Exchange rate risk in a newsvendor framework with normally distributed exchange rate error

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Abstract - *In the international business between two firms of two different countries when there is a fixed time duration between the payments made while placing the order and the order is realized, there is a possibility of exchange rate fluctuation and this affects the optimal pricing and order quantity decisions of the parties involved. In this paper we demonstrate the effect of exchange rate fluctuation under the normal distribution when the buyer or seller undertakes to share the exchange rate risk under the additive demand error in the news vendor framework. The observations under the normally distributed exchange rate error are compared with the exchange rate effect under the generalized beta distribution error in the model given in [1]. This is elaborated through numerical example using maple software through nonlinear optimization techniques, to test the sensitivity of the model and to compare the two scenarios of the buyer and seller.*

Keywords - *transaction exposure, exchange rate error, newsvendor problem, optimal pricing and quantity, normal distribution, generalized beta distribution.*

I. INTRODUCTION

Let there be two firms from two different countries having different currencies are into a business. If due to unexpected changes in the global economy the exchange rate between the two currencies gets affected then there exists a financial risk and this risk is known as foreign exchange risk (or exchange rate risk). A transaction exposure arises only when there exists a time lag between the time of the order is placed and the payment is made when the goods arrived. This is because of the purchase price to buyer/ retailer on the settlement day may differ from that when it was incurred, if the debt is denominated in the manufacturer currency. The model of Arcelus, Gor and Srinivasan [1] in news vendor framework gives the optimum ordering and pricing strategies for buyer/seller, when the exchange rate between the two countries doing the business faces transaction exposure. The complete derivation of optimum policies and expected profit of the exchange rate risk model for additive demand error is given in Sanjay Patel, Ravi Gor [2]. Our main contribution in this paper is to explain the effect of normal distribution in the exchange rate error under the linear demand with additive error in news vendor setting. The effect of normally distributed error is also compared with the generalized beta distribution error in the exchange rate.

II. LITERATURE REVIEW

In this paper we have basically followed the model given in [1]. The cases of foreign exchange transaction exposure when a firm has an accounts receivable or payable denominated in a foreign currency has been reported in [3]. In the global trading the general rule is that either the buyer or the seller has to bear what is commonly known in international finance as transaction exposure, and it is explained in [4] and [5]. The very important newsvendor framework with pricing introduced in [6] and the price dependent demand forms in the additive and multiplicative error structures given in [7] and [8] have been used. The expected profit and optimal policies are mathematically derived in [2] when demand is linear with additive error and in [9] for iso-elastic demand with multiplicative demand error. We have also developed more general hybrid demand model of foreign exchange transaction exposure in [11].

III. THE FOREIGN EXCHANGE RISK MODEL

Suppose a retailer wants to order q units from a foreign manufacturer of a certain product. The retailer does not know the demand (D) of the product, which is uncertain. But it partly depends upon the price(p) and partly random. In this paper we take the price dependent demand with additive error which can be defined as $D(p,\in) = g(p)+\in$, where $\in$ is the additive error in the demand and it follows some distribution(say $f(\in)$) with mean μ in some interval [A,B] and $g(p)$ is the deterministic demand. [Generally g(p) is taken as decreasing linear function of p say, $g(p) = a-bp$ in additive demand error case with the restrictions *a, b* >0]

Suppose the exchange rate in the seller's currency is 'r' when the order is placed. Let the cost of one unit of the product be w in the seller's

currency. If the buyer pays immediately then he has to pay *wr* per unit of the product in his currency.

When there is a time lag between the order is placed and the amount is paid for the product when it is acquired by the buyer, there exists transaction exposure risk since the exchange rate (r) may get fluctuate. So the buyer has to pay more or less according to the existing rate at the time of the arrival of the product. Generally the fluctuation in the exchange rate r is very small and random. We model the future exchange rate(FER) as FER= current exchange rate + fluctuation in the exchange rate. The fluctuation in the exchange rate is always some percentage of r, hence we can take the FER= r +$r\epsilon_r$ =$r(1+\epsilon_r)$.Where ϵ_r is also a random variable together with the random variable *D*. We assume that ϵ_r lies in [-*a*, *a*]. Here 0<*a*<1.The fluctuation ϵ_r is unknown but its distribution is known say $\psi(\epsilon_r)$. In this paper we consider truncated normal distribution for ϵ_r with support [*a*,*b*].

