ \cdots \\ \mathrm{z}_r(sin\theta)^{2d_r} \end{array}\right)\mathrm{d}\theta = \sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a_1\frac{\left(\frac{j+1}{k}\right)_n(-x)^j}{(1/k)_n}$$

$$\binom{n}{j}\frac{\pi\Gamma(1+\rho)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i+1}}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}$$

$$\aleph_{U_{02}:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} 2^{-2d_1}z_1 \\ \cdots \\ \cdots \\ 2^{-2d_r}z_r \end{array}\middle|\begin{array}{c} \cdots \mathrm{A}:\mathrm{C} \\ \cdots \\ (\frac{-\mu\pm\rho}{2}\text{-h j-d}\eta_{G,g} - \sum_{i=1}^{s}K_if_i;d_1,\cdots,d_r),B:D \end{array}\right) \tag{2.2}$$

where $U_{02} = p_i;q_i+2;\tau_i;R$, valid under the same conditions as needed for (2.1)

Integral 3

$$\int_0^{\pi/2} cos2\rho\theta(cos\theta)^{\mu}V_n(1-2xsin^{2h}\theta;k)\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(sin\theta)^{2d})S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} \mathrm{y}_1(sin\theta)^{2f_1} \\ \cdots \\ \mathrm{y}_s(sin\theta)^{2f_s} \end{array}\right)$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(sin\theta)^{2d_1} \\ \cdots \\ \mathrm{z}_r(sin\theta)^{2d_r} \end{array}\right)\mathrm{d}\theta = \sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}(-)^j a_1\binom{n}{j}\frac{(1+n)_{kj}}{(1)_{kj}}x^{kj}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1/2+\rho)\Gamma(1/2-\rho)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i+1}}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} 2^{-2d_1}z_1 \\ \cdots \\ \cdots \\ 2^{-2d_r}z_r \end{array}\middle|\begin{array}{c} (\text{-}\mu-2hj-2d\eta_{G,g}-2\sum_{i=1}^{s}K_if_i;2d_1,\cdots,2d_r),A:C \\ \cdots \\ (\frac{-\mu\pm\rho}{2}\text{-h j-d}\eta_{G,g} - \sum_{i=1}^{s}Kf_i;d_1,\cdots,d_r),B:D \end{array}\right) \tag{2.3}$$

where $U_{12} = p_i+1;q_i+2;\tau_i;R$, valid under the same conditions as needed for (3.1)

Integral 4

$$\int_0^{\pi/2} cos\rho\theta(cos\theta)^{\mu}V_n(1-2xsin^{2h}\theta;k)\aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix} y_1(sin\theta)^{2f_1}\\ \cdots \\ y_s(sin\theta)^{2f_s}\end{pmatrix}$$

$$\aleph^{0,n:V}_{U:W}\begin{pmatrix} z_1(sin\theta)^{2d_1}\\ \cdots \\ z_r(sin\theta)^{2d_r}\end{pmatrix}d\theta=\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}(-)^j a_1\binom{n}{j}\frac{(1+n)_{kj}}{(1)_{kj}}x^{kj}$$

$$\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\pi\Gamma(1+\rho)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i+1}}$$

$$\aleph^{0,n:V}_{U_{02}:W}\left(\begin{array}{c|c} 2^{-2d_1}z_1 & \cdots A:C\\ \cdots & \cdots\\ \cdots 2^{-2d_r}z_r & (\frac{-\mu\pm\rho}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}K_if_i;d_1,\cdots,d_r),B:D\end{array}\right) \tag{2.4}$$

where $U_{02}=p_i;q_i+2;\tau_i;R$, valid under the same conditions as needed for (3.1)

Proof

To obtain (2.1), express the biorthogonal polynomials $U_n(1-2xsin^{2h}\theta;k)$ as given by (1.16), a general class of polynomials occurring in the integrand of (2.1) as defined in (1.5),series representation of the Aleph-function by (1.3) and the multivariable Aleph-function by its Mellin-Barnes contour integral with the help of (1.7). Now we interchange the order of summation and integrations (which is permissible under the conditions stated above), evaluate the inner integral with the help of a result recently obtained in ([8], (2.3.5)), and reinterpreting the multiple contour integral so obtained in the form of multivariable Aleph-function with the help of (1.7), we obtain the desired result.
The integrals (2.2) to (2.4) can be developed in the similar method with the help of integrals ([8], (2.3.5)and (2.3.6))

