Lehmer 3 –Mean Labeling of Some New Disconnected Graphs

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Abstract :- *A graph G=(V,E) with P vertices and q edges is called Lehmer -3 mean graph ,if it is possible to label vertices x ЄV with distinct label f(x) from 1,2,3,…………..q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=*$\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ *(or)* $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$*, then the edge labels are distinct. In this case f is called Lehmer -3 mean labeling of G. In this paper we investigate Lehmer -3 mean labeling of some standard graphs*

Keywords: - *Graph, Path, Cycle, Comb, Crown, Ladder.*

I .INTRODUCTION

A graph considered here are finite undirected and simple. The vertex set and edge set of a graph are denoted by V(G) and E(G) respectively. For detailed survey Gallian survey [1] is refered and standerd terminologies and notations are followed from Harary [2]. We will find the brief summary of definitions and informations necessary for the present investigation.

Definition 1.1

A graph G=(V,E) with P vertices and q edges is called Lehmer -3 mean graph, if it is possible to label vertices x ЄV with distinct label f(x) from 1,2,3,………….q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=$\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ (or) $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$, then the edge labels are distinct. In this case f is called Lehmer -3 mean labeling of G.

Definition 1.2

A path P_n is obtained by joining u_i to the consecutive vertices u_{i+1} for $1\leq i\leq n$

Definition 1.3

Comb is a graph obtained by joining a single pendant edge to each vertex of a path

Definition 1.4

A closed path is called a cycle of G.

Definition 1.5

Crown is a graph obtained by joining a single pendant edge to each vertex of a cycle

Definition 1.6

A product graph $P_m x P_n$ is called a planar grid $P_2 x P_n$ is called a ladder.

Definition 1.7

$P_n \odot K_{1,2}$ is a graph obtained by attaching $K_{1,2}$ to each vertex of P_n

Definition 1.8

$P_n \odot K_{1,3}$ is a graph obtained from the path attaching $K_{1,3}$ to each of its vetices

Definition 1.9

$P_n \odot K_3$ is a graph connected by a complete graph K_3 in its each vertex

II. Main Results

Theorem:2.1

$(C_n \odot k_1) \cup P_m$ is a Lehmer-3 mean graph.

Proof:

Let $C_n \odot k_1$ be the crown with $u_1, u_2, \ldots\ldots u_n$,v_1 as the cycle and v_i be the pendent vertices adjacent to u_i, $1 \leq i \leq n$

Let $w_1, w_2, \ldots\ldots w_m$ be the path

Let G be the graph obtained by the union of $(C_n \odot k_1) \cup P_m$.

Define a function $f: V(G) \rightarrow \{1,2,3,\ldots..q+1\}$ by

$f(ui)=i \quad ; \quad 1 \leq i \leq n$

$f(v_i)=n+i; \quad 1 \leq i \leq n$

$f(w_j)=2n+j \quad ; \quad 1 \leq j \leq m$

Then the distinct edge labels are $f(u_i u_{i+1})=i \quad ; \quad 1 \leq i \leq n$

$f(u_i v_i)=n+1 \quad ; \quad 1 \leq i \leq n$

$f(w_j w_{j+1})=2n+j \quad ; \quad 1 \leq j \leq m-1$

Thus $(C_n \odot K_1) \cup P_m$ forms Lehmer-3 mean graph.

Example: 2.2

The Lehmer-3 mean graph of $(C_3 \odot k_1) \cup P_4$ is given below.

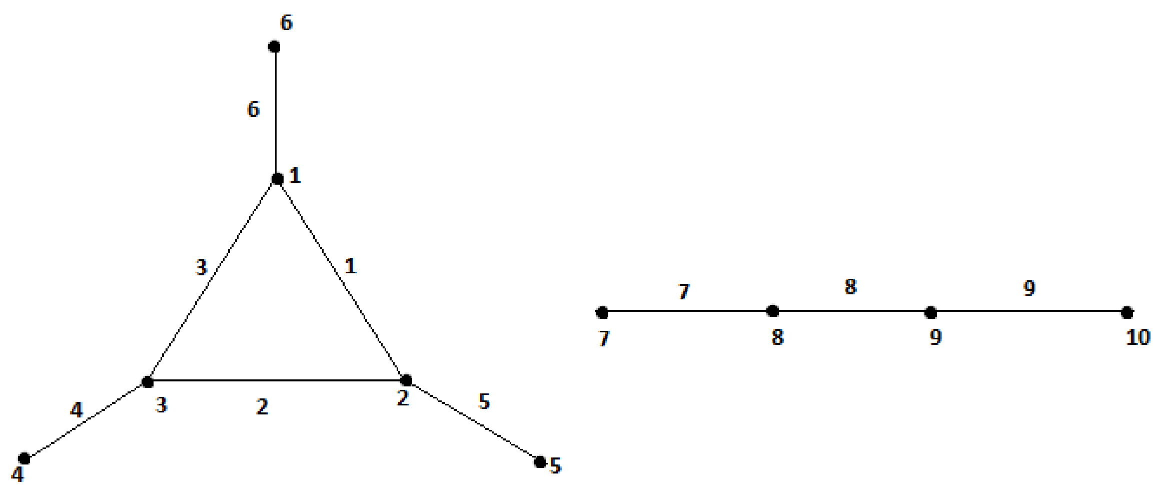

Figure - 1

Theorem: 2.3

$(C_n \odot k_1) \cup (P_m \odot K_1)$ is a Lehmer-3 mean graph.