If the fluctuation ϵ_r is positive buyer has to pay more and if it is negative seller will get less. So the question arises here is that who will bare the exchange rate risk? Buyer/retailer OR seller/manufacturer? In this paper we discuss the two situations under the additive demand error. In each case the retailer's optimal policy is to determine the optimum order(*q*) and selling price(*p*) of the product so that his expected profit is maximum. At the same time we obtain the manufacturer's optimal policies as well.

IV. ASSUMPTIONS AND NOTATIONS

The following assumptions are made in the exchange rate risk model:
(i) We apply the standard newsvendor problem assumptions.
(ii) The global supply chain consists of single retailer- single manufacturer.
(iii) The demand error is additive.
(iv) Only one of the two-retailer or manufacturer- bears the exchange rate risk.

The following notations are used in the paper:
q = order quantity
p = selling price per unit
D = demand of the product= no. of units required
ϵ = demand error
v = salvage value per unit
s = penalty cost per unit for shortage
c = cost of manufacturing per unit for manufacturer
w_r = purchase cost for retailer
ϵ_r = exchange rate error
Π = profit function.

V. THE TWO LAYOUTS

Case-1: The buyer bears the exchange rate risk

In the case-1 we assume that the buyer bears the exchange rate risk and seller does not. Thus the seller will get *w* per unit at any point of

time and the buyer will have to pay according to the existing exchange rate. So the buyer will be paying $wr(1+\epsilon_r)$ per unit, on the settlement day or when the product is acquired by him. This amount in terms of seller currency is $wr(1+\epsilon_r)/r$ =$w(1+\epsilon_r)$ =w_r (say). Thus w_r is the purchase cost to buyer in seller's currency.

Now the retailer/ buyer will choose the selling price p and the order quantity q so as to maximize his expected profit. The profit function for the retailer is given by,

$\Pi(p,q)$ =[revenue from q items]–[expenses for the q items].

$$\Pi(p,q) = \begin{cases} [pD+v(q-D)]-[qw_r] \ if \ D \le q \ (overstocking) \\ [pq]-[s(D-q)+qw_r] \ if \ D > q \quad (shortage) \end{cases} \quad (1)$$

Note that all the parameters p, v, s, w_r are taken in manufacturer's currency and the salvage value v is taken as an income from the disposal of each of the q-D leftover.

Since the demand, $D(p,\epsilon) = g(p)+\epsilon$ the retailer's profit function (1) for ordering q units and keeping selling price p is given by,

$$\Pi(p,q) = \begin{cases} p(g(p)+\epsilon)+v(q-g(p)-\epsilon)-qw_r \ if \ D \le q \\ pq-s(g(p)-q+\epsilon)-qw_r \ if \ D > q \end{cases}$$

Put $g(p)=g$ and define $z = q - g(p) = q - g$ i.e. $q= z + g$, for the additive demand error.

Now $D \le q \Leftrightarrow g+\epsilon \le q \Leftrightarrow \epsilon \le q - g \Leftrightarrow \epsilon \le z$

and similarly $D > q \Leftrightarrow \epsilon > z$.

$$\Pi(z,p) = \begin{cases} p(g+\epsilon)+v(z-\epsilon)-w_r(z+g) \ if \ \epsilon \le z \\ p(z+g)-s(\epsilon-z)-w_r(z+g) \ if \ \epsilon > z \end{cases} \quad (2)$$

The equation (2) describes the profit function for the retailer in the manufacturer currency. Note that the parameter q is replaced by z. Now the retailer wants to find the optimal order quantity q say $q*$ and optimal price $p= p*$ to maximize his expected profit. In order to do this he must find optimal values of the price p and the parameter z, say $p*$ and $z*$ respectively which maximizes his expected profit so that he can determine the optimal order $q*=z*+g(p*)$. The profit Π is a function of the random variable ϵ with support [A, B]. Thus the retailer's expected profit is given by,

$$E \ \Pi(z,p) \ = \int_A^B \Pi(z,p) f(u)du .$$

(Here we take $\epsilon=u$ for simplicity in (2)).
Then we get the expected profit in terms of the parameters z and p as,

$$E \ \Pi(z,p) \ = \int_A^z p(g+u)+v(z-u)-w_r(z+g) \ f(u)du$$
$$+ \int_z^B p(z+g)-s(u-z)-w_r(z+g) \ f(u)du \quad (3)$$

