Fuzzy theory based solution of multi objective resource allocation problem: A case study of food factory

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Abstract: *In developing unorganised business sector it is necessary to utilise the resources in optimal way. Analysis of the resources like manpower, material, marketing etc. is essential to entrepreneur. This study addresses the issue that an unorganised business sector faces during the production, their objectives are clear but knowingly or un-knowingly they may over use resources. Here we have collected data form food factory which is producing multi object in a day and massive amount of resource is used during the pre production. Problem has been formulated in LPP and applied linear and exponential function to get efficient solution. Certain adjustment parameter are derived which can further assist entrepreneur to take decision in future. Here LINGO15 is deployed to solve the MLPP. Such a complex problem can be divided in to multi stage and further analyse for more simplicity.*

Key words: *LPP, multi objective, resource management, LINGO15.*

1. Introduction

Generally, manufacturing environment can be characterised by the following features: dynamic, global, and customer-driven. The trend to tailor-make products to meet customer needs has led to reduction in batch quantities, increase in product varieties, and shorter product life cycles. The need for flexibility, efficiency, and quality has imposed a major change in manufacturing industries. Consequently, it becomes increasingly difficult for manufacturers to predict customer demands precisely and to formulate effective production and inventory plans. In this connection, most companies do not practise the incorporation of uncertainties (e.g. production and demand uncertainties) into their production planning processes, owing to the difficulties arising in prediction [1]. However, some effective means must be introduced to minimise the negative effects of uncertainties in designing manufacturing systems to reduce costs and other wastage. Despite the large amount of literature on manufacturing systems design, very few papers have considered the impact of uncertainties in the design process. Bitran and Tirupati [2] considered the resource allocation problem according to the level of uncertainties. Tang [3] investigated the impact of uncertainties on production and inventory in a multistage production system. Wein [4] suggested another approach to managing uncertainties when only a single production line is used, by integrating inventory control and scheduling. However, his model did not include the changeover time for changing the production set-ups because of alterations to the production sequence. Erlebacher and Singh [5] considered the problem of allocating processing time variability in a synchronous assembly line.

Major four recourse namely: manpower, machine power, material, money and method are considered as resources, whether it is use or not it had some cost behind it. If we have to stock it at some place then some cost is required. If we manage all this recourses in optimal way then it can optimize our profit or loss. Certain data like unit per man hour, efficiency, plan conformance etc can actually be derived and further used to plan accordingly. Some time entrepreneur wish to have multi objective for their business i.e. They may aim to minimize cost max profit, minimize risk, minimize the use of manpower maximize the use of machine power etc. This multi objective aim may perplex the entrepreneur while taking the decision.

In this study we have consider a food manufacturing industries which produce multi product multi time using and our target is to optimize their multi objective cost and risk so multi-objective problem is formulated in next coming subsequent section and its solution is obtained by fuzzy programming technique to solve multi objective linear programming problem.

1.1. Single objective linear programming base resource allocation problem.

Some entrepreneur aims for single objective at a time, such single objective recourse allocation problem can be formulated in LPP as follows [7]

Minimum $Z=\sum_{i=1}^{n} c_i x_i$

Subject to

$\sum_{i=1}^{m} a_i x_i \ (\leq, =, \geq) B_j, \ j = 1,2,3 \ldots n$
(n such constraints)

Where $x_i \geq 0$ .... (1)

Where, c_{ij}= associated cost vector, B_j= resource vector, a_i= associated quantity of resource required for production of x_i

This single objective linear programming base resource allocation problem has constraint which can be either in equality or inequality form.

1.2. Multi objective linear programming problem.

Most of the entrepreneur now a day's do not have a aim of single objective but they wish to target multi objective i.e.` they not only try to minimize cost but try to minimize some recourse so that their business can grow in best of manner. In competitive world entrepreneur need to be aware of competition and should monopolised business. Their important objective could be to minimize risk using the same set of constraints. Such general multi objective linear programming problem can be defined as under[8].

Minimum $z_r=\sum_{i=1}^{n} c_i^r x_i$, *r=1, 2, 3, 4 ...k*

Subject to

$\sum_{i=1}^{m} a_i x_i \ (\leq, =, \geq) B_j, \ j = 1,2,3 \ldots n$ (n such constraints)

Where $x_i \geq 0$ .... (2)

1.2. Formulation Resource allocation problem of sandwich factory in MOLPP

In the following sections, the considered sandwich factory resource allocation problem and formulated in Multi objective linear programming problem. First the process system of sandwich industry is introduced. Then the objective as well as constraints are stated, namely to reduce the production costs on the long run by assigning the products and minimize the risk.