3. Fourier series

Formula 1

$$(sin\theta)^{\mu}U_n(1-2xsin^{2h}\theta;k)\aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix} y_1(sin\theta)^{2f_1}\\ \cdots \\ y_s(sin\theta)^{2f_s}\end{pmatrix}$$

$$\aleph^{0,n:V}_{U:W}\begin{pmatrix} z_1(sin\theta)^{2d_1}\\ \cdots \\ z_r(sin\theta)^{2d_r}\end{pmatrix}=\sum_{m=0}^{\infty}\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{\left(\frac{j+1}{k}\right)_n(-x)^j}{(1/k)_n}$$

$$\binom{n}{j}\frac{\Gamma(1/2+m)\Gamma(1/2-m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{\pi 2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i-1}}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}$$

$$\aleph^{0,n+1:V}_{U_{12}:W}\left(\begin{array}{c|c} 2^{-2d_1}z_1 & (\text{-}\mu-2hj-2d\eta_{G,g}-2\sum_{i=1}^{s}K_if_i;2d_1,\cdots,2d_r),A:C\\ \cdots & \cdots\\ \cdots 2^{-2d_r}z_r & (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}Kf_i;d_1,\cdots,d_r),B:D\end{array}\right)cosm2\theta \tag{3.1}$$

where $U_{12}=p_i+1;q_i+2;\tau_i;R$, valid under the same conditions as needed for (3.1)

Formula 2

$$(cos\theta)^{\mu}U_n(1-2xsin^{2h}\theta;k)\aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_r(sin\theta)^{2d_r}\end{array}\right)=\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{\left(\frac{j+1}{k}\right)_n(-x)^j}{(1/k)_n}$$

$$\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1+m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_i f_i-1}}$$

$$\aleph^{0,\mathfrak{n}:V}_{U_{02}:W}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdots\\ 2^{-2d_r}z_r\end{array}\left|\begin{array}{c}\cdots\mathrm{A}:\mathrm{C}\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}K_if_i;d_1,\cdots,d_r),B:D\end{array}\right.\right)cosm\theta \qquad (3.2)$$

where $U_{02}=p_i;q_i+2;\tau_i;R$, valid under the same conditions as needed for (3.1)

Formula 3

$$(cos\theta)^{\mu}\ V_n(1-2xsin^{2h}\theta;k)\ \aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_r(sin\theta)^{2d_r}\end{array}\right)=\sum_{m=0}^{\infty}\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}(-)^j a_1\binom{n}{j}\frac{(1+n)_{kj}}{(1)_{kj}}x^{kj}$$

$$\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1/2+m)\Gamma(1/2-m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{\pi 2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_i f_i-1}}$$

$$\aleph^{0,\mathfrak{n}+1:V}_{U_{12}:W}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdots\\ 2^{-2d_r}z_r\end{array}\left|\begin{array}{c}(\text{-}\mu-2hj-2d\eta_{G,g}-2\sum_{i=1}^{s}K_if_i;2d_1,\cdots,2d_r),A:C\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}Kf_i;d_1,\cdots,d_r),B:D\end{array}\right.\right)cosm2\theta \qquad (3.3)$$

where $U_{12}=p_i+1;q_i+2;\tau_i;R$, valid under the same conditions as needed for (3.1)

Formula 4

$$(cos\theta)^{\mu}V_n(1-2xsin^{2h}\theta;k)\aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_r(sin\theta)^{2d_r}\end{array}\right)=\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{\left(\frac{j+1}{k}\right)_n(-x)^j}{(1/k)_n}$$

$$\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1+m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i-1}}$$

$$\aleph^{0,\mathfrak{n}:V}_{U_{02}:W}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdot\cdot\cdot\\ 2^{-2d_r}z_r\end{array}\middle|\begin{array}{c}\cdots\ \mathrm{A}:\mathrm{C}\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}K_if_i;d_1,\cdots,d_r),B:D\end{array}\right)cosm\theta \tag{3.4}$$

where $U_{02}=p_i;q_i+2;\tau_i;R$, valid under the same conditions as needed for (3.1)

Proof of (3.1)

To develop the formula (3.1), we first consider

$$f(\theta)=(sin\theta)^\mu U_n(1-2xsin^{2h}\theta;k)\aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_r(sin\theta)^{2d_r}\end{array}\right)=\sum_{m=0}^{\infty}A_m cos2m\theta,(0<\theta<\pi/2) \tag{3.5}$$