Proof:

Let G be the given graph.

$C_n \odot k_1$ is the crown with $u_1,u_2,\ldots\ldots u_n,u_1$ as the cycle and v_i be the pendent vertices adjacent to u_i, $1\le i\le n$.

Let $P_m \odot K_1$ be the comb with $x_1,x_2,\ldots\ldots x_m$ as the vertices and y_j be the pendent vertices adjacent to x_j, $1\le j\le m$.

Define a function $f:V(G) \rightarrow \{1,2,\ldots\ldots q+1\}$ by

$f(u_i)=i$; $1\le i\le n$

$f(v_i)=n+i$; $1\le i\le n$

$f(x_j)=2n+(2j-1)$; $1\le j\le m$

$f(y_j)=2n+(2j)$; $1\le j\le m$

Then the distinct edge labeling are $f(u_iu_{i+1})=i$; $1\le i\le n$

$f(u_iv_i)=n+1$; $1\le i\le n$

$f(x_jx_{j+1})=2n+2j$; $1\le j\le m-1$

$f(x_jy_j)=2n+(2j-1)$; $1\le j\le m$

Hence G forms a Lehmer-3 mean graph

Example:2.4

$(C_5 \odot k_1) \cup (P_4 \odot K_1)$ is a Lehmer-3 mean graph.

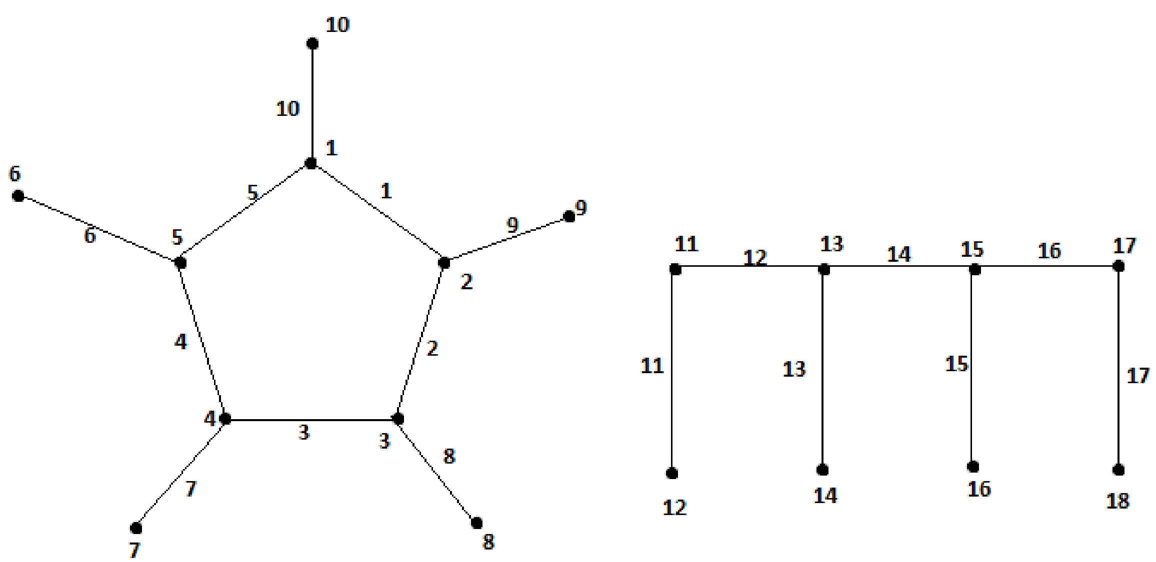

Figure - 2

Theorem:2.5

$(C_n \odot K_1) \cup C_m$ is a Lehmer -3 mean graph.

Proof:

Let G be a graph obtained by $(C_n \odot K_1) \cup C_m$.

Let C_nAK_1 be the crown with $u_1,u_2,\ldots\ldots u_n$ as a cycle and v_i be the pendent vertices adjacent to $u_i, 1\leq i\leq n$.

Let C_m be a cycle with $w_j, 1\leq j\leq m$ as the vertices.

Define a function $f:V(G)\rightarrow\{1,2\ldots\ldots q+1\}$ by A

$f(u_i)=2i-1$; $1\leq i\leq n$

$f(v_i)=2i$; $1\leq i\leq n$

$f(w_j)=2n+j$; $1\leq j\leq m$

Thus the edge labels are distinct.

Hence $(C_n\odot K_1)\cup C_m$ is a Lehmer -3 mean graph.

Example:2.6

$(C_5\odot K_1)\cup C_6$ is a Lehmer-3 mean graph.

