4-cordiality of Some New Path Related Graphs

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Abstract—*For an Abelian Group* $< A, * >$ *a graph* $G = (V(G), E(G))$ *is said to be A-cordial if there is a mapping* $f{:}V(G)\rightarrow A$ *which satisfies the conditions* $|v_f(a)-v_f(b)|\leq 1$ *and* $|e_f(a)-e_f(b)|\leq 1$, *for all* $a,b \in A$, *when the edge* $e=uv$ *is labeled as* $f(u)*f(v)$. *Where* $v_f(a)$ *is the number of vertices with label a and* $e_f(a)$ *is the number of edges with label a. If we consider an Abelian Group* $< A,* > = < Z_k, +_k >$ *then it is called k-cordial labeling. In this research paper we proved that* $Z\text{-}P_n$, *braid graph* $B(n)$, *triangular ladder* TL_n *and irregular quadrilateral snake* $I(QS_n)$ *are 4-cordial for all n.*

Keywords—*A-cordial Labeling;* $Z\text{-}P_n$; *Braid Graph* $B(n)$; *Triangular Ladder* TL_n; *Irregular Quadrilateral Snake* $I(QS_n)$.

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I. INTRODUCTION

In this research paper we consider finite, connected, undirected and simple graphs. In the graph $G=(V(G),E(G))$ the cardinality of the vertex set is called order of G and the cardinality of the edge set is called the size of G. They are denoted by $|V(G)|$ and $|E(G)|$ respectively. In *Graph Labeling* we assign numerical values to vertices or edges or both subject to certain conditions.

Definition 1.1 Let $< A, * >$ be any Abelian group. A graph is said to be A-*cordial* if there is a mapping $f{:}V(G)\rightarrow A$ which satisfies the following two conditions when the edge $e=uv$ is labeled as $f(u)*f(v)$

(i) $|v_f(a)-v_f(b)|\leq 1$; for all $a,b \in A$,

(ii) $|e_f(a)-e_f(b)|\leq 1$; for all $a,b \in A$,

Where,

$v_f(a)$=the number of vertices with label a;

$v_f(b)$=the number of vertices with label b;

$e_f(a)$=the number of edges with label a;

$e_f(b)$=the number of edges with label b.

We note that if $A=< Z_k, +_k >$ that is additive group of modulo k then the labeling is known as k-cordial labeling.

Here, we consider $< Z_4, +_4 >$ that is additive group of modulo 4 then the labeling is known as 4-cordial labeling.

The concept of A-cordial labeling was introduced by Hovey [3] and proved the following results:

- All the connected graphs are 3-cordial.
- All the trees are 3-cordial and 4-cordial.
- Cycles are k-cordial for all odd k.

Youssef[10] obtained the following results:

- C_{2k}^{+1} is not $(2k + 1)$-cordial for $k > 1$.
- K_n is 4-cordial $\leftrightarrow n \leq 6$.
- $C_n^{\,2}$ is 4-cordial$\leftrightarrow n \not\equiv 2(mod4)$.
- $K_{m,n}$ is 4-cordial $\leftrightarrow m$ *or* $n \not\equiv 2(mod4)$.

Rathod and Kanani[4] proved the following results:

- All the wheels W_n are 4-cordial.
- All the fans f_n are 4-cordial.
- All the friendship graphs F_n are 4-cordial.
- All the gear graphs G_n are 4-cordial.
- All the double fans Df_n are 4-cordial.
- All the helms H_n are 4-cordial.

Rathod and Kanani[5] also proved the following results:

- The middle graph $M(P_n)$ of path Pn is 4-cordial.
- The total graph $T(P_n)$ of path Pn is 4-cordial.
- The splitting graph $S'(P_n)$ of path is 4-cordial.
- The square graph $P_n^{\,2}$ of path P_n is 4-cordial.
- The triangular snake TS_n is 4-cordial.

In[6] Rathod and Kanani have derived the following results:

- The square graph of Path $P_n^{\,2}$ is k-cordial.
- The pan graph C_n^{+1} is k-cordial for all even k and $n = k + j,\ 0\leq j \leq k - 1$.
- The pan graph C_n^{+1} is k-cordial for all even k and

$n = 2tk + j$, where $t \in N \cup \{0\}$ and $0 \le j \le k - 1$.

- The pan graph C_n^{+1} is k-cordial for all even k and
  $n = 2tk + k + j$, where $t \in N$ and $0 \le j \le k - 1$.

We consider the following useful definitions to understand the results of this research paper.

Definition 1.2 The graph $Z\text{-}P_n$ is obtained from the pair of paths P_n' and P_n''. Let $v_1, v_2, ..., v_n$ be the vertices of path P_n' and $u_1, u_2, ..., u_n$ are the vertices of path P_n''. To find $Z\text{-}P_n$ join i^{th} vertex of path P_n' with $(i + 1)^{th}$ vertex of path P_n'' for all $1 \le i \le n - 1$.