The equation (3.5) is valid since $f(\theta)$ is continuous and bounded variation in the open interval $(0,\pi/2)$. Now multiply both the sides of (3.5) by $cos2p\theta$, integrate with respect to θ from 0 to $\pi/2$ and use the result (2.1), we get

$$A_m=\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{(\frac{j+1}{k})_n(-x)^j}{(1/k)_n}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}\binom{n}{j}$$

$$\frac{\Gamma(1/2+m)\Gamma(1/2-m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{\pi 2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i-1}}$$

$$\aleph^{0,\mathfrak{n}+1:V}_{U_{12}:W}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdot\cdot\cdot\\ 2^{-2d_r}z_r\end{array}\middle|\begin{array}{c}(\text{-}\mu-2hj-2d\eta_{G,g}-2\sum_{i=1}^{s}K_if_i;2d_1,\cdots,2d_r),A:C\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}Kf_i;d_1,\cdots,d_r),B:D\end{array}\right) \tag{3.6}$$

Now on subtituting the value of A_m in (3.5), we obtain the desired result. The Fourier serie (3.2) to (3.4) can be developed by the similar method.

4. Multivariable I-function

If $\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$ the Aleph-function of several variables degenere to the I-function of several variables. The Fourier expansions have been derived in this section for multivariable I-functions defined by Sharma et

Formula 1

$$(sin\theta)^\mu U_n(1-2xsin^{2h}\theta;k)\aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$I^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_r(sin\theta)^{2d_r}\end{array}\right)=\sum_{m=0}^{\infty}\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{(\frac{j+1}{k})_n(-x)^j}{(1/k)_n}$$

$$\binom{n}{j}\frac{\Gamma(1/2+m)\Gamma(1/2-m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{\pi 2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i-1}}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}$$

$$I_{U_{12}:W}^{0,\mathrm{n}+1:V}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdot\cdot\cdot\\ 2^{-2d_r}z_r\end{array}\middle|\begin{array}{c}(\text{-}\mu-2hj-2d\eta_{G,g}-2\sum_{i=1}^{s}K_if_i;2d_1,\cdots,2d_r),A:C\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}Kf_i;d_1,\cdots,d_r),B:D\end{array}\right)cosm2\theta \quad (4.2)$$

valid under the same notations and same conditions as needed for (3.1)

Formula 2

$$(cos\theta)^{\mu}U_n(1-2xsin^{2h}\theta;k)\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(sin\theta)^{2d})S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$I_{U:W}^{0,\mathrm{n}:V}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_r(sin\theta)^{2d_r}\end{array}\right)=\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{\left(\frac{j+1}{k}\right)_n(-x)^j}{(1/k)_n}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1+m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i-1}}$$

$$I_{U_{02}:W}^{0,\mathrm{n}:V}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdot\cdot\cdot\\ 2^{-2d_r}z_r\end{array}\middle|\begin{array}{c}\cdots\ \mathrm{A:C}\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}K_if_i;d_1,\cdots,d_r),B:D\end{array}\right)cosm\theta \quad (4.3)$$

valid under the same notations and same conditions as needed for (3.1)

Formula 3

$$(cos\theta)^{\mu}V_n(1-2xsin^{2h}\theta;k)\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z(sin\theta)^{2d})S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$I_{U:W}^{0,\mathrm{n}:V}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_r(sin\theta)^{2d_r}\end{array}\right)=\sum_{m=0}^{\infty}\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}(-)^ja_1\binom{n}{j}\frac{(1+n)_{kj}}{(1)_{kj}}x^{kj}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1/2+m)\Gamma(1/2-m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{\pi 2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i-1}}$$

$$I_{U_{12}:W}^{0,\mathrm{n}+1:V}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdot\cdot\cdot\\ 2^{-2d_r}z_r\end{array}\middle|\begin{array}{c}(\text{-}\mu-2hj-2d\eta_{G,g}-2\sum_{i=1}^{s}K_if_i;2d_1,\cdots,2d_r),A:C\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}Kf_i;d_1,\cdots,d_r),B:D\end{array}\right)cosm2\theta \quad (4.4)$$

valid under the same notations and same conditions as needed for (3.1)

Formula 4

$$(cos\theta)^{\mu}V_n(1-2xsin^{2h}\theta;k)\aleph^{M,N}_{P_i,Q_i,c_i;r'}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$I^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_r(sin\theta)^{2d_r}\end{array}\right)=\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{\left(\frac{j+1}{k}\right)_n(-x)^j}{(1/k)_n}$$

$$\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1+m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^r K_if_i-1}}$$

$$I^{0,\mathfrak{n}:V}_{U_{02}:W}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdots\\ 2^{-2d_r}z_r\end{array}\left|\begin{array}{c}\cdots \mathrm{A}:\mathrm{C}\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^s K_if_i;d_1,\cdots,d_r),B:D\end{array}\right.\right)cosm\theta \quad (4.5)$$

5. Aleph-function of two variables

If $r=2$, we obtain the Aleph-function of two variables defined by K.Sharma [5], and we have the following Fourier expansions.