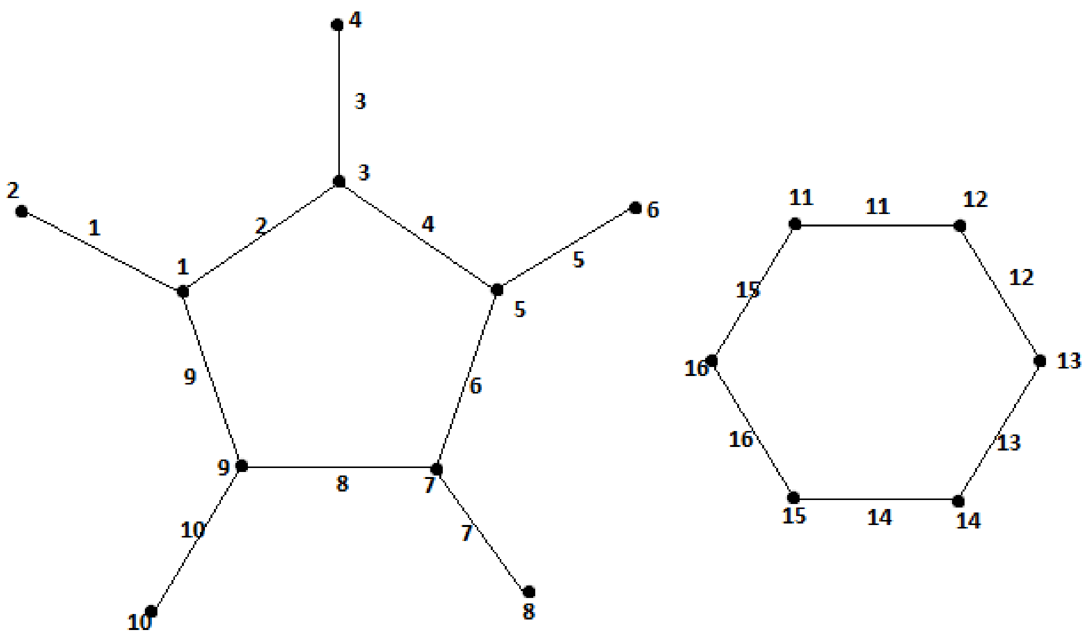

Figure - 3

Theorem:2.7

$(C_n\odot K_1)\cup(P_m\odot K_{1,2})$ be a Lehmer -3 mean graph.

Proof :

Let G be a graph obtained from the union of $(C_n\odot K_1)\cup(P_m\odot K_{1,2})$.

Let $C_n\odot K_1$ be a crown with vertices $u_1,u_2\ldots.u_n; v_1,v_2\ldots\ldots.v_n$.

Let $P_m\odot K_{1,2}$ is a path attaching $K_{1,2}$ with vertices $w_1,w_2,\ldots\ldots.w_m$ $:x_1,x_2\ldots\ldots.x_m.$

Define a function $f:V(G)\rightarrow\{1,2,\ldots..q+1\}$ by

$f(u_i)=2i-1$; $1\leq i\leq n$

$f(v_i)=2i$; $1\le i\le n$

$f(w_j)= 2n+(3i-2)$; $1\le j\le m$

$f(x_j)= 2n+(3i-1)$; $1\le j\le m$

$f(y_j) = 2n+3i$; $1\le j\le m$

Thus we obtained distinct edge labelings.

Hence $(C_n\odot K_1) \cup (P_m\odot K_{1,2})$ is a Lehmer -3 mean graph.

Example:2.8

$(C_4\odot K_1) \cup (P_4\odot K_{1,2})$ is a Lehmer -3 mean graph.

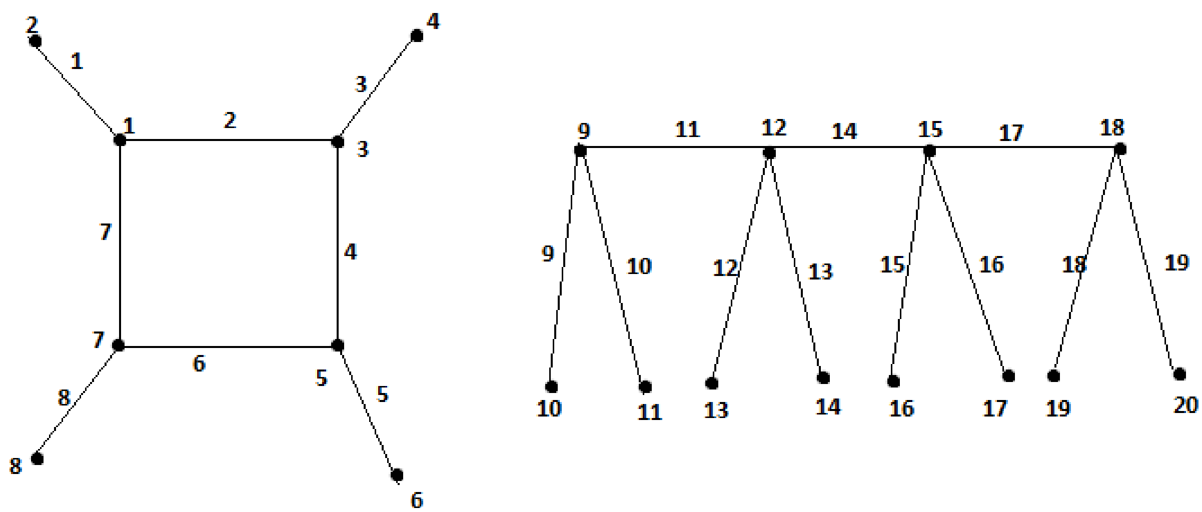

Figure - 4

Theorem:2.9

$(C_n\odot K_1) \cup (P_m\odot K_{1,3})$ be a Lehmer -3 mean graph.