Define $\Lambda(z) = \int\limits_{A}^{z} (z-u) f(u) du$ [expected leftovers]

and

$$\Phi(z) = \int\limits_{Z}^{B} (u-z) f(u) du \quad \text{[expected shortages]}.$$

Then the expected profit of the retailer as a function of z and p is given by

$$E\, \Pi(z,p) = (p-w_r)(g+\mu) - (w_r-v)\Lambda - (p+s-w_r)\Phi \quad (4)$$

as derived in r[2].

Where $\mu = \int\limits_{A}^{B} u f(u) du$ in the equation (4) and it

gives the expected value of the randomness u in the demand D.

We use Whitin's method given in [10] to maximize the expected profit function. In this method first we keep p fixed in (4) and use the second order optimality conditions $\dfrac{\partial E}{\partial z} = 0$ and $\dfrac{\partial^2 E}{\partial z^2} < 0$ to find the

optimum value of z^* as a function of p. Then we substitute the value of z^* in the expected profit (4) so that it becomes a function of single variable p and hence the optimal p^* can also be obtained. The authors have already derived the optimal policies given below, in [2].

$$z^* = F^{-1}\left(\frac{p+s-w_r}{p+s-v} \right) \quad (5)$$

Where $F(z) = \int\limits_{A}^{z} f(u)\, du$ is the CDF.

This z^* gives the optimum solution for maximum profit as a function of p.

Now substitute this z^* in $E[\Pi(z,p)]$ and obtain optimum p^* using the second order optimality criteria. Hence the retailer's optimal order $q = q^*$ is given by,

$$q^* = g(p^*) + z^* = g(p^*) + F^{-1}\left(\frac{p^* + s - w_r}{p^* + s - v} \right) \quad (6)$$

Also the manufacturer's profit when the buyer bears the risk is [(selling price of seller)-(cost of purchase to seller]× no. of units sold,
$\Pi_m = (w-c)q^*.$ (7)

Case-2: Seller bears the exchange rate risk
In the case-2 we assume that the manufacturer bears the exchange rate risk and retailer does not. Thus the retailer pays w per unit in manufacturer's currency at any point of time and the manufacturer will get according to the existing exchange rate. So the manufacturer will be getting $wr/(r(1+\in_r)) = w_m$ per unit on the settlement day in his currency. Now the retailer's profit function, his expected profit and optimal policies to get maximum expected profit can

be obtained by replacing wr by w in case-1. So we get the retailer's profit as,

$$\Pi(p,q) = \begin{cases} [pD + v(q-D)] - [qw] & \text{if } D \le q \text{ (overstocking)} \\ [pq] - [s(D-q) + qw] & \text{if } D > q \quad \text{(shortage)} \end{cases} \quad (8)$$

And his expected profit as,

$$E\, \Pi(z,p) = (p-w)(g+\mu) - (w-v)\Lambda - (p+s-w)\Phi \quad (9)$$

The optimal value of z is given by

$$z^* = F^{-1}\left(\frac{p+s-w}{p+s-v} \right) \quad \text{and hence the optimum}$$

order quantity is,

$$q^* = g(p^*) + z^* = g(p^*) + F^{-1}\left(\frac{p^* + s - w}{p^* + s - v} \right) (10)$$

Also the manufacturer's profit when the buyer bears the risk is [(selling price of seller)-(cost of purchase to seller]× no. of units sold.
$\Pi_m = (w_m - c)q^*.$ (11)

VI. SENSITIVITY ANALYSIS

We assume linear demand with additive demand error u which follows the uniform distribution $f(u)$ with support [A,B]. Then we compute the optimum policies and maximum expected profit of the retailer and manufacturer obtained above, using MAPLE software when anyone of them bears the exchange rate risk. We compute the optimum values by using normal distribution $\psi(\in_r)$ in the exchange rate error $\in_r$ with support [-0.1,0.1]. In case-1 and case-2 we also compare it with the policies obtained in [1], for the generalized beta distribution in the exchange rate error for each of the case positive, negative and symmetrical beta distribution. We have also tested the sensitivity of the model by comparing uniform and beta distribution in the exchange rate error in [12] and [13] for additive and multiplicative demand errors.