Definition 1.3 The Braid Graph $B(n)$ is obtained from the pair of paths P_n' and P_n''. Let $v_1, v_2, ..., v_n$ be the vertices of path P_n' and $u_1, u_2, ..., u_n$ are the vertices of path P_n''. To find braid graph join i^{th} vertex of path P_n' with $(i+1)^{th}$ vertex of path P_n'' and i^{th} vertex of path P_n'' with $(i+2)^{th}$ vertex of path P_n' with the new edges for all $1 \le i \le n - 2$.

Definition 1.4 The *Triangular Ladder* TL_n is obtained from the ladder $L_n = P_n \times P_2$ $(n \ge 2)$ by adding the edges $u_i v_{i+1}$ for all $1 \le i \le n-1$, where the consecutive vertices of two copies of paths are $v_1, v_2, ..., v_n$ and $u_1, u_2, ..., u_n$ and the edges are $u_i v_i$.

Definition 1.5 The *Irregular Quadrilateral Snake* $I(QS_n)$ is obtained from the path P_n. Let $u_1, u_2, ..., u_n$ be the vertices of path P_n and $v_1, v_2, ..., v_{n-2}$ & $w_1, w_2, ..., w_{n-2}$ are the newly added vertices. To obtain irregular quadrilateral snake join the vertices $u_i v_i$, $w_i u_{i+2}$ and $v_i w_i$ for all $1 \le i \le n - 2$.

Here, all terminologies are considered from Gross and Yellen[2].

II. MAIN RESULTS

Theorem 2.1 The graph $Z\text{-}P_n$ is 4-cordial for all n.

Proof. Let $G = Z\text{-}P_n$ be the graph obtained from the pair of paths P_n' and P_n''. Let $v_1, v_2, ..., v_n$ be the vertices of path P_n' and $u_1, u_2, ..., u_n$ are the vertices of path P_n''. To find $Z\text{-}P_n$ join i^{th} vertex of path P_n' with $(i + 1)^{th}$ vertex of path P_n'' for all $1 \le i \le n - 1$. We note that $|V(G)| = 2n$ and $|E(G)| = 3n - 3$.

Define 4-cordial labeling $f : V(G) \to Z_4$ as follows:

$f(v_i) = 0;$ $\quad i \equiv 1, 6 (mod 8);$
$f(v_i) = 1;$ $\quad i \equiv 4, 7 (mod 8);$
$f(v_i) = 2;$ $\quad i \equiv 2, 5 (mod 8);$
$f(v_i) = 3;$ $\quad i \equiv 0, 3 (mod 8);$ $\quad 1 \le i \le n,$
$f(u_i) = 0;$ $\quad i \equiv 4, 7 (mod 8);$
$f(u_i) = 1;$ $\quad i \equiv 2, 5 (mod 8);$
$f(u_i) = 2;$ $\quad i \equiv 0, 3 (mod 8);$
$f(u_i) = 3;$ $\quad i \equiv 1, 6 (mod 8);$ $\quad 1 \le i \le n,$

Let $n = 8p + q$, $\quad p, q \in N \cup \{0\}$.

TABLE 1

q	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1)+1 = e_f(2)+1 = e_f(3)+1$
1	$v_f(0) = v_f(1)+1 = v_f(2)+1 = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
2	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)+1$
3	$v_f(0)+1 = v_f(1)+1 = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2)+1 = e_f(3)+1$
4	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1)+1 = e_f(2)+1 = e_f(3)+1$
5	$v_f(0)+1 = v_f(1) = v_f(2) = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)+1$
7	$v_f(0) = v_f(1) = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2)+1 = e_f(3)+1$

From the Table 1 we can see that the labeling pattern defined above satisfies all the conditions of 4-cordiality. Hence, the graph $Z\text{-}P_n$ is 4-cordial for all n.

Illustration 2.2 The graph $Z\text{-}P_6$ and its 4-cordial labeling is shown in *Figure 1*.

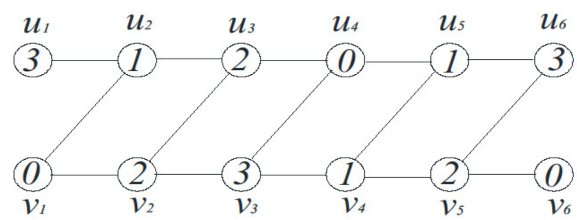

Fig. 1 4-cordial labeling of Z-P_6

Theorem 2.3 The Braid graph $B(n)$ is 4-cordial for all n.