Proof :

Let G be a graph obtained from $(C_n\odot K_1)\cup(P_m\odot K_{1,3})$.

The vertices of $C_n\odot K_1$be $u_1,u_2\ldots.u_n$; $v_1,v_2\ldots\ldots..v_n$.

$(P_m\odot K_{1,3})$ be a graph with vertices $x_1,x_2,\ldots\ldots..x_m$; $y_1,y_2,\ldots\ldots..y_m$; $z_1,z_2,\ldots..z_m$.

Define a function $f:V(G)\to\{1,2,\ldots\ldots..q+1\}$ by

$f(u_i)=2i-1$; $1\le i\le n$

$f(v_i)= 2i$; $1\le i\le n$

$f(w_j)=2n +(4j-3)$; $1\le j\le m$

$f(x_j)= 2n+(4j-2)$; $1\le j\le m$

$f(y_j)= 2n+(4j-1)$; $1 \le j \le m$

$f(z_j)= 2n+(4j)$; $1 \le j \le m$

Thus we get distinct edge labels.

Thus $(C_n \odot K_1) \cup (P_m \odot K_{1,3})$ is a Lehmer -3 mean graph.

Example:2.10

The Lehmer -3 mean labeling of $(C_5 \odot K_1) \cup (P_3 \odot K_{1,3})$ is given below

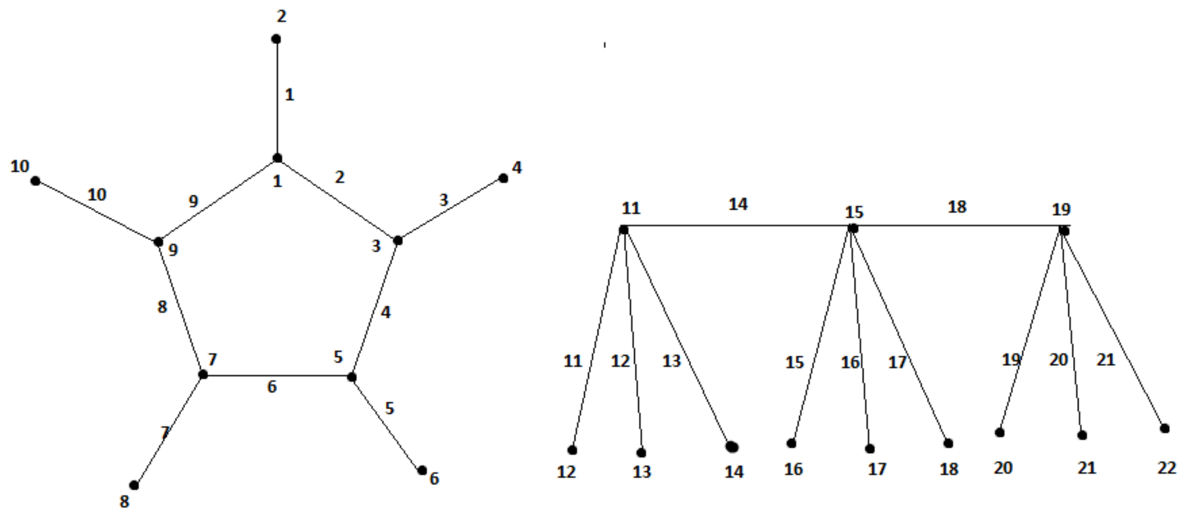

Figure - 5

Theorem:2.11

$(C_n \odot K_1) \cup (P_m \odot K_3)$ is a Lehmer- 3 mean graph

Proof:

Let G be a graph obtained by $(C_n \odot K_1) \cup (P_m \odot K_3)$

Let $(C_n \odot K_1)$ be a graph with vertices $u_1,u_2....u_n$; $v_1,v_2........v_n$respectively

Let $(P_m \odot K_3)$ be a graph obtained by joining P_m with K_3 with vertices w_j, x_j, y_j; $1 \le j \le m$ respectively

