Intuitionistic Anti-Fuzzy Hx Ring

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Abstract - *In this paper, we define the notion of intuitionistic anti fuzzy HX subring of a HX ring and some of their related properties are investigated. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic anti-fuzzy HX subring and discuss some of its properties. We discuss the concept of an image, pre-image of an intuitionistic anti-fuzzy HX subring and homomorphic, anti homomorphic properties of an intuitionistic anti- fuzzy HX subring are discussed.*

Keywords - Intuitionistic fuzzy set, fuzzy HX ring, intuitionistic anti-fuzzy HX subring, homomorphism and anti homomorphism of an intuitionistic anti-fuzzy HX subring, image and pre-image of an intuitionistic anti-fuzzy HX subring.

I. INTRODUCTION

In 1965, Zadeh [12] introduced the concept of fuzzy subset μ of a set X as a function from X into the closed unit interval [0, 1] and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic, set theory, group theory, ring theory, real analysis, measure theory etc. In .1967, Rosenfeld [11] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [5] introduced the concept of HX group. In 1982 Wang-jin Liu[7] introduced the concept of fuzzy ring and fuzzy ideal. With the successful upgrade of algebraic structure of group many researchers considered the algebraic structure of some other algebraic systems in which ring was considered as first. In 1988, Professor Li Hong Xing [6] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2,3] gave the structures of HX ring on a class of ring. R.Muthuraj et.al[10]., introduced the concept of fuzzy HX ring. In this paper we define a new algebraic structure of an intuitionistic anti-fuzzy HX subring of a HX ring and investigate some related properties. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic anti-fuzzy HX subring and discuss some of its properties. Also we introduce the image and pre-image of an intuitionistic anti -fuzzy HX subring and discuss some of its properties.

2 PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R ,+, ·) is a Ring, e is the additive identity element of R and xy, we mean x.y

2.1 Definition [3]

Let R be a ring. In $2^R - \{\phi\}$, a non-empty set $\vartheta \subset 2^R - \{\phi\}$ with two binary operation ' + ' and '.' is said to be a HX ring on R if ϑ is a ring with respect to the algebraic operation defined by

i. A + B = {a + b / a ∈ A and b ∈ B} , which its null element is denoted by Q, and the negative element of A is denoted by – A.
ii. AB = {ab / a ∈ A and b ∈ B},
iii. A (B + C) = AB + AC and (B + C) A = BA + CA.

3.1 Intuitionistic anti-fuzzy HX subring of a HX Ring

In this section we define the concept of an intuitionistic anti fuzzy HX subring of a HX ring and discuss some related results.

3.1.1 Definition

Let R be a ring. Let H = { ⟨x, μ(x), η(x)⟩ / x∈R } be an intuitionistic fuzzy set defined on a ring R, where μ : R→[0,1], η : R→[0,1] such that $0 \leq \mu(x) + \eta(x) \leq 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring .An intuitionistic fuzzy subset $\lambda^H = \{ \langle A, \lambda^{\mu}(A), \lambda^{\eta}(A)\rangle / A \in \Re$ and $0 \leq \lambda^{\mu}(A) + \lambda^{\eta}(A) \leq 1\}$ of ℜ is called an intuitionistic fuzzy HX subring on ℜ or an intuitionistic fuzzy subring induced by H and if the following conditions are satisfied. For all A , B ∈ℜ,

i. $\lambda^{\mu}(A - B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,
ii. $\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,
iii. $\lambda^{\eta}(A - B) \leq \max\{\lambda^{\eta}(A), \lambda^{\eta}(B)\}$,
iv. $\lambda^{\eta}(AB) \leq \max\{\lambda^{\eta}(A), \lambda^{\eta}(B)\}$,

where $\lambda^{\mu}(A) = \max\{\mu(x) /$ for all $x \in A \subseteq R\}$ and $\lambda^{\eta}(A) = \min\{\eta(x) /$ for all $x \in A \subseteq R\}$.

3.1.2 Definition

Let R be a ring. Let H = { ⟨x, μ(x), η(x)⟩ / x∈R } be an intuitionistic fuzzy set defined on a ring R, where μ : R→[0,1], η : R→[0,1] such that $0 \leq \mu(x) + \eta(x) \leq 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. An intuitionistic fuzzy subset $\lambda_H = \{ \langle A, \lambda_{\mu}(A), \lambda_{\eta}(A)\rangle / A \in \Re$ and $0 \leq \lambda_{\mu}(A) + \lambda_{\eta}(A) \leq 1\}$ of ℜ is called an intuitionistic anti-fuzzy HX subring of ℜ or an intuitionistic anti-fuzzy subring induced by H if the following conditions are satisfied. For all A,B ∈ℜ,

i. $\lambda_{\mu}(A - B) \leq \max\{\lambda_{\mu}(A), \lambda_{\mu}(B)\}$,
ii. $\lambda_{\mu}(AB) \leq \max\{\lambda_{\mu}(A), \lambda_{\mu}(B)\}$,

iii. $\lambda_\eta(A-B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$,
iv. $\lambda_\eta(AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$.

where $\lambda_\mu(A) = \min\{\mu(x) /$ for all $x \in A \subseteq R\}$ and $\lambda_\eta(A) = \max\{\eta(x) /$ for all $x \in A \subseteq R\}$.