Recall the general normal probability density function is given by,

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma} \right)^2}, \quad -\infty < x < \infty \quad \text{with}$$

mean μ and standard deviation σ. The truncated normal distribution for the error support (a,b) is

given by $\psi(x) = \dfrac{\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma} \right)^2}}{\Phi(b') - \Phi(a')}, \quad a < x < b$,

where $\Phi(x) = \int\limits_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy$ is the CDF of the standard normal density function,

$a' = \dfrac{a-\mu}{\sigma}, b' = \dfrac{b-\mu}{\sigma}$. And the four parameter beta density function is given by,

$$f(y/a,b,\alpha,\beta) = \frac{(y-a)^{\alpha-1}(b-y)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}}$$

where $a \le y \le b, \alpha, \beta > 0$ and by taking $y = a + (b-a)x$ we get its transformation in the standard beta distribution as ,

$$f(x/0,1,\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}.$$

Where $0 \le x \le 1, \alpha, \beta > 0$ and

$$B(\alpha,\beta) = \int_0^1 t^{1-\alpha}(1-t)^{1-\beta}dt.$$

We assume the following parameter values:
Demand support =[A,B]=[-3500,1500]

Mean Demand = $\mu = \frac{A+B}{2} = -1000$

Linear demand $g(p) = a - bp, a = 100000, b = 1500$
v =Salvage value = 10
s=Penalty cost = 5
c= cost of manufacturing per unit = 20
r=current exchange rate = 45

The following computation is done through Maple software. We have assumed that the mean μ=0.0001 for the exchange rate error in the interval [-0.1,0.1] and the support of the exchange rate covers 6 standard deviations of the normal distribution and hence $\sigma = \frac{0.1-(-0.1)}{6} = \frac{2}{6} = \frac{1}{3} \approx 0.33$

Case-1 *Retailer Bears the risk*

Distribution	Parameters of the dist.	p*	q*	Seller's selling price w*	Optimum exp.profit of buyer	Optimum exp.profit of seller
Beta	α=1 , β=3	53.45	18047	43.82	195075	429886
Normal		53.7	17639	42.13	185952	390476
Variation%		0.467727	2.26076356	3.8566864	4.6766628	9.16754674
		↑	↓	↓	↓	↓
Beta	α=3 , β=1	53.95	17234	40.61	177080	355344
Normal		53.7	17639	42.13	185952	390476
Variation%		0.463392	2.3500058	3.7429205	5.0101649	9.88675762
		↓	↑	↑	↑	↑
Beta	α=1 , β=1	53.7	17640	42.13	185970	390548
Normal		53.7	17639	42.13	185952	390476
Variation%		0	0.00566893	0	0.009679	0.01843563
			↓		↓	↓
Beta	α=2 , β=5	53.49	17989	43.56	193761	423989
Normal		53.7	17639	42.13	185952	390476
Variation%		0.392597	1.94563344	3.2828283	4.0302228	7.9042145
		↑	↓	↓	↓	↓
Beta	α=5 , =2	53.91	17292	40.82	178336	360142
Normal		53.7	17639	42.13	185952	390476
Variation%		0.389538	2.0067083	3.2092112	4.2705903	8.42278879
		↓	↑	↑	↑	↑