Proof. Let $G = B(n)$ be the braid graph obtained from the pair of paths P_n' and P_n''. Let $u_1, u_2, ..., u_n$ be the vertices of path P_n' and $v_1, v_2, ..., v_n$ are the vertices of path P_n''. To find braid graph join i^{th} vertex of path P_n' with $(i+1)^{th}$ vertex of path P_n'' and i^{th} vertex of path P_n'' with $(i+2)^{th}$ vertex of path P_n' with the new edges for all $1 \le i \le n - 2$. We note that $|V(G)| = 2n$ and $|E(G)| = 4n - 5$.

Define 4-cordial labeling $f: V(G) \to Z_4$ we consider the following two cases:

<u>**Case 1**</u>: If $n \le 3$.

$f(u_i) = 1;$ $\quad i \equiv 2, 3(mod4);$
$f(u_i) = 3;$ $\quad i \equiv 1(mod8);$ $\quad 1 \leq i \leq 3,$
$f(v_i) = 0;$ $\quad i \equiv 1(mod4);$
$f(v_i) = 2;$ $\quad i \equiv 2, 3(mod4);$ $\quad 1 \leq i \leq 3.$

Case 2: If $n \geq 4$.
$f(u_1) = 3;$
$f(u_2) = 3;$
$f(u_3) = 0;$
$f(u_4) = 0;$
$f(u_i) = 1;$ $\quad i \equiv 1, 3(mod4);$
$f(u_i) = 3;$ $\quad i \equiv 0,2(mod4);$ $\quad 5 \leq i \leq n,$
$f(v_i) = 1;$
$f(v_i) = 1;$
$f(v_i) = 2;$
$f(v_i) = 2;$
$f(v_i) = 0;$ $\quad i \equiv 1,3(mod4);$
$f(v_i) = 2;$ $\quad i \equiv 0,2(mod4);$ $\quad 5 \leq i \leq n,$

Let $n = 4p + q,$ $\quad p, q \in N \cup \{0\}$.

TABLE 2

q	*Vertex conditions*	*Edge conditions*
0,2	$v_f(0)= v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)+1$
1,3	$v_f(0)= v_f(1) = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1)= e_f(2)= e_f(3)+1$

From the Table 2 we can see that the labeling pattern defined above satisfies all the conditions of 4-cordiality. Hence, the braid graph $B(n)$ is 4-cordial for all n.

Illustration 2.4 (a) The Braid graph $B(3)$ and its 4-cordial labeling is shown in *Figure 2*.

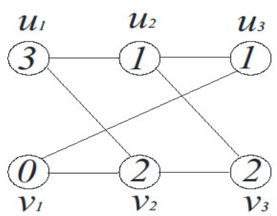

Fig. 2 4-cordial labeling of Braid graph B(3)

(b) The Braid graph $B(7)$ and its 4-cordial labeling is shown in *Figure 3*.

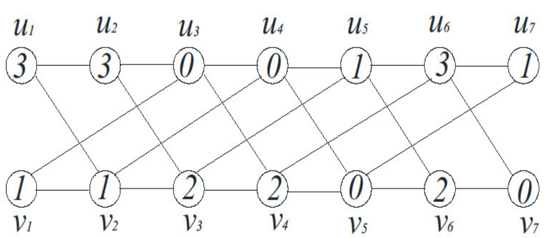

Fig. 3 4-cordial labeling of Braid graph B(7)

Theorem 2.5 The Triangular Ladder TL_n is 4-cordial for all n.

Proof. Let $G = TL_n$ be the triangular ladder obtained from the ladder $L_n=P_n \times P_2$ $(n \geq 2)$ by adding the edges u_iv_{i+1} for all $1 \leq i \leq n-1$, where the consecutive vertices of two copies of paths are $v_1, v_2, ..., v_n$ and $u_1, u_2, ..., u_n$ and the edges are u_iv_i. We note that $|V(G)| = 2n$ and $|E(G)| = 4n -3$.

Define 4-cordial labeling $f : V(G) \rightarrow Z_4$ as follows:

$f(u_i) = 1;$ $\quad i \equiv 0, 2(mod4);$
$f(u_i) = 3;$ $\quad i \equiv 1,3(mod8);$ $\quad 1 \leq i \leq n,$
$f(v_i) = 0;$ $\quad i \equiv 0,2(mod4);$
$f(v_i) = 2;$ $\quad i \equiv 1, 3(mod4);$ $\quad 1 \leq i \leq n,$

Let $n = 2p + q,$ $\quad p, q \in N \cup \{0\}$.