3.1.3 Remark

For an intuitionistic anti-fuzzy HX subring λ_μ of a HX ring $\mathfrak{R}$, the following result is obvious. For all $A, B \in \mathfrak{R}$,

i. $\lambda_\mu(A) \leq \lambda_\mu(0)$ and $\lambda_\mu(A) = \lambda_\mu(-A)$,
ii. $\lambda_\mu(A-B) = 0$ implies that $\lambda_\mu(A) = \lambda_\mu(B)$.
iii. $\lambda_\eta(A) \geq \lambda_\eta(0)$ and $\lambda_\eta(A) = \lambda_\eta(-A)$,
iv. $\lambda_\eta(A-B) = 0$ implies that $\lambda_\eta(A) = \lambda_\eta(B)$.

3.1.4 Theorem

If H is an intuitionistic anti-fuzzy subring of a ring R then the intuitionistic fuzzy subset λ_H is an intuitionistic anti-fuzzy HX subring of a HX ring $\mathfrak{R}$.

Proof

Let H be an intuitionistic anti-fuzzy subring of R.

i. $\max\{\lambda_\mu(A), \lambda_\mu(B)\} = \max\{\min\{\mu(x)/\text{for all } x \in A \subseteq R\}, \min\{\mu(y)/\text{for all } y \in B \subseteq R\}$
$= \max\{\mu(x_0), \mu(y_0)\}$
$\geq \mu(x_0 - y_0)$
$\geq \min\{\mu(x-y) / \text{for all } x-y \in A-B \subseteq R\}$
$\geq \lambda_\mu(A-B)$
$\lambda_\mu(A-B) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$.

ii. $\max\{\lambda_\mu(A), \lambda_\mu(B)\} = \max\{\min\{\mu(x)/\text{for all } x \in A \subseteq R\}, \min\{\mu(y)/\text{for all } y \in B \subseteq R\}$
$= \max\{\mu(x_0), \mu(y_0)\}$
$\geq \mu(x_0 \, y_0)$
$\geq \min\{\mu(xy) / \text{for all } xy \in AB \subseteq R\}$
$\geq \lambda_\mu(AB)$
$\lambda_\mu(AB) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$.

iii. $\min\{\lambda_\eta(A), \lambda_\eta(B)\} = \min\{\max\{\mu(x)/\text{for all } x \in A \subseteq R\}, \max\{\mu(y)/\text{for all } y \in B \subseteq R\}$
$= \min\{\mu(x_0), \mu(y_0)\}$
$\leq \eta(x_0 - y_0)$
$\leq \max\{\eta(x-y) / \text{for all } x-y \in A-B \subseteq R\}$
$\leq \lambda_\eta(A-B)$
$\lambda_\eta(A-B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$.

iv $\min\{\lambda_\eta(A), \lambda_\eta(B)\} = \min\{\max\{\mu(x)/\text{for all } x \in A \subseteq R\}, \max\{\mu(y)/\text{for all } y \in B \subseteq R\}$
$= \min\{\mu(x_0), \mu(y_0)\}$
$\leq \eta(x_0 \, y_0)$

$\leq \max\{\eta(x-y) / \text{for all } xy \in AB \subseteq R\}$
$\leq \lambda_\eta(AB)$
$\lambda_\eta(AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$.

Hence, λ_H is an intuitionistic anti-fuzzy HX subring of a HX ring $\mathfrak{R}$.

3.1.5 Theorem

Let G and H be any two intuitionistic fuzzy sets on R. Let γ_G and λ_H be any two intuitionistic anti-fuzzy HX subrings of a HX ring $\mathfrak{R}$ then their intersection, $\gamma_G \cap \lambda_H$ is also an intuitionistic anti-fuzzy HX subring of a HX ring $\mathfrak{R}$.

Proof

Let $G = \{\langle x, \alpha(x), \beta(x)\rangle / x \in R\}$ and $H = \{\langle x, \mu(x), \eta(x)\rangle / x \in R\}$ be any two intuitionistic fuzzy sets defined on a ring R.Then, $\gamma_G = \{\langle A, \gamma_\alpha(A), \gamma_\beta(A)\rangle / A \in \mathfrak{R}\}$ and $\lambda_H = \{\langle A, \lambda_\mu(A), \lambda_\eta(A)\rangle / A \in \mathfrak{R}\}$ be any two intuitionistic anti-fuzzy HX subrings of a HX ring $\mathfrak{R}$.