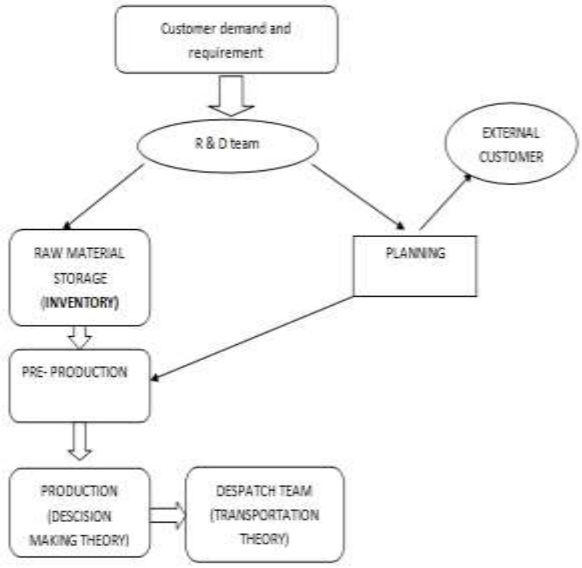

Fig: 1 Process Diagram

System: Figure 1 explains the process in which sandwich factory works. In this paper we have directly focus onto production section we have assume that pre products for all the production assembly is available and there is no downtime regards to resource for the product. For production certain resource like man power, material, machinery is required and it should be available before production. Also some pre production is also requiring before the production which we have to assume that, it is available before production. Here we can apply inventory control models for inventory which needs to be stored before preproduction, in production we have applied linear programming problem and solved it using LINGO. In despatch section we can apply transportation problem. So the sandwich factory problem can be divided into three

1) Inventory control model

2) linear programming problem

3) Transportation problem.

1.2.1 Mathematical Formulation:

For mathematical formulation we assume that sandwich industry is producing 'n' product and it require some resource like man power, machine power, and material to make such n product. Here we assume that

X1= product 1

X2= Product2

. .

. .

Xn= Product n

c_j, r_j j= 1,2 ...,n, associated cost vectors (Cost, Risk)

B_j be the associated resource vector (Like Bread, Tomato, Cheese, etc.)

Here we assume that the costs associated to produce such product are c_j, r_j j= 1,2 ...,n,and B_j be the associated resource vector then the Objective function: cost function Z1 and risk function Z2 for 'n' product can formulated as follows:

Min Z1= $c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Mini Z= $r_1x_1 + r_2x_2 + \cdots + r_nx_n$

Other all the constrain associated with different resources can be formulated as follows

Subject to constraints are

Bread constraint

$a_1x_1 + a_1x_2 + \cdots + a_nx_n(\leq, =, \geq)\, B_1$

Here, $a_i\, is\, amount\, of\, bread\, required\, x_i$ here

B_1 = Total amount of bread required for entire production

Tomato constraint

$b_1x_1 + b_2x_2 + \cdots + b_nx_n \leq, =, \geq\ B_2$

Here, $b_i\, is\, amount\, of\, tomato\, required\, x_i$ here

B_2 =Total amount of tomato required for total production

Cheese constraints

$c_1x_1 + c_2x_2 + \cdots + c_nx_n \leq, =, \geq\ B_3$

Here, $c_i\, is\, amount\, of\, cheese\, required\, x_i$ here

B_3= total amount of cheese required for total production

Potato constraints

$d_1x_1 + dx_2 + \cdots + d_nx_n \leq, =, \geq\ B_4$

Here, $d_i\, is\, amount\, of\, bread\, required\, x_i$ here

B_4 = Total amount of bread required for total production

Tiki constraints

$e_1x_1 + e_2x_2 + \cdots + e_nx_n \leq, =, \geq\ B_5$ (tiki constraints)

Here, $e_i\, is\, amount\, of\, Tiki\, required\, x_i$ here

B_5 = Total amount of tiki required for total production

Manpower constraints

$f_1x_1 + f_2x_2 + \cdots + f_nx_n \leq, =, \geq\ B_6$

Here, $f_i\, is\, amount\, of\, man\, power\, required\, x_i$ here

B_6 = Total amount of man power required for total production

Machine constraints

$g_1x_1 + g_2x_2 + \cdots + a_nx_n \leq, =, \geq\ B_7$

Here, $g_i\, is\, amount\, of\, bread\, required\, x_i$ here

B_7 =Total amount of bread required for total production

Chicken constraints

$h_1x_1 + h_2x_2 + \cdots + h_nx_n \leq, =, \geq\ B_8$

Here, $h_i\, is\, amount\, of\, chicken\, required\, x_i$ here

B_8 =Ttotal amount of chicken required for total production

	Z_1	Z_2	………	Z_k
$f_1(x_1, x_2, x_3........x_n)$	Z_{11}	Z_{21}	………	Z_{k1}
$f_2(x_1,x_2, x_3........x_n)$	Z_{12}	Z_{22}	………	Z_{2k}
……			………	
$f_n(x_1,x_2, x_3........x_n)$	Z_{1n}	Z_{2n}	………	Z_{kn}