Define a function f: $V(G) \rightarrow \{1,2,........q+1\}$ defined by

$f(u_i)= 2i-1$; $1 \le i \le n$

$f(v_i)=2i$; $1 \le i \le n$

$f(w_j)=2n+(4j-3)$; $1 \le j \le m$

$f(x_j)=2n+(4j-2)$; $1 \le j \le m$

$f(y_j)= 2n+(4j-1)$; $1 \le j \le m$

Thus we get the distinct edge labeling

Hence $(C_n \odot K_1) \cup (P_m \odot K_3)$ is a Lehmer- 3 mean graph

Example: 2.12

The labeling pattern of $(C_4 \odot K_1) \cup (P_4 \odot K_3)$ is given below

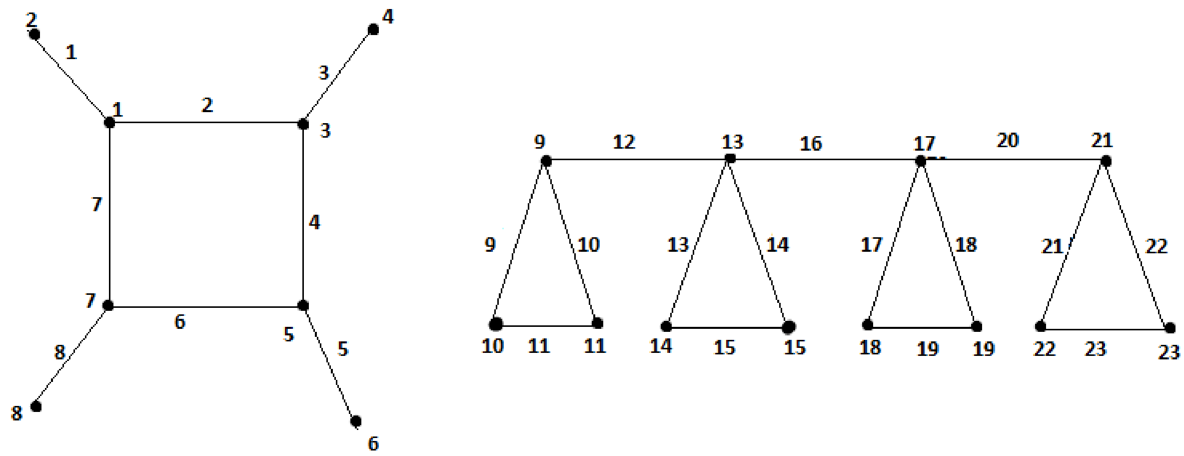

Figure-6

Theorem:2.13

$(C_n \odot k_1) \cup L_m$ is a Lehmer -3 mean graph.

Proof :

Let G be a graph obtained by $(C_n \odot k_1) \cup L_m$

Let $(C_n \odot k_1)$ be a graph with vertices $u_1, u_2 \ldots . u_n; v_1, v_2 \ldots \ldots . v_n$

Let L_m be a ladder with m vertices as $w_1, w_2, \ldots \ldots . w_m$; $x_1, x_2 \ldots \ldots . x_m$.

Define a function $f:V(G) \rightarrow \{1,2,\ldots\ldots.q+1\}$ defined by

$f(u_i)= 2i-1$; $1 \le i \le n$

$f(v_i)=2i$; $1 \le i \le n$

$f(w_1)=2n+1$

$f(w_j)=2n+(3j-3)$; $2 \le j \le m$

$f(x_1)=2n+2$

$f(x_j)=2n+(3j-2)$; $2 \le j \le m$

Thus the distinct edge labels are obtained.

Hence $(C_n \odot k_1) \cup L_m$ is a Lehmer -3 mean graph.

Example:2.14

The Lehmer -3 mean labeling of $(C_4\odot k_1) \cup L_4$ is given below.

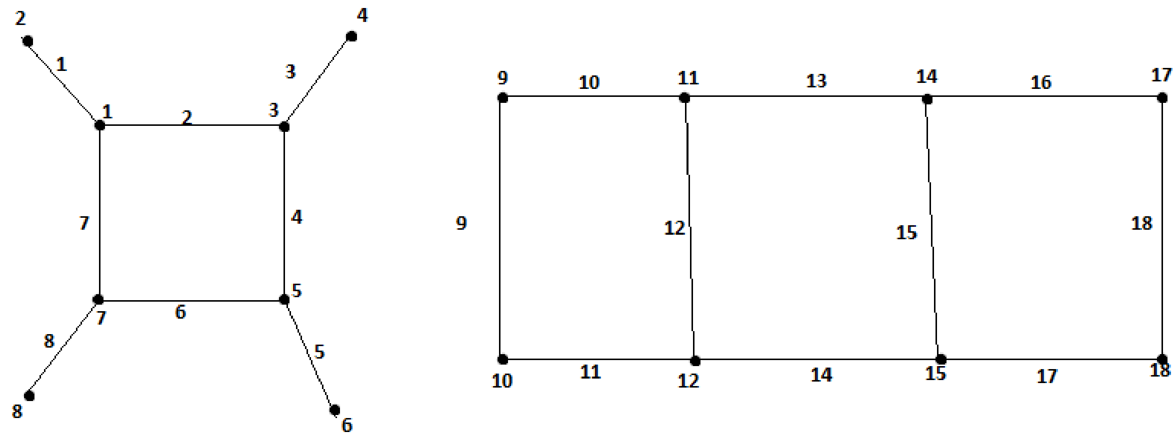

Figure -7

Theorem:2.15

$(C_n\odot K_{1,2})\cup P_m$ is a Lehmer -3 mean graph.

Proof :

Let G be a graph obtained by $(C_n\odot K_{1,2})\cup P_m$

Let $(C_n\odot K_{1,2})$ is a graph with vertices $u_1,u_2\ldots.u_n$; $v_1,v_2\ldots\ldots..v_n$ and $w_1,w_2,\ldots\ldots..w_n$ respectively.