$\gamma_G \cap \lambda_H = \{\langle A, (\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\beta \cup \lambda_\eta)(A)\rangle / A \in \mathfrak{R}\}$

i. $(\gamma_\alpha \cap \lambda_\mu)(A-B) = \min\{\gamma_\alpha(A-B), \lambda_\mu(A-B)\}$
$\leq \min\{\max\{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max\{\lambda_\mu(A), \lambda_\mu(B)\}\}$
$= \max\{\min\{\gamma_\alpha(A), \lambda_\mu(A)\}, \min\{\gamma_\alpha(B), \lambda_\mu(B)\}\}$
$= \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}$
$(\gamma_\alpha \cap \lambda_\mu)(A-B) \leq \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}$.

ii. $(\gamma_\alpha \cap \lambda_\mu)(AB) = \min\{\gamma_\alpha(AB), \lambda_\mu(AB)\}$
$\leq \min\{\max\{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max\{\lambda_\mu(A), \lambda_\mu(B)\}\}$
$= \max\{\min\{\gamma_\alpha(A), \lambda_\mu(A)\}, \min\{\gamma_\alpha(B), \lambda_\mu(B)\}\}$
$= \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}$
$(\gamma_\alpha \cap \lambda_\mu)(AB) \leq \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap)(B)\}$.

iii. $(\gamma_\beta \cup \lambda_\eta)(A-B) = \max\{\gamma_\beta(A-B), \lambda_\eta(A-B)\}$
$\geq \max\{\min\{\gamma_\beta(A), \gamma_\beta(B)\}, \min\{\lambda_\eta(A), \lambda_\eta(B)\}\}$
$= \min\{\max\{\gamma_\beta(A), \lambda_\eta(A)\}, \max\{\gamma_\beta(B), \lambda_\eta(B)\}\}$
$= \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}$
$(\gamma_\beta \cup \lambda_\mu)(A-B) \geq \min\{(\gamma_\beta \cup \gamma_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}$.

iv. $(\gamma_\beta \cup \lambda_\eta)(AB) = \max\{\gamma_\beta(AB), \lambda_\eta(AB)\}$
$\geq \max\{\min\{\gamma_\beta(A), \gamma_\beta(B)\}, \min\{\lambda_\eta(A), \lambda_\eta(B)\}\}$
$= \min\{\max\{\gamma_\beta(A), \lambda_\eta(A)\}, \max\{\gamma_\beta(B), \lambda_\eta(B)\}\}$
$= \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}$
$(\gamma_\beta \cup \lambda_\mu)(AB) \geq \min\{(\gamma_\beta \cup \gamma_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}$.

Hence, $\gamma_G \cap \lambda_H$ is an intuitionistic anti-fuzzy HX subring of a HX ring $\mathfrak{R}$.

3.1.6 Theorem

Let G and H be any two intuitionistic fuzzy sets on R. Let γ_G and λ_H be any two intuitionistic anti-fuzzy HX subrings of a HX ring $\mathfrak{R}$ then their union, $\gamma_G \cup \lambda_H$ is also an intuitionistic anti-fuzzy HX subring of a HX ring $\mathfrak{R}$.

Proof

Let G = { ⟨x, $\alpha(x)$, $\beta(x)$⟩ / x∈R } and H = { ⟨x, $\mu(x)$, $\eta(x)$⟩ / x∈R } be any two intuitionistic fuzzy sets defined on a ring R.

Then, γ_G = { ⟨A, $\gamma_\alpha(A)$, $\gamma_\beta(A)$⟩ / A∈ℜ } and λ_H = { ⟨A, $\lambda_\mu(A)$, $\lambda_\eta(A)$⟩ / A∈ℜ } be any two intuitionistic anti-fuzzy HX subrings of a HX ring ℜ.Then,

$\gamma_G \cup \lambda_H$ = { ⟨A, $(\gamma_\alpha \cup \lambda_\mu)(A)$, $(\gamma_\beta \cap \lambda_\eta)(A)$⟩ / A∈ℜ }

Let A,B ∈ ℜ

i. $(\gamma_\alpha \cup \lambda_\mu)(A-B) = \max\{\gamma_\alpha(A-B), \lambda_\mu(A-B)\}$
$\leq \max\{\max\{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max\{\lambda_\mu(A), \lambda_\mu(B)\}\}$
$= \max\{\max\{\gamma_\alpha(A), \lambda_\mu(A)\}, \max\{\gamma_\alpha(B), \lambda_\mu(B)\}\}$
$= \max\{(\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(B)\}$