$x_1, x_2, \dots \dots, x_n \geq 0$

So the general multi-objective mathematical model for the sandwich factory can be written as

Min Z1= $c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Mini Z2= $r_1x_1 + r_2x_2 + \cdots + r_nx_n$

Subject to constraints are

$a_1x_1 + a_2x_2 + \cdots + a_nx_n \leq, =, \geq\ B_1$ (Bread constraints)

$b_1x_1 + b_2x_2 + \cdots + b_nx_n \leq, =, \geq\ B_2$ (tomato constraints)

$c_1x_1 + c_2x_2 + \cdots + c_nx_n \leq, =, \geq\ B_3$ (cheese constraints)

$d_1x_1 + dx_2 + \cdots + d_nx_n \leq, =, \geq\ B_4$ (Potato constraints)

$e_1x_1 + e_2x_2 + \cdots + e_nx_n \leq, =, \geq\ B_5$ (tiki constraints)

$f_1x_1 + f_2x_2 + \cdots + f_nx_n \leq, =, \geq$ (manpower constraints)

$g_1x_1 + g_2x_2 + \cdots + a_nx_n \leq, =, \geq$ (machine constraints)

$h_1x_1 + h_2x_2 + \cdots + h_nx_n \leq, =, \geq\ B_8$ (chicken constraints)

$where\ x_1, x_2, \dots \dots, x_n \geq 0$ .... (3)

2: Method to solve LPP using fuzzy linear and exponential membership function:

Step 1: The formulated linear programming problem first solve by using single objective function. Here we have use LINGO15 software to solve this single objective LPP and derive optimal solution say f_1 (x_1, x_2, x_3........x_n), for first objective Z_1 and then obtain other objective value with the same solution. Procedure repeat same for Z_2......... Z_k

.

Step 2: Corresponding to above data we can construct matrix which can give various alternate optimal value.

Here,

Zij : indicated optimal solution of objective i using solution of objective j, i= 1, 2, ..., k and j=1,2,3...n.

Here two different membership function are utilized to find efficient solution of this multi-objective resource allocation problem.

Fuzzy linear membership function is defined as follows[8]

$$\sigma^l(z_k) = \begin{cases} 0, & if \quad z_k \leq L_k \\ 1 - \frac{z_k - L_k}{U_k - L_k}, & if \quad L_K < Z_k < U_k \\ 1, & if \quad z_k \geq U_k \end{cases}$$

Here for any Z_k, minimum (Z_{k1},Z_{k2}.........,Z_{kn}) = L_k and Maximum (Z_{k1},Z_{k2}.........,Z_{kn}) = U_k

Corresponding to data adjustment parameter: σ can be obtained which will give us degree of satisfaction of both the objectives.

By using linear membership function The LPP described in (3) can be converted into crisp model as follows

Maximum σ

Subject to

$Z_k + \sigma(U_k - L_k) \leq U_k$

$\sum_{i=1}^{m} a_ix_i\ (\leq, =, \geq) B_j,\ j = 1,2,3 \dots n$ (n such constraints) Where $x_i \geq 0$

Where $x_{ij} \geq 0$

Fuzzy exponential membership function is defined as follows[8]

$$\sigma^l(z_k) = \begin{cases} 1, & if \quad z_k \leq L_K \\ \frac{e^{-s\beta_k(x)} - e^{-s}}{1 - e^{-s}}, & if \quad L_K < Z_k < U_k \\ 0, & if \quad z_k \geq U_k \end{cases}$$

Where $\beta_k(x) = \frac{z_k - L_k}{U_k - L_k}$, $k = 1,2,3 \dots., K$

Corresponding to this exponential membership function the linear programming problem will be as under

Maximum = σ

Subject to

$e^{-s\beta_k(x)} - (1 - e^{-s})\,\sigma \ge e^{-s})$

$\sum_{i=1}^{m} a_i x_i\ (\le, =, \ge) B_j,\ j = 1,2,3 \ldots n$ (n such constraints) Where $x_i \ge 0$

Solution of this model will give you an efficient solution with respect to different shape parameter s.

