# Common Weight Decompositions of Some Classes of Graphs

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***ABSTRACT:*** *A difference labeling of a graph G is realized by assigning distinct integer values to its vertices and then associating with each edge uv the absolute difference of those values assigned to its end vertices. A decomposition of labeled graph into parts, each part containing the edge having a common- weight is called a common – weight decomposition. In this paper we investigate the existence of difference labeling of* $P_{2n}(+)N_m$, *Braid graph, Mongolian Ger and Alternative Quadrilateral Snake.*

**KEYWORDS:** *Difference labeling, common weight decomposition.*

## 1. INTRODUCTION

In this paper, we consider only finite simple undirected graph. In which G has vertex set $V = V(G)$ and edge set $E = E(G)$ the set of vertices adjacent to vertex $u$ of $G$ is denoted by $N = N(u)$. For the notation and terminology we referred to Bondy and Murthy [2].

A difference labeling of a graph G is realized by assigning distinct integer values to its vertices and then associating with each edge $uv$ the absolute difference of those values assigned to its end vertices. The concept of difference labelings was introduced by Bloom and Ruiz [1] and was further investigated by Arumugam and Meena [6]. Meena and Vaithilingam [7] have investigated the existence difference labelings whereas crown graph, grid graph, pyramid graph, fire cracker, banana trees, gear graph, ladder, fan graph, friendship graph, helm graph, wheel graph and $P_{2n}(+)N_m$. In addition, various labelings graphs problem have been examined by Jeyanthi and Saratha Devi [8]; and Manisha [9].

**Definition: 1.1.** Let $G = (V, E)$ be a graph. A difference labeling of G is an injection f from V to the set of non-negative integers with weight function f* on $E$ given by $f^*(uv) = |f(u) - f(v)|$ for every $uv$ edge in $G$.A graph with a difference labeling defined on it is called a labeled graph.

**Definition: 1.2.** A decomposition of labeled graph into parts, each part containing the edge having a common-weight is called a common – weight decomposition.

**Definition: 1.3.** A common weight decomposition of $G$ in which each part contains m edges is called $m$- equitable.

**Definition: 1.4.** Specified Parts Decomposition Problem is a given graph G with edge set E(G) and a collection of edge – disjoint linear forests $F_1, F, F_3, \ldots, F_k$ containing a total of $|E|$ edges , does there exists a common weight decomposition of G whose parts are respectively isomorphic to $F_1, F, F_3, \ldots, F_k$.

**Definition: 1.4.** Let $P_{2n}(+)N_m$ be the graph with $p = 2n + m$ and $q = 2(m + n) - 1$ and vertex set $V(P_{2n}(+)N_m) = \{u_1, u_2 \ldots, u_{2n}, v_1, v_2 \ldots, v_m\}$. where $V(P_{2n}) = \{u_1, u_2 \ldots, u_{2n}\}$ and $V(N_m)) = \{v_1, v_2 \ldots, v_m\}$ and the edge set $E(P_{2n}(+)N_m) = E(P_{2n}) \cup \{(u_1,v_1)(u_1,v_2)(u_1,v_3),\ldots,(u_1,v_m),(u_{2n},v_1),(u_{2n},v_2),\ldots,(u_{2n},v_m)\}$.This graph has $m$ cycles of length $2n + 1$ and many 4-cycles.

**Definition: 1.5.** For each $n \geq 2$, the braid graph B($n$) is defind as follows, $V(B(n)) = \{x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n\}$and
$E(B(n)) = \{x_i x_{i+1} | 1 \leq i \leq n-1\}\} U \{y_i y_{i+1} | 1 \leq i \leq n-1\} U \{(x_i, y_i) | 1 \leq i \leq n\}$.

**Definition: 1.6.** For any integers $m > 2$ and $n > 1$, *the Mongolian* Ger is the graph M$(m, n)$ with vertex set V$(M(m,n)) = \{u, x_{11}, x_{12}, \dots, x_{1m,} x_{21}, x_{22}, \dots, x_{2m}, \dots, x_{n1,} x_{n2}, \dots, x_{nm}\}$ and edge set E$(M(m,n)) = \{(u, x_{1i}): i = 1, 2, \dots, m\} \cup \{(x_{ij,} x_{ij+1}): \quad i = 1, 2, \dots, n, \ j = 1, 2, \dots, m\} \cup \{(x_{ij}, x_{i+1j}): i = 1, 2, \dots, n-1, j = 1, 2, \dots, m\}$. The graph $M(m, n)$ has $p = mn + 1$ and $q = 2mn$.