$(\gamma_\alpha \cup \lambda_\mu)(A-B) \leq \max\{(\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(B)\}$.

ii. $(\gamma_\alpha \cup \lambda_\mu)(AB) = \max\{\gamma_\alpha(AB), \lambda_\mu(AB)\}$
$\leq \max\{\max\{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max\{\lambda_\mu(A), \lambda_\mu(B)\}\}$
$= \max\{\max\{\gamma_\alpha(A), \lambda_\mu(A)\}, \max\{\gamma_\alpha(B), \lambda_\mu(B)\}\}$
$= \max\{(\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(B)\}$

$(\gamma_\alpha \cup \lambda_\mu)(AB) \leq \max\{(\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(B)\}$.

iii. $(\gamma_\beta \cap \lambda_\eta)(A-B) = \min\{\gamma_\beta(A-B), \lambda_\eta(A-B)\}$
$\geq \min\{\min\{\gamma_\beta(A), \gamma_\beta(B)\}, \min\{\lambda_\eta(A), \lambda_\eta(B)\}\}$
$= \min\{\min\{\gamma_\beta(A), \lambda_\eta(A)\}, \min\{\gamma_\beta(B), \lambda_\eta(B)\}\}$
$= \min\{(\gamma_\beta \cap \gamma_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}$

$(\gamma_\beta \cap \lambda_\mu)(A-B) \geq \min\{(\gamma_\beta \cap \gamma_\eta)(A), (\gamma_\beta \cap \lambda_\eta)(B)\}$.

iv. $(\gamma_\beta \cap \lambda_\eta)(AB) = \min\{\gamma_\beta(AB), \lambda_\eta(AB)\}$
$\geq \min\{\min\{\gamma_\beta(A), \gamma_\beta(B)\}, \min\{\lambda_\eta(A), \lambda_\eta(B)\}\}$
$= \min\{\min\{\gamma_\beta(A), \lambda_\eta(A)\}, \min\{\gamma_\beta(B), \lambda_\eta(B)\}\}$
$= \min\{(\gamma_\beta \cap \gamma_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}$

$(\gamma_\beta \cap \lambda_\mu)(AB) \geq \min\{(\gamma_\beta \cap \gamma_\eta)(A), (\gamma_\beta \cap \lambda_\eta)(B)\}$.

Hence, $\gamma_G \cup \lambda_H$ is an intuitionistic anti-fuzzy HX subring of a HX ring ℜ.

3.1.7 Definition

Let G = { ⟨x, $\alpha(x)$, $\beta(x)$⟩ / x∈R } and H = { ⟨x, $\mu(x)$, $\eta(x)$⟩ / x∈R } be any two intuitionistic fuzzy sets defined on a ring R. . Let $\Re_1 \subset 2^R{}_1 - \{\phi\}$ and $\Re_2 \subset 2^R{}_2 - \{\phi\}$ be any two HX rings. Let γ_G = { ⟨A, $\gamma_\alpha(A)$, $\gamma_\beta(A)$⟩ / A∈ℜ } and λ_H = {⟨A, $\lambda_\mu(A)$, $\lambda_\eta(A)$⟩ /A∈ℜ } be any two intuitionistic fuzzy subsets of a HX ring ℜ, then the cartesian anti-product of γ_G and λ_H is defined as $(\gamma_G \times \lambda_H)$ = {⟨(A,B), $(\gamma_\alpha \cup \lambda_\mu)(A, B)$, $(\gamma_\beta \cap \lambda_\eta)(A, B)$⟩ /(A, B) ∈ $\Re_1 \times \Re_2$},where, $(\gamma_\alpha \cup \lambda_\mu)(A, B) = \max\{\gamma_\alpha(A), \lambda_\mu(B)\}$, for all (A, B) ∈ $\Re_1 \times \Re_2$, $(\gamma_\beta \cap \lambda_\eta)(A, B) = \min\{\gamma_\beta(A), \lambda_\eta(B)\}$, for all (A, B) ∈ $\Re_1 \times \Re_2$.

3.1.8 Theorem

Let G and H be any two intuitionistic fuzzy sets of R_1 and R_2 respectively. Let $\Re_1 \subset 2^R{}_1 - \{\phi\}$ and $\Re_2 \subset 2^R{}_2 - \{\phi\}$ be any two HX rings. If γ^G and λ^H are any two intuitionistic anti-fuzzy HX subrings of $\Re_1$ and $\Re_2$ respectively then, $\gamma^G \times \lambda^H$ is also an intuitionistic anti-fuzzy HX subring of a HX ring $\Re_1 \times \Re_2$.

Proof

It is clear.

3.1.9 Theorem

Let H be an intuitionistic fuzzy set defined on R. Let λ^H be an intuitionistic fuzzy HX subring of ℜ if and only if $(\lambda^H)^c$ is an intuitionistic anti-fuzzy HX subring of ℜ.