Let P_m be a path with m vertices $x_1,x_2\ldots\ldots.x_m$.

Define a function f: $V(G)\rightarrow\{1,2,\ldots\ldots q+1\}$ by

$f(u_i)=3i-2$; $1\leq i\leq n$

$f(v_i)=3i-1$; $1\leq i\leq n$

$f(w_i)=3i$; $1\leq i\leq n$

$f(x_j)= 3n+j$; $1\leq j\leq m$

The edge labelings are distinct.

Hence $(C_n\odot K_{1,2})\cup P_m$ is a Lehmer -3 mean graph.

Example:2.16

$(C_4\odot K_{1,2})$U P_5 is a Lehmer -3 mean graph.

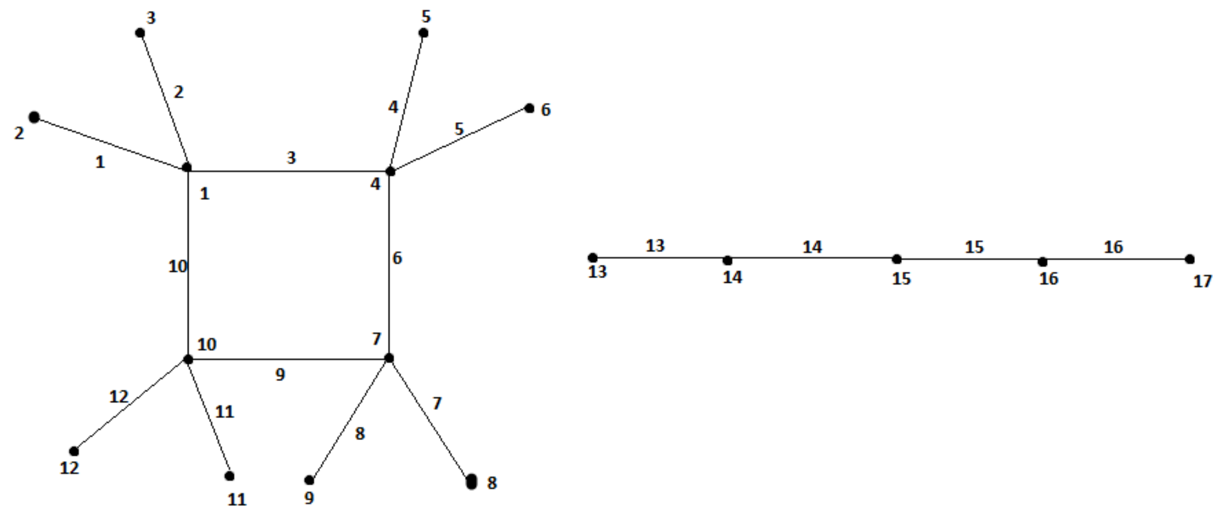

Figure -8

Theorem:2.17

$(C_n \odot K_{1,2}) \cup (P_m \odot K_1)$ is a Lehmer- 3 mean graph

Proof:

Let G be a graph obtained by $(C_n \odot K_{1,2}) \cup (P_m \odot K_1)$

Let $(C_n \odot K_{1,2})$ be a graph with vertices $u_1,u_2….u_n$; $v_1,v_2……..v_n$; $w_1,w_2,……..w_n$respectively

Let $(P_m \odot K_1)$ be a comb with vertices $x_1,x_2,…….x_m$ and $y_1,y_2,……y_m$ respectively Define a function f: V(G)→ {1,2,……..q+1} defined by

$f(u_i)= 3i-2 \quad ; \quad 1 \le i \le n$

$f(v_i)=3i-1 \quad ; \quad 1 \le i \le n$

$f(w_i)=3i \quad ; \quad 1 \le i \le n$

$f(x_j)=3n+(2j-1) \quad ; \quad 1 \le j \le m$

$f(y_j)= 3n+2j \quad ; \quad 1 \le j \le m$

Thus the edge labelings are distinct

Hence $(C_n \odot K_{1,2}) \cup (P_m \odot K_1)$ is a Lehmer- 3 mean graph.