Formula 1

$$(sin\theta)^{\mu}U_n(1-2xsin^{2h}\theta;k)\aleph^{M,N}_{P_i,Q_i,c_i;r}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_2(sin\theta)^{2d_2}\end{array}\right)=\sum_{m=0}^{\infty}\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{\left(\frac{j+1}{k}\right)_n(-x)^j}{(1/k)_n}$$

$$\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1/2+m)\Gamma(1/2-m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{\pi 2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^r K_if_i-1}}$$

$$\aleph^{0,\mathfrak{n}+1:V}_{U_{12}:W}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdots\\ 2^{-2d_2}z_2\end{array}\left|\begin{array}{c}(\text{-}\mu-2hj-2d\eta_{G,g}-2\sum_{i=1}^s K_if_i;2d_1,2d_2),A:C\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^s Kf_i;d_1,d_2),B:D\end{array}\right.\right)cosm2\theta \quad (5.1)$$

valid under the same notations and same conditions as needed for (3.1) with $r=2$

Formula 2

$$(cos\theta)^{\mu}U_n(1-2xsin^{2h}\theta;k)\aleph^{M,N}_{P_i,Q_i,c_i;r}(z(sin\theta)^{2d})S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_2(sin\theta)^{2d_2}\end{array}\right)=\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{\left(\frac{j+1}{k}\right)_n(-x)^j}{(1/k)_n}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1+m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i-1}}$$

$$\aleph_{U_{02}:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdots\\ 2^{-2d_2}z_2\end{array}\left|\begin{array}{c}\cdots\mathrm{A}:\mathrm{C}\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}K_if_i;d_1,d_2),B:D\end{array}\right.\right)cosm\theta \quad (5.2)$$

valid under the same notations and same conditions as needed for (3.1) with $r=2$

Formula 3

$$(cos\theta)^{\mu}V_n(1-2xsin^{2h}\theta;k)\aleph_{P_i,Q_i,c_i;r}^{M,N}(z(sin\theta)^{2d})S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_2(sin\theta)^{2d_2}\end{array}\right)=\sum_{m=0}^{\infty}\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}(-)^ja_1\binom{n}{j}\frac{(1+n)_{kj}}{(1)_{kj}}x^{kj}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1/2+m)\Gamma(1/2-m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{\pi 2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i-1}}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdots\\ \cdots\\ 2^{-2d_2}z_2\end{array}\left|\begin{array}{c}(\text{-}\mu-2hj-2d\eta_{G,g}-2\sum_{i=1}^{s}K_if_i;2d_1,2d_2),A:C\\ \cdots\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}Kf_i;d_1,d_2),B:D\end{array}\right.\right)cosm2\theta \quad (5.3)$$

valid under the same notations and same conditions as needed for (3.1) with $r=2$

Formula 4

$$(cos\theta)^{\mu}V_n(1-2xsin^{2h}\theta;k)\aleph_{P_i,Q_i,c_i;r}^{M,N}(z(sin\theta)^{2d})S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c}\mathrm{y}_1(sin\theta)^{2f_1}\\ \cdots\\ \mathrm{y}_s(sin\theta)^{2f_s}\end{array}\right)$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1(sin\theta)^{2d_1}\\ \cdots\\ \mathrm{z}_2(sin\theta)^{2d_2}\end{array}\right)=\sum_{j=0}^{n}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}a_1\frac{\left(\frac{j+1}{k}\right)_n(-x)^j}{(1/k)_n}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(\eta_{G,g})}{B_G g!}\binom{n}{j}\frac{\Gamma(1+m)z^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}}{2^{\mu+2hj+2d\eta_{G,g}+2\sum_{i=1}^{r}K_if_i-1}}$$

$$\aleph_{U_{02}:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}2^{-2d_1}z_1\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ 2^{-2d_2}z_2\end{array}\left|\begin{array}{c}\cdot\cdot\cdot\ \mathrm{A}:\mathrm{C}\\ \cdot\cdot\cdot\\ (\frac{-\mu\pm m}{2}\text{-h j-d}\eta_{G,g}-\sum_{i=1}^{s}K_if_i;d_1,d_2),B:D\end{array}\right.\right)cosm\theta \tag{5.4}$$

valid under the same notations and same conditions as needed for (3.1) with $r=2$

6. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions o several variables such as multivariable I-function ,multivariable Fox's H-function, Fox's H-function , Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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