Proof

Let λ^H be an intuitionistic fuzzy HX subring of ℜ.

Let A,B ∈ ℜ

i. $\lambda^\mu(A-B) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$
⇔
$1-\lambda^\mu(A-B) \leq 1-\min\{\lambda^\mu(A), \lambda^\mu(B)\}$
⇔ $(\lambda^\mu)^c(A-B) \leq \max\{(1-\lambda^\mu(A)),(1-\lambda^\mu(B))\}$
⇔ $(\lambda^\mu)^c(A-B) \leq \max\{(\lambda^\mu)^c(A), (\lambda^\mu)^c(B)\}$.

ii. $\lambda^\mu(AB) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$
⇔ $1-\lambda^\mu(AB) \leq 1-\min\{\lambda^\mu(A), \lambda^\mu(B)\}$
⇔ $(\lambda^\mu)^c(AB) \leq \max\{(1-\lambda^\mu(A)),(1-\lambda^\mu(B))\}$
⇔ $(\lambda^\mu)^c(AB) \leq \max\{(\lambda^\mu)^c(A), (\lambda^\mu)^c(B)\}$.

iii $\lambda^\eta(A-B) \leq \max\{\lambda^\eta(A), \lambda^\eta(B)\}$
⇔ $1-\lambda^\eta(A-B) \geq 1-\max\{\lambda^\eta(A), \lambda^\eta(B)\}$
⇔ $(\lambda^\eta)^c(A-B) \geq \min\{(1-\lambda^\eta(A)),(1-\lambda^\eta(B))\}$
⇔ $(\lambda^\eta)^c(A-B) \geq \min\{(\lambda^\eta)^c(A), (\lambda^\eta)^c(B)\}$.

iv. $\lambda^\eta(AB) \leq \max\{\lambda^\eta(A), \lambda^\eta(B)\}$
⇔ $1-\lambda^\eta(AB) \geq 1-\max\{\lambda^\eta(A), \lambda^\eta(B)\}$
⇔ $(\lambda^\eta)^c(AB) \geq \min\{(1-\lambda^\eta(A)),(1-\lambda^\eta(B))\}$
⇔ $(\lambda^\eta)^c(AB) \geq \min\{(\lambda^\eta)^c(A), (\lambda^\eta)^c(B)\}$.

Hence, $(\lambda^H)^c$ is an intuitionistic anti-fuzzy HX subring of ℜ.

3.1.10 Definition

Let H be an intuitionistic fuzzy set of R. Let ℜ ⊂ $2^R - \{\phi\}$ be a HX ring. Let λ_H be an intuitionistic fuzzy set of ℜ. We define the following “necessity” and possibility” operations:

□λ_H = { ⟨ A, $\lambda_\mu(A)$, $1-\lambda_\mu(A)$ ⟩ / A∈ℜ}

◊λ_H = { ⟨ A, $1-\lambda_\eta(A)$, $\lambda_\eta(A)$ ⟩ / A∈ℜ}.

3.1.11 Theorem

Let H be an intuitionistic fuzzy set on R. Let λ_H be an intuitionistic anti-fuzzy HX subring of a HX ring $\Re$ then $\Box\lambda_H$ is an intuitionistic anti-fuzzy HX subring of a HX ring $\Re$.

Proof

Let λ_H be an intuitionistic anti-fuzzy HX subring of a HX ring $\Re$. Then, For all A,B $\in\Re$,

i. $\lambda_\mu\ (A - B) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$,
ii. $\lambda_\mu\ (AB) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$,
iii. $\lambda_\eta(A - B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$,
iv. $\lambda_\eta(AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$.

Now,

$$\lambda_\mu\ (A - B) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$$
$$1-\lambda_\mu(A - B) \geq 1 - \max\{\lambda_\mu(A), \lambda_\mu(B)\}$$
$$\geq \min\{1 - \lambda_\mu(A), 1 - \lambda_\mu(B)\}.$$

That is,

$$1-\lambda_\mu(A - B) \geq \min\{1- \lambda_\mu(A), 1 - \lambda_\mu(B)\}.$$

We have, $\lambda_\mu(AB) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$

$$1-\lambda_\mu\ (AB) \geq 1 - \max\{\lambda_\mu(A), \lambda_\mu(B)\}$$
$$\geq \min\{1 - \lambda_\mu(A), 1 - \lambda_\mu(B)\}.$$

That is $1-\lambda_\mu(AB) \geq \min\{1 - \lambda_\mu(A), 1 - \lambda_\mu(B)\}$.

Hence, $\Box\lambda_H$ is an intuitionistic anti-fuzzy HX subring of a HX ring $\Re$.