3.0. Case study

In this paper we have taken the case study of a sandwich factory XYZ. This factory have production capacity of 200000 sandwich per day and it requires tremendous amount of resource like manpower, machine power, material, money and proper method to achieve target per day. This factory is divided in to units and each unit are divided into lines. So in total approximate 30 lines require 1000 people and huge amount of machinery to achieve daily target. Here we have taken the data from 1 line and similarly we can get the data for 30 lines. One line that work in two shifts require manpower, machine power, material following data have been collected.

Table-1 cost of product per unit

product	Cost per unit
Cheese s/w	20
Cheese burger	30
Allo tiki	20
Chicken pasta	40

Table-1 cost of product per unit

Cost table Indicates cost per item

product	Cheese s/w	Cheese burger	Allo tiki	Chicken pasta
Bread Rs. Per unit	3	7	4	10
Tomato Rs. Per unit	2	1	2	2
Potato Rs. Per unit	2	0	0	0
Cheese Rs. Per unit	4	5	5	5
Rent Rs. Per unit	3	2	2	2
Tiki Rs. Per unit	0	5	4	0
Chicken Rs. Per unit	0	0	0	10
Pasta Rs. Per unit	0	0	0	5
Manpower Rs. Per unit	2	3	2	3
Machine Rs. Per unit	2	3	0	3
Other Rs. Per unit	4	4	1	2

Table-2: Cost table Indicates cost per item

Requirement of resource to produce all type of sandwich

product	Cheese s/w	Cheese burger	Allo tiki	Chicken pasta	Availability
Bread	1	1	1	1	210
Tomato	30	30	50	50	4000
Potato	30	0	0	0	15000
Cheese	20	25	25	25	4000
Tiki	0	50	50	0	110
Chicken	0	0	0	40	4000
Pasta	0	0	0	25	3000
Manpower	3	3	3	3	960
Machine	2	2	0	2	600
Other	50	50	50	50	210

Table-3: Requirement of resource to produce all type of sandwich

By considering all this data the MOLPP can be formulated as follows

x_1= Number of Chess s/w sandwich

x_2 = Number of Cheese burger sandwich

x_3 = Number of Allo tiki sandwich

x_4 =Number of Chicken pasta sandwich

By using this variables objectives related with cost and risk can be formulated as follows

Cost = $(3+2+2+4+3+4+2+0+2+2+4)\,x_1 + (7+1+0+5+2+5+0+0+3+3+4)x_2 + (4+2+0+5+2+5+0+0+2+0+1)\,x_3$

$+ (10+2+0+5+2+0+10+5+3+3+2)\,x_4$

Risk = $15x_1 + 17.5x_2 + 7x_3 + 10x_4$

Both the objective can be written in notation form as follows

Minz1 = $20x_1 + 30x_2 + 20x_3 + 40x_4$
Minz2 = $15x_1 + 17.5x_2 + 7x_3 + 10x_4$

All the constraints of the problem related with resource can be written as

$x_1 + x_2 + x_3 + x_4 <= 210$ (Bread constraint)
$30x_1 + 30x_2 + 50x_3 + 50x_4 >= 4000$ (tomato constraint)
$20x_1 + 25x_2 + 25x_3 + 25x_4 >= 4000$ (cheese constraint)
$30x_1 <= 15000$ (potato constraint)
$0x_1 + 1x_2 + 1x_3 + 0.0x_4 < = 110$ (tiki constraint)
$3x_1 + 3x_2 + 3x_3 + 3x_4 < = 960$ (manpower constraint)
$2x_1 + 2x_2 + 2x_3 + 2x_4 < = 600$ (machine constraint)
$0.04x_4 <= 4000$ (chicken constraint)
$0.025x_4 <= 3000$ (Bread constraint)
$x_1 >= 20$ (minimum constraint)
$x_2 >= 20$ (minimum constraint)
$x_3 >= 20$ (minimum constraint)
$x_4 >= 20$ (minimum constraint)
$x_1 >= 0,\ x_2 >= 0,\ x_3 >= 0,\ x_4 >= 0$

Solution Methods:

By using LINGO solution of 1st and 2nd objective individually can obtained as follows

Global optimal solution found.