**Definition: 1.7.** An Alternative Quadrilateral Snake A$(Q\, S_n)$ is obtained from a path $u_1, u_2 \dots, u_n$ by joining $u_i u_{i+1}$ (alternatively) to new vertices $v_i$ and $v_{i+1}$ and then joining $v_i v_{i+1}$. That is every alternative edge of a path is replaced by a cycle $C_{4,r}$.

## 2. MAIN RESULTS

**Theorem: 2.1.** There exists a labeling which realizes a common weight decomposition of the graph $P_{2n}(+)N_m$ for $m = 2n - 1$ into $m - 1$ copies of $2P_2$, a copy of $P_3$ and a copy $P_{2n}$.

**Proof:**

Let $G = P_{2n}(+)N_m$ be the graph with $p = 2n + m$ and $q = 2(m + n) - 1$ with vertex set

$V(P_{2n}(+)N_m) = \{u_1, u_2 \dots, u_{2n}, v_1, v_2 \dots, v_m\}$. where V$(P_{2n}) = \{u_1, u_2 \dots, u_{2n}\}$ and V$(N_m)) = \{v_1, v_2 \dots, v_m\}$, and the edge set E$(P_{2n}(+)N_m)$ = E$(P_{2n})$U$\{(u_1,v_1)(u_1,v_2)(u_1,v_3),..,(u_1,v_m),(u_{2n},v_1),(u_{2n},v_2),..,(u_{2n},v_m)\}$.

Define a vertex labeling $f: VG) \to \{0,1,2, \dots, 2n + m - 1\}$ as follows

$f(u_i) = 2i - 2, \qquad 1 \le i \le 2n$

$f(v_j) = 2i - 1, \qquad 1 \le i \le m$

Then the set of edges $S_1, S_2, S_3$ forms a common weight decomposition of $P_{2n}(+)N_m$ graph into $(m - 1)$ copies of $2P_2$, a copy of $P_3$ and a copy $P_{2n}$.

Where

$S_1 = \{u_1 v_i, u_{2n} v_{m-(i-1)}\} \qquad$ for $i \neq n$

$S_2 = \{u_1 v_n u_{2n}\}$

$S_3 = \{u_1, u_2 \dots, u_{2n}\}$

**Theorem: 2.2.** There exists a labeling which realizes a common weight decomposition of the Braid graph into two maximum matching and a copy of $2P_n$ for $n \ge 2$.

**Proof:**

Let G = $B_n$ be the graph with vertex set V$(B_n) = \{x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n\}$ and the edge set E$(B_n) = \{x_i x_{i+1} | 1 \le i \le n-1\}\}$U$\{y_i y_{i+1} | 1 \le i \le n-1\}$U$\{(x_{i,} y_i) | 1 \le i \le n\}$ be the edge set of the graph.

Here $|V(B_n)| = 2n$ and $|E(B_n)| = 4(n - 1)$.

Define a vertex labeling $f: V(G) \to \{0,1,2, \dots, 2n - 1\}$ as follows

$f(x_i) = 2i - 2, \qquad 1 \le i \le n$

$f(y_i) = 2i - 1, \qquad 1 \le i \le n$

Then the set of edges $S_1, S_2, S_3$ forms a common weight decomposition of Braid graph $B(n)$ into two maximum matchings and a copy of $2P_n$

Where

$$S_1 = \{x_i y_{i+1} \,/\, 1 \le i \le n-1\}$$

$$S_2 = \{y_i x_{i+1} \,/\, 1 \le i \le n-1\}$$

$$S_3 = \{x_1 x_2 \dots x_n, y_1\, y_2 \dots y_n\}$$

**Theorem: 2.3.** There exists a labeling which realizes a common weight decomposition of the Mongolian Ger graph $M(m,n)$ into $m$ copies of $P_n$, $n$ copies of $P_m$, a copy of $nP_2$ and $m$ copies of $P_2$.

**Proof:**

Let $G$ be a Mangolian Ger. For any integers $m > 2$ and $n > 1$, the Mongolian Ger is the graph $M(m,n)$ with vertex set $V(M(m,n)) = \{u, x_{11}, x_{12}, \dots, x_{1m} x_{21}, x_{22}, \dots, x_{2m}, \dots, x_{n1,} x_{n2}, \dots, x_{nm}\}$ and the edge set $E(M(m,n)) = \{(u, x_{1i}) : i = 1,2,\dots,m\} \cup \{(x_{ij}, x_{ij+1}) : i = 1,2,\dots,n, j = 1,2,\dots,m\} \cup \{(x_{ij}, x_{i+1j}) : i = 1,2,\dots,n-1, j = 1,2,\dots,m\}$. The graph $M(m,n)$ has $p = mn + 1$ and $q = 2mn$.