Case-2 *Manufacturer Bears the risk*

Distribution	Parameters of the dist.	p*	q*	Buyer's purchase cost w_r*	Optimum exp. profit of buyer	Optimum exp. profit of seller
Beta	α=1 , β=3	53.45	18047	41.62	195075	429886
Normal		53.7	17639	42.14	185952	390476
Variation%		0.4677268	2.26076356	1.24939933	4.67666282	9.1675467
		↑	↓	↑	↓	↓
Beta	α=3 , β=1	53.95	17234	42.63	177080	355344
Normal		53.7	17639	42.14	185952	390476
Variation%		0.463392	2.3500058	1.14942529	5.0101649	9.8867576
		↓	↑	↓	↑	↑
Beta	α=1 , β=1	53.7	17640	42.13	185970	390548
Normal		53.7	17639	42.14	185952	390476
Variation%		0	0.00566893	0.02373606	0.00967898	0.0184356
			↓	↑	↓	↓
Beta	α=2 , β=5	53.49	17989	41.7	193761	423989
Normal		53.7	17639	42.14	185952	390476
Variation%		0.3925967	1.94563344	1.05515588	4.0302228	7.9042145
		↑	↓	↑	↓	↓
Beta	α=5 , β=2	53.91	17292	42.56	178336	360142

Normal		53.7	17639	42.14	185952	390476
Variation%		0.3895381	2.0067083	0.98684211	4.27059035	8.4227888
		↓	↑	↓	↑	↑

VII. CONCLUSION

We demonstrate normally distributed exchange rate fluctuation when the retailer or manufacturer undertakes to share the exchange rate risk and the demand error is modelled in the additive form in the news vendor framework. We also have compared the exchange rate effect with the generalized beta distribution error, which is evident from the above tables.

REFERENCES

[1] F.J. Arcelus, Ravi Gor, G. Srinivasan, *Foreign exchange transaction exposure in a newsvendor setting*, European Journal of Operational Research, 227(2013) 552-557

[2] Sanjay Patel and Ravi Gor, *Exchange rate risk sharing contract model*, IOSR Journal of Mathematics(IOSR-JM),e-ISSN: 2278-5728, p-ISSN: 2319-765X.,Volume11,Issue 2, Version-I (Mar-Apr. 2015),PP 47-52

[3] Goel, M., 2012. *Management of transaction exposure: a comparative analysis of MNCs in India*. International Journal of Service Science, Management, Engineering and Technology 3, 37–54.

[4] Eitemann, D.K., Stonehill, A.I., Moffett, M.H., 2010. *Multinational Business Finance*,12th ed. Prentice Hall, Boston.

[5] Shubita, M.F., Harris, P., Malindretos, J., Bobb, L.M., 2011. *Foreign exchange exposure: an overview*. International Research Journal of Finance and Economics 78, 171–177.

[6] Petruzzi, N.C., Dada, M., 1999. *Pricing and the newsboy problem: a review with extensions.* Operations Research 47, 183–194.

[7] Mills, E.S., 1958. *Uncertainty and price theory,* Quarterly Journal of Economics 73, 116–130.

[8] Karlin, S., Carr, C.R., 1962. *Prices and optimal inventory policy*, In: Arrow, K.J., Scarf, H. (Eds.), Studies in Applied Probability and Management Science. Stanford University Press, Palo Alto, CA, pp. 159–172.

[9] Sanjay Patel and Ravi Gor, *Transaction exposure risk modelled in a newsvendor framework under the multiplicative demand error*, IOSR Journal of Mathematics(IOSR-JM),e-ISSN: 2278-5728, p-ISSN: 2319-765X., Volume11,Issue 2, Version-I (Mar-Apr. 2015),P 53-59

[10] Whitin, T. M., 1955. *Inventory control and price theory*, Management Sci. 2 61-68.

[11] Sanjay Patel and Ravi Gor. *Foreign exchange transaction exposure within a newsvendor frame work under hybrid demand distribution,* IOSR Journal of Mathematics(IOSR-JM), *e-ISSN: 2278-5728, p-ISSN: 2319-765X* ,Volume11,Issue 5, Version-I (Sep-Oct. 2015),P 51-58.

[12] Sanjay Patel and Ravi Gor, *A newsvendor framework of exchange rate risk with uniformly distributed exchange rate error*, International Journal of Research in Engineering and Technology,e-ISSN:2319-1163 ,p-ISSN: 2321-7308,May-2016,Volume-5,Issue-5, 412-416

[13] Sanjay Patel and Ravi Gor, *A newsvendor framework of transaction exposure model with uniformly distributed exchange rate error and isoelastic demand* , International Journal of Science, Engineering and Technology Research,e-ISSN:2278-7798,June-2016,Volume-5,Issue-6, 1904-1908