3.1.12 Theorem

Let H be an intuitionistic fuzzy set on R. Let λ_H be an intuitionistic anti-fuzzy HX subring of a HX ring $\Re$ then $\Diamond\lambda_H$ is an intuitionistic anti-fuzzy HX subring of a HX ring $\Re$.

Proof

Let λ_H be an intuitionistic anti-fuzzy HX subring of a HX ring $\Re$. Then, For all A,B $\in\Re$,

i. $\lambda_\mu\ (A - B) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$,
ii. $\lambda_\mu\ (AB) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$,
iii. $\lambda_\eta(A - B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$,
iv. $\lambda_\eta(AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$.

Now,

$$\lambda_\eta(A - B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$$
$$1-\lambda_\eta(A - B) \leq 1 - \min\{\lambda_\eta(A), \lambda_\eta(B)\}$$
$$\leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$$

That is,

$$1-\lambda_\eta(A - B) \leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$$

We have,

$$\lambda_\eta\ (AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$$
$$1-\lambda_\eta(AB) \leq 1 - \min\{\lambda_\eta(A), \lambda_\eta(B)\}$$
$$\leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$$

That is, $1-\lambda_\eta(AB) \leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}$.

Hence, $\Diamond\lambda_H$ is an intuitionistic anti-fuzzy HX subring of a HX ring $\Re$.

3.2 Homomorphism and anti homomorphism

In this section, we introduce the concept of an anti-image, anti-pre-image of an intuitionistic anti-fuzzy HX subring of a HX ring and discuss its properties under homomorphism and anti homomorphism.

3.2.1 Definition

Let R_1 and R_2 be any two rings. Let $\Re_1 \subset 2^{R_1} - \{\phi\}$ and $\Re_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings defined on R_1 and R_2 respectively. Let H and G be any two intuitionistic fuzzy subsets in R_1 and R_2 respectively. Let λ_H and γ_G be any two intuitionistic anti-fuzzy HX subrings of HX rings $\Re_1$ and $\Re_2$ respectively induced by H and G. Let f: $\Re_1 \to \Re_2$ be a mapping then the image of λ_H denoted as f (λ_H) is an intuitionistic fuzzy subset of $\Re_2$ and is defined as for each $U\in\Re_2$,

$$(f(\lambda_\mu))(U) = \begin{cases} \inf\{\lambda_\mu(X): X\in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

$$f(\lambda_\eta))(U) = \begin{cases} \sup\{\lambda_\eta(X): X\in f^{-1}(U)\} & \text{if } f^{-1}(U) \neq \phi \\ 1, & \text{otherwise} \end{cases}$$

Also the pre-image of γ_G denoted as $f^{-1}(\gamma_G)$ under f is an intuitionistic fuzzy subset of $\Re_1$ defined as for each $X \in \Re_1$, $(f^{-1}(\gamma_\alpha))(X) = \gamma_\alpha(f(X))$ and $(f^{-1}(\gamma_\beta))(X) = \gamma_\beta(f(X))$.

3.2.2 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let H be an intuitionistic fuzzy subset of R_1. Let λ_H be an intuitionistic anti-fuzzy HX subring of $\Re_1$ then f (λ_H) is an intuitionistic anti-fuzzy HX subring of $\Re_2$, if λ_H has a infimum property and λ_H is f-invariant.

Proof

Let H = $\{\langle x, \mu(x), \eta(x)\rangle \ / \ x\in R_1\}$ be an intuitionistic fuzzy sets defined on a ring R_1.Then, $\lambda_H = \{\langle X, \lambda_\mu(X), \lambda_\eta(X)\rangle \ / \ X\in\Re_1\}$ be an intuitionistic anti-fuzzy HX subring of a HX ring $\Re_1$. Then, $f(\lambda_H) = \{\langle f(X), f(\lambda_\mu)(f(X)), f(\lambda_\eta)(f(X))\rangle \ / \ X\in\Re_1\}$.There exist X, Y$\in \Re_1$ such that f(X) , f(Y) $\in \Re_2$,

i. $(f(\lambda_\mu))(f(X)-f(Y)) = f(\lambda_\mu))(f(X-Y))$
$= \lambda_\mu(X-Y)$
$\leq \max\{\lambda_\mu(X), \lambda_\mu(Y)\}$
$= \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}$

$(f(\lambda_\mu))(f(X) - f(Y)) \leq \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}$

ii. $(f(\lambda_\mu))(f(X)f(Y)) = (f(\lambda_\mu))(f(XY))$
$= \lambda_\mu(XY)$
$\leq \max\{\lambda_\mu(X), \lambda_\mu(Y)\}$
$= \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}$

$(f(\lambda_\mu))(f(X)f(Y)) \leq \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}$.