Define the vertex labeling $f: V(G) \to \{0,1,\dots mn+1\}$ as follows,

$$f(u) = 0$$

$$f(x_{1j}) = 2j-1, \qquad for\ 1 \le j \le m$$

$$f(x_{ij}) = (2j-1) + (2i-2)m, \qquad for\ 1 \le j \le m\ and\ 1 \le i \le n$$

Then the set of edges $S_1, S_2, S_3, S_4$ forms a common- weight decomposition of the Mangolian Ger $M(m,n)$ decomposed into $m$ copies of $P_n$, $n$ copies of $P_m$, a copy of $nP_2$ and $m$ copies of $P_2$.

Where $S_1 = \{x_{1j} x_{2j} \dots x_{nj} \,/\, 1 \le j \le m\}$

$$S_2 = \{x_{i1} x_{i2} \dots x_{im} \,/\, 1 \le i \le n\}$$

$$S_3 = \{x_{i1}\, x_{im} | 1 \le i \le n\}$$

$$S_4 = \{u x_{11}, u x_{12}, \dots u x_{1m}\}$$

**Theorem: 2.4.** There exists a labeling which realizes a common weight decomposition of the Alternative Quadrilateral Snake $A(QS_n)$ for $n \ge 2$ and $n$ is even into a perfect matching, a copy of $P_n \cup (n/2)P_2$.

**Proof:**

Let $G = A(QS_n)$ be the graph with vertex set $V(G) = \{u_1, u_2 \dots, u_n, v_1, v_2 \dots, v_n\}$ and the edge set $E(G) = \{u_i u_{i+1} | 1 \le i \le n-1\}\} \cup \{v_{2i-1} v_{2i} \,|\, 1 \le i \le \frac{n}{2}\} \cup \{(u_i, v_i) | 1 \le i \le n\}$.

Define a vertex labeling $f: V(G) \to \{0,1,\dots,2n-1\}$ as follows

$$f(u_i) = i-1, \qquad 1 \le i \le n$$

$$f(v_i) = i-1+n, \qquad 1 \le i \le n$$

Then the set of edges $S_1, S_2$ forms a common weight decomposition of Alternative Quadrilateral Snake $A(QS_n)$ for $n \ge 2$ and $n$ is even into a perfect matchings, a copy of $P_n \cup (\frac{n}{2})P_2$.

Where $S_1 = \{u_i\, v_i | 1 \le i \le n\}$

$$S_2 = \{u_1u_2 \dots u_n\} \cup \left\{v_{2i-1} \cup v_{2i} \middle| 1 \le i \le \frac{n}{2}\right\}$$

**Theorem: 2.5.** There exists a labeling which realizes a common weight decomposition of the Alternative Quadrilateral Snake $A(QS_n)$ for $n$ is even, $n \ge 2$ into two perfect matchings and a copy of $(\frac{n}{2} - 1)P_2$.

**Proof:**

Let $G = A(QS_n)$ be the graph with vertex set V(G) = $\{u_1, u_2 \dots, u_n, v_1, v_2 \dots, v_n\}$ and the edge set E(G) = $\{u_iu_{i+1} | 1 \le i \le n-1\}\}$U$\{v_{2i-1}v_{2i} \,|\, 1 \le i \le \frac{n}{2}\}$U$\{(u_{i,}v_i) | 1 \le i \le n\}$.

Define a vertex labeling $f: V(G)) \to \{0,1, \dots, 2n-1\}$ as follows

$f(u_1) = 0$

$f(u_i) = i$, if $i$ is odd, $i = 2m+1$ when $m$ is odd, $3 \le i \le n-1$

$f(u_i) = i-1$, if $i$ is odd, $5 \le i \le n-1$, $i = 2m+1$ where $m$ is even

$f(u_i) = i-1$, if i is even, $2 \le i \le n$, $i = 2m$ where $m$ is odd

$f(u_i) = i-2$, if $i$ is even, $4 \le i \le n$, $i = 2m$ where $m$ is even

$f(v_i) = f(u_i) + n$, $1 \le i \le n$

Then the set of edges $S_{1,}S_{2,}S_{3,}$ forms a common weight decomposition of Alternative Quadrilateral Snake $A(Qs_n)$ into two perfect matchings and a copy of $(\frac{n}{2} - 1)P_2$.

Where $S_1 = \{u_i v_i | 1 \le i \le n\}$

$S_2 = \{u_iu_{i+1}, v_iv_{i+1} | 1 \le i \le n-1, i \; is \; odd\}$

$S_3 = \{u_iu_{i+1} | 2 \le i \le n-2, \; i \; is \; even\}$

## 3. CONCLUSION

In this paper we investigate the existence of difference labelings for some classes of graphs. The specified parts decomposition problem can be investigated for other classes of graphs.

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