Objective Min Z_1: 3950, where

$x_1 = 37.50$, $x2 = 20$, $x_3 = 90$, $x_4 = 20$

Objective Min Z_2: 1620, where

$x_1 = 20$, $x_2 = 20$, $x_3 = 90$, $x_4 = 34$

Using this method described above we can find matrix which can give various alternate optimal value as follows

z_k / $f_i(x_1, x_2, x_3 \ldots\ldots x_n)$	z_1 (cost)	z_2 (minimum risk)
$f_1(37,20,90,20)$	3950	1735
$f_1(20,20,90,34)$	4160	1620

By considering maximum and minimum value objective function of cost and minimum

z_k / $f_i(x_1, x_2, x_3 \ldots\ldots x_n)$	z_1 (cost)	z_2 (minimum risk)
$f_1(37,20,90,20)$	3950 (L_1)	1735 (u_2)
$f_1(20,20,90,34)$	4160 (u_1)	1620 (L_2)

Using the linear membership function we get the following model with additional constraints as follows

Maximum: σ
Subject to
$20x1 + 30x2 + 20x3 + 40x4 + \sigma(210) \leq 4160$; $(Z_1 + \sigma(U_1 - L_1) \leq U_1)$
$15x1 + 17.5x2 + 7x3 + 10\,x4 + \sigma(115) \leq 1735$; $(Z_2 + \sigma(U_2 - L_2) \leq U_2)$
$x_1 + x_2 + x_3 + x_4 <= 210$ (Bread constraint)
$30x_1 + 30x_2 + 50x_3 + 50x_4 >= 4000$ (tomato constraint)
$20x_1 + 25x_2 + 25x_3 + 25x_4 >= 4000$ (cheese constraint)
$30x_1 <= 15000$ (potato constraint)
$0x_1 + 1x_2 + 1x_3 + 0.0x_4 < = 110$ (tiki constraint)
$3x_1 + 3x_2 + 3x_3 + 3x_4 < = 960$ (manpower constraint)
$2x_1 + 2x_2 + 2x_3 + 2x_4 < = 600$ (machine constraint)
$0.04x_4 <= 4000$ (chicken constraint)
$0.025x_4 <= 3000$ (Bread constraint)
$x_1 >= 20$ (minimum constraint)
$x_2 >= 20$ (minimum constraint)
$x_3 >= 20$ (minimum constraint)
$x_4 >= 20$ (minimum constraint)
$x_1 >= 0,\ x_2 >= 0,\ x_3 >= 0,\ x_4 >= 0$

By using LINGO15 solution can be obtain as follows

Objective valueσ: 0.4842105,
$x_1 = 28.47368$, $x2 = 20$, $x3 = 90$, $x_4 = 27.22105$

Using this exponential membership function and solving it with lingo we get following LPP

Maximum = σ

Subject to:

$$e^{-1\left(\frac{20*x1+30*x2+20*x3+40*x4-3950}{210}\right)}-(1-e^{-1})\sigma \geq e^{-1},$$

$$e^{-1\left(\frac{15*x1+17.5*x2+7*x3+10*x4-1620}{115}\right)}-(1-e^{-1})\sigma \geq e^{-1}$$

$x_1 + x_2 + x_3 + x_4 <= 210$ (Bread constraint)
$30x_1 + 30x_2 + 50x_3 + 50x_4 >= 4000$ (tomato constraint)
$20x_1 + 25x_2 + 25x_3 + 25x_4 >= 4000$ (cheese constraint)
$30x_1 <= 15000$ (potato constraint)
$0x_1 + 1x_2 + 1x_3 + 0.0x_4 < =110$ (tiki constraint)
$3x_1 + 3x_2 + 3x_3 + 3x_4 < =960$ (manpower constraint)
$2x_1 + 2x_2 + 2x_3 + 2x_4 < =600$ (machine constraint)
$0.04x_4 <= 4000$ (chicken constraint)
$0.025x_4 <= 3000$ (Bread constraint)
$x_1 >= 20$ (minimum constraint)
$x_2 >= 20$ (minimum constraint)
$x_3 >= 20$ (minimum constraint)
$x_4 >= 20$ (minimum constraint)
$x_1 >= 0; x_2 >= 0; x_3 >= 0; x_4 >= 0$

Solution of this LPP will be given by

Objective value: $\sigma = 0.6071889$
$x_1 = 28.47368$, $x_2 = 20.00000$
$x_3 = 90.00000$, $x_4 = 27.22105$

4. CONCLUSION

Fuzzy theory based solution of multi objective resource allocation problem has been derived . We got high degree of satisfaction using exponential membership function compare to linear membership function for this resource allocation problem.

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