iii. $(f(\lambda_\eta))(f(X) - f(Y)) = (f(\lambda_\eta))(f(X-Y))$
$= \lambda_\eta(X-Y)$
$\geq \min\{\lambda_\eta(X), \lambda_\eta(Y)\}$
$= \min\{(f(\lambda_\eta))(f(X))$,

$(f(\lambda_\eta))(f(Y))\}$

$(f(\lambda_\eta))(f(X)-f(Y)) \geq \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}$

iv. $(f(\lambda_\eta))(f(X)f(Y)) = (f(\lambda_\eta))(f(XY))$
$= \lambda_\eta(XY)$
$\geq \min\{\lambda_\eta(X), \lambda_\eta(Y)\}$
$= \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}$

$(f(\lambda_\eta))(f(X)f(Y)) \geq \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}$.

Hence, $f(\lambda_H)$ is an intuitionistic anti-fuzzy HX subring of $\mathfrak{R}_2$.

3.2.3 Theorem

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \to \mathfrak{R}_2$ be a homomorphism on HX rings. Let G be an intuitionistic fuzzy subset of R_2. Let γ_G be an intuitionistic anti-fuzzy HX subring of $\mathfrak{R}_2$, then $f^{-1}(\gamma_G)$ is an intuitionistic anti-fuzzy HX subring of $\mathfrak{R}_1$.

Proof

Let $G = \{\langle y, \alpha(y), \beta(y)\rangle / y \in R_2\}$ be an intuitionistic fuzzy sets defined on a ring R_2.

Then, $\gamma_G = \{\langle Y, \gamma_\alpha(Y), \gamma_\beta(Y)\rangle / Y \in \mathfrak{R}_2\}$ be an intuitionistic anti-fuzzy HX subring of a HX ring $\mathfrak{R}_2$. Then, $f^{-1}(\gamma_G) = \{\langle X, f^{-1}(\gamma_\alpha)(X), f^{-1}(\gamma_\beta)(X)\rangle / X \in \mathfrak{R}_1\}$.

For any $X, Y \in \mathfrak{R}_1$, $f(X), f(Y) \in \mathfrak{R}_2$,

i. $(f^{-1}(\gamma_\alpha))(X-Y) = \gamma_\alpha(f(X-Y))$
$= \gamma_\alpha(f(X)-f(Y))$
$\leq \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\}$
$= \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}$

$(f^{-1}(\gamma_\alpha))(X-Y) \leq \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}$.

ii. $(f^{-1}(\gamma_\alpha))(XY) = \gamma_\alpha(f(XY))$
$= \gamma_\alpha(f(X)f(Y))$
$\leq \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\}$
$= \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}$

$(f^{-1}(\gamma_\alpha))(XY) \leq \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}$

iii. $(f^{-1}(\gamma_\beta))(X-Y) = \gamma_\beta(f(X-Y))$
$= \gamma_\beta(f(X)-f(Y))$
$\geq \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\}$
$= \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}$

$(f^{-1}(\gamma_\beta))(X-Y) \geq \min\{(f^{-1}(\eta_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}$.

iv. $(f^{-1}(\gamma_\beta))(XY) = \gamma_\beta(f(XY))$
$= \gamma_\beta(f(X)f(Y))$
$\geq \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\}$
$= \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}$

$(f^{-1}(\gamma_\beta))(XY) \geq \min\{(f^{-1}(\eta_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}$.

Hence, $f^{-1}(\gamma_G)$ is an intuitionistic anti-fuzzy HX subring of $\mathfrak{R}_1$.

3.2.4 Theorem

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti homomorphism onto HX rings. Let H be an intuitionistic fuzzy subset of R_1. Let λ_H be an intuitionistic anti-fuzzy HX subring of $\mathfrak{R}_1$ then $f(\lambda_H)$ is an intuitionistic anti-fuzzy HX subring of $\mathfrak{R}_2$, if λ_H has a infimum property and λ_H is f-invariant.

Proof

Let $H = \{\langle x, \mu(x), \eta(x)\rangle / x \in R_1\}$ be an intuitionistic fuzzy sets defined on a ring R_1. Then, $\lambda_H = \{\langle X, \lambda_\mu(X), \lambda_\eta(X)\rangle / X \in \mathfrak{R}_1\}$ be an intuitionistic anti-fuzzy HX subring of a HX ring $\mathfrak{R}_1$. Then, $f(\lambda_H) = \{\langle f(X), f(\lambda_\mu)(f(X)), f(\lambda_\eta)(f(X))\rangle / X \in \mathfrak{R}_1\}$. There exist $X, Y \in \mathfrak{R}_1$ such that $f(X), f(Y) \in \mathfrak{R}_2$,

i. $(f(\lambda_\mu))(f(X)-f(Y)) = (f(\lambda_\mu))(f(X-Y))$
$= \lambda_\mu(X-Y)$
$\leq \max\{\lambda_\mu(X), \lambda_\mu(Y)\}$
$= \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}$