TABLE 3

q	*Vertex conditions*	*Edge conditions*
0	$v_f(0)= v_f(1) = v_f(2) = v_f(3)$	$e_f(0)+1 = e_f(1) = e_f(2)+1= e_f(3)+1$
1	$v_f(0)+1= v_f(1)+1= v_f(2) = v_f(3)$	$e_f(0)+1= e_f(1)= e_f(2)+1= e_f(3)+1$

From the Table 3 we can see that the labeling pattern defined above satisfies all the conditions of 4-cordiality. Hence, triangular ladder TL_n is 4-cordial for all n.

Illustration 2.6 The Triangular Ladder TL_6 and its 4-cordial labeling is shown in *Figure 4.*

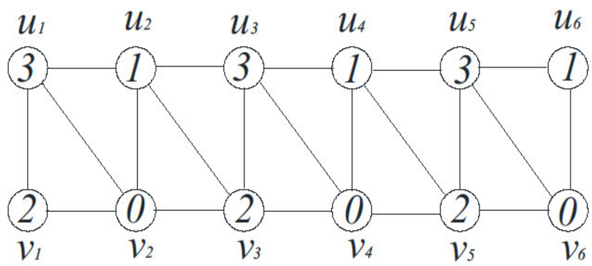

Fig. 4 4-cordial labeling of Triangular Ladder TL_6

Theorem 2.7 The Irregular Quadrilateral Snake $IQ(S_n)$ is 4-cordial for all n.

Proof. Let $G = IQ(S_n)$ be the irregular quadrilateral snake of the path P_n. Let $u_1, u_2, ..., u_n$ be the vertices of path P_n and $v_1, v_2, ..., v_{n-2}$ & $w_1, w_2, ..., w_{n-2}$ are the newly added vertices. To find irregular quadrilateral snake join the vertices u_iv_i, w_iu_{i+2} and v_iw_i for all $1 \leq i \leq n - 2$. We note that $|V(G)| = 3n-4$ and $|E(G)| = 4n -7$.

Define 4-cordial labeling $f: V(G) \rightarrow Z_4$ as follows:

$f(u_i) = 0; \quad i \equiv 0(mod4);$
$f(u_i) = 1; \quad i \equiv 2(mod4);$
$f(u_i) = 2; \quad i \equiv 3(mod4);$
$f(u_i) = 3; \quad i \equiv 1(mod4); \quad 1 \le i \le n,$
$f(v_i) = 0; \quad i \equiv 3(mod4);$
$f(v_i) = 1; \quad i \equiv 1(mod4);$
$f(v_i) = 2; \quad i \equiv 0, 2(mod4); \quad 1 \le i \le n\text{-}2,$
$f(w_i) = 0; \quad i \equiv 1(mod4);$
$f(w_i) = 1; \quad i \equiv 3(mod4);$
$f(w_i) = 2; \quad i \equiv 0, 2(mod4); \quad 1 \le i \le n\text{-}2.$

Let $n = 8p + q, \quad p, q \in N \cup \{0\}$.

TABLE 4

q	Vertex conditions	Edge conditions
0,4	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0)+1 = e_f(1)+1 = e_f(2)+1 = e_f(3)$
1,5	$v_f(0) = v_f(1) = v_f(2)+1 = v_f(3)$	$e_f(0)+1 = e_f(1)+1 = e_f(2)+1 = e_f(3)$
2,6	$v_f(0)+1 = v_f(1) = v_f(2)+1 = v_f(3)$	$e_f(0) = e_f(1)+1 = e_f(2)+1 = e_f(3)+1$
3,7	$v_f(0)+1 = v_f(1) = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1)+1 = e_f(2)+1 = e_f(3)+1$

From the Table 4 we can see that the labeling pattern defined above satisfies all the conditions of 4-cordiality. Hence, irregular quadrilateral snake $IQ(S_n)$ is 4-cordial for all n.

Illustration 2.8 The Irregular Quadrilateral Snake $IQ(S_{11})$ and its 4-cordial labeling is shown in *Figure5*.

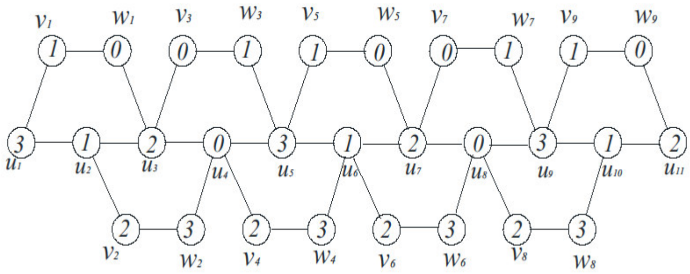

Fig. 5 4-cordial labeling of Irregular Quadrilateral Snake $IQ(S_{11})$

III. CONCLUSIONS

Graph labeling technique is a wide area of research. In this research paper, we investigate some new results on 4-cordiality of graphs. For better understanding of labeling pattern, we have given some illustration. To investigate more graph families which admit k-cordial labeling is an open area of research.

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