$(f(\lambda_\mu))(f(X)-f(Y)) \leq \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}$

ii. $(f(\lambda_\mu))(f(X)f(Y)) = (f(\lambda_\mu))(f(YX))$
$= \lambda_\mu(YX)$
$\leq \max\{\lambda_\mu(Y), \lambda_\mu(X)\}$
$= \max\{\lambda_\mu(X), \lambda_\mu(Y)\}$
$= \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}$

$(f(\lambda_\mu))(f(X)f(Y)) \leq \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}$.

iii. $(f(\lambda_\eta))(f(X)-f(Y)) = (f(\lambda_\eta))(f(X-Y))$
$= \lambda_\eta(X-Y)$
$\geq \min\{\lambda_\eta(X), \lambda_\eta(Y)\}$
$= \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}$

$(f(\lambda_\eta))(f(X)-f(Y)) \geq \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}$

iv. $(f(\lambda_\eta))(f(X)f(Y)) = (f(\lambda_\eta))(f(YX))$
$= \lambda_\eta(YX)$
$\geq \min\{\lambda_\eta(Y), \lambda_\eta(X)\}$
$= \min\{\lambda_\eta(X), \lambda_\eta(Y)\}$
$= \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}$

$(f(\lambda_\eta))(f(X)f(Y)) \geq \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}$.

Hence, $f(\lambda_H)$ is an intuitionistic anti-fuzzy HX subring of $\mathfrak{R}_2$

3.2.5 Theorem

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti homomorphism on HX rings. Let G be an intuitionistic fuzzy subset of R_2. Let γ_G be an intuitionistic anti-fuzzy HX subring of $\mathfrak{R}_2$, then $f^{-1}(\gamma_G)$ is an intuitionistic anti-fuzzy HX subring of $\mathfrak{R}_1$.

Proof

Let $G = \{\langle y, \alpha(y), \beta(y)\rangle / y \in R_2\}$ be an intuitionistic fuzzy sets defined on a ring R_2. Then, $\gamma_G = \{\langle Y, \gamma_\alpha(Y), \gamma_\beta(Y)\rangle / Y \in \mathfrak{R}_2\}$ be an intuitionistic anti-fuzzy HX subring of a HX ring $\mathfrak{R}_2$.

Then, $f^{-1}(\gamma_G) = \{ \langle X, f^{-1}(\gamma_\alpha)(X), f^{-1}(\gamma_\beta)(X)\rangle / X\in\Re_1 \}$.For any X, Y $\in \Re_1$, f(X) , f(Y) $\in \Re_2$,

i.
$$\begin{aligned}(f^{-1}(\gamma_\alpha))(X-Y) &= \gamma_\alpha(f(X-Y)) \\ &= \gamma_\alpha(f(X)-f(Y)) \\ &\le \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\} \\ &= \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\ (f^{-1}(\gamma_\alpha))(X-Y) &\le \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}.\end{aligned}$$

ii.
$$\begin{aligned}(f^{-1}(\gamma_\alpha))(XY) &= \gamma_\alpha(f(XY)) \\ &= \gamma_\alpha(f(Y)f(X)) \\ &\le \max\{\gamma_\alpha(f(Y)), \gamma_\alpha(f(X))\} \\ &= \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\} \\ &= \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\ (f^{-1}(\gamma_\alpha))(XY) &\le \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}\end{aligned}$$

iii.
$$\begin{aligned}(f^{-1}(\gamma_\beta))(X-Y) &= \gamma_\beta(f(X-Y)) \\ &= \gamma_\beta(f(X)-f(Y)) \\ &\ge \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\} \\ &= \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\} \\ (f^{-1}(\gamma_\beta))(X-Y) &\ge \min\{(f^{-1}(\eta_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}.\end{aligned}$$

iv.
$$\begin{aligned}(f^{-1}(\gamma_\beta))(XY) &= \gamma_\beta(f(XY)) \\ &= \gamma_\beta(f(Y)f(X)) \\ &\ge \min\{\gamma_\beta(f(Y)), \gamma_\beta(f(X))\} \\ &= \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\} \\ &= \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\} \\ (f^{-1}(\gamma_\beta))(XY) &\ge \min\{(f^{-1}(\eta_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}.\end{aligned}$$

Hence, $f^{-1}(\gamma_G)$ is an intuitionistic anti-fuzzy HX subring of $\Re_1$.

4. Conclusion

In this paper we introduce the concept of intuitionistic anti -fuzzy HX ring and discuss the basic results on intuitionistic anti-fuzzy HX ring.

Also we introduce the image and pre-image of an intuitionistic anti-fuzzy HX subring and discuss some of its properties.

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