Example: 2.18

$(C_6 \odot K_{1,2}) \cup (P_4 \odot K_1)$ is a Lehmer- 3 mean graph

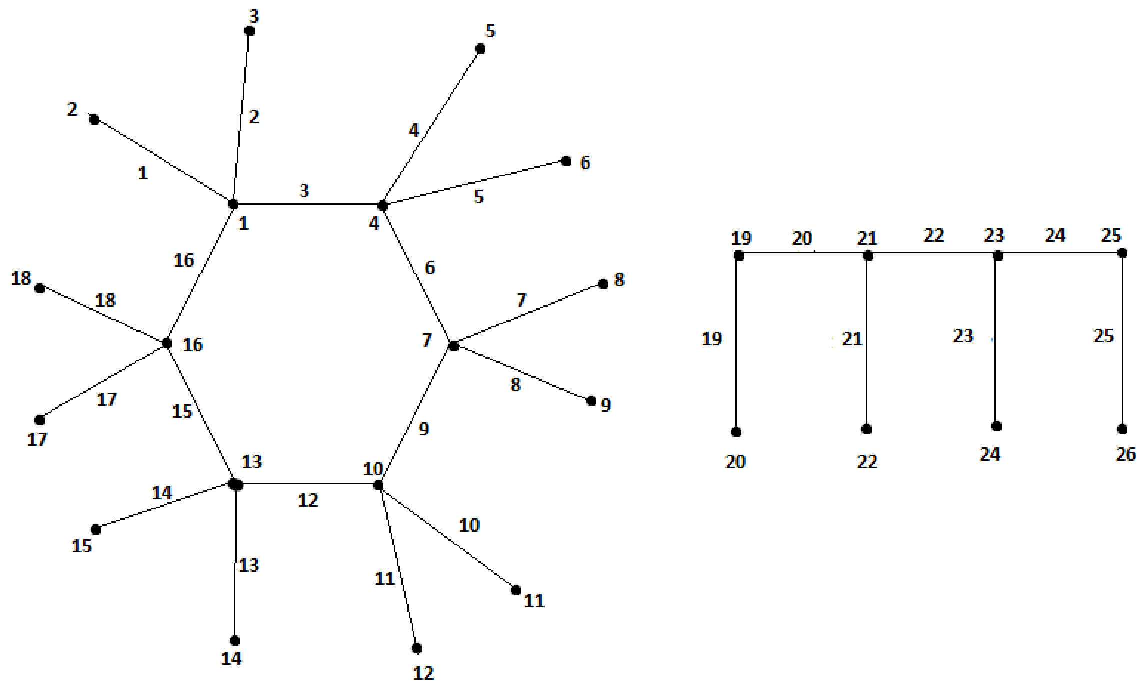

Figure-9

Theorem:2.19

$(C_n \odot K_{1,2}) \cup (P_m \odot K_{1,2})$ be a Lehmer -3 mean graph.

Proof :

Let G be a graph obtained from the union of $(C_n \odot K_{1,2})$ and $(P_m \odot K_{1,2})$.

Let $C_n \odot K_{1,2}$ be a graph with vertices $u_1, u_2 \ldots u_n$; $v_1, v_2 \ldots\ldots v_n$ and $w_1, w_2, \ldots\ldots w_n$

Let $(P_m \odot K_{1,2})$ be a graph with vertices xj,yj,zj; $1 \le j \le m$.

Define a function $f: V(G) \rightarrow \{1,2,\ldots\ldots q+1\}$ by

$f(u_i) = 3i-2$; $1 \le i \le n$

$f(v_i) = 3i-1$; $1 \le i \le n$

$f(w_i) = 3i$; $1 \le i \le n$

$f(x_j) = 3n+(3j-2)$; $1 \le j \le m$

$f(y_j) = 3n+(3j-1)$; $1 \le j \le m$

$f(z_j) = 3n+3j$; $1 \le j \le m$.

we obtain distinct edge labelings.

Thus $(C_n \odot K_{1,2}) \cup (P_m \odot K_{1,2})$ is a Lehmer -3 mean graph.

Example:2.20

$(C_5 \odot K_{1,2}) \cup (P_4 \odot K_{1,2})$ is a Lehmer -3 mean graph.

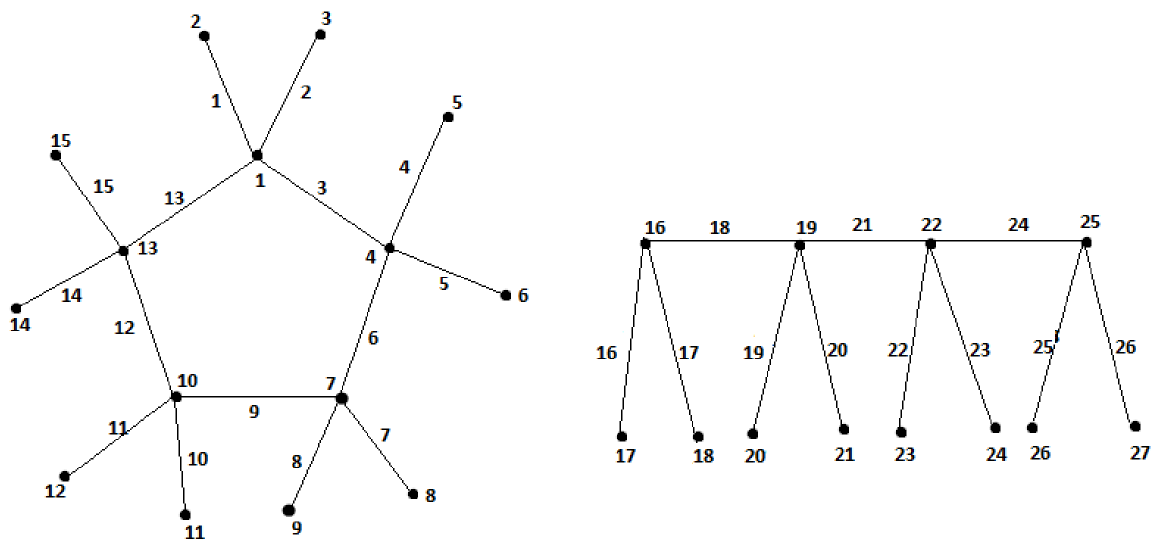

Figure-10

Theorem:2.21

$(C_n \odot K_{1,2}) \cup (P_m \odot K_{1,3})$ is a Lehmer- 3 mean graph

Proof:

Let $(C_n \odot K_{1,2}) \cup (P_m \odot K_{1,3})$ be a graph obtained from the union of $(C_n \odot K_{1,2})$ and $(P_m \odot K_{1,3})$

Let $u_1,u_2\ldots.u_n$; $v_1,v_2\ldots\ldots..v_n$; $w_1,w_2,\ldots\ldots..w_n$ be the vertices of $(C_n \odot K_{1,2})$ and let $x_1,x_2,\ldots.x_m$; $y_1,y_2,\ldots..y_m$; $z_1,z_2,\ldots..z_m$; $t_1,t_2,\ldots.t_m$ be the vertices of $(P_m \odot K_{1,3})$

Define a function $f: V(G) \rightarrow \{1,2,\ldots\ldots..q+1\}$ defined by

$f(u_i)= 3i-2$; $1 \le i \le n$

$f(v_i)=3i-1$; $1 \le i \le n$

$f(w_i)=3i$; $1 \le i \le n$

$f(x_j)=3n+(4j-3)$; $1 \le j \le m$

$f(y_j)= 3n+(4j-2)$; $1 \le j \le m$

$f(z_j)=3n+(4j-1)$; $1 \le j \le m$

$f(t_j)= 3n+(4j)$; $1 \le j \le m$

Thus we obtain distinct edge labels

Hence $(C_n \odot K_{1,2}) \cup (P_m \odot K_{1,3})$ is a Lehmer- 3 mean graph

Example:2.22

$(C_5 \odot K_{1,2}) \cup (P_4 \odot K_{1,3})$ is a Lehmer- 3 mean graph

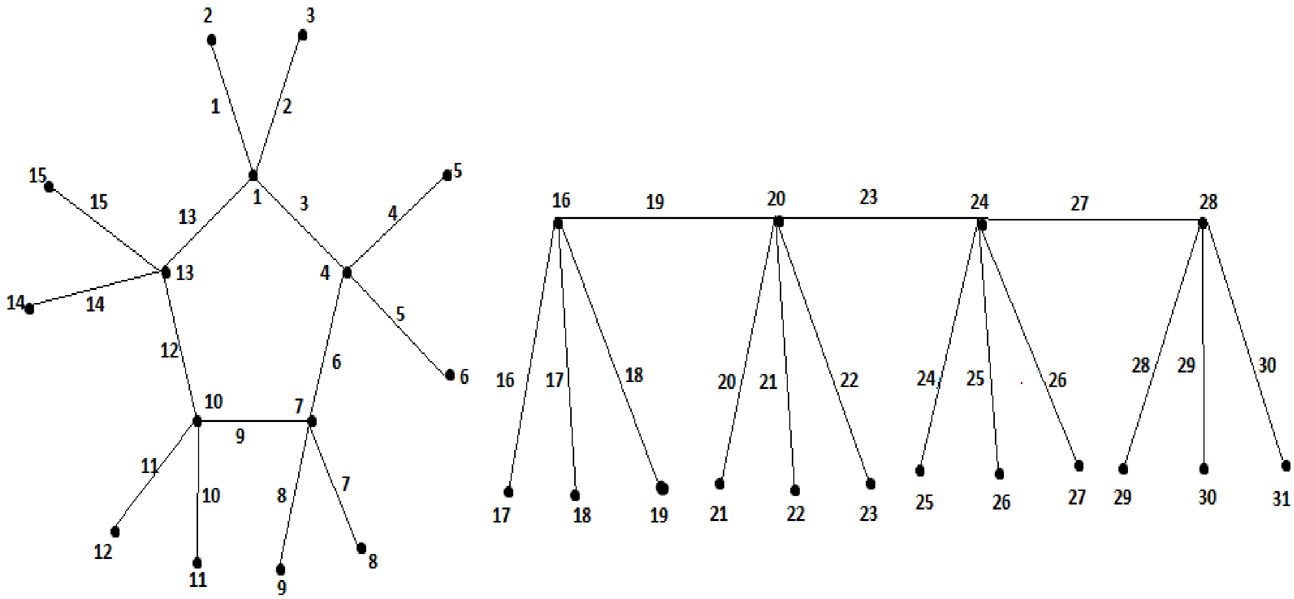

Figure-11

Theorem:2.23

$(C_n \odot K_{1,2})$ U$(P_m \odot K_3)$ is a Lehmer- 3 mean graph

Proof:

Let $(C_n \odot K_{1,2})$ be a graph with vertices $u_1,u_2….u_n$; $v_1,v_2……..v_n$; $w_1,w_2,……..w_n$respectively

Let $(P_m \odot K_3)$ be a graph with vertices x_j,y_j,z_j; $1 \le j \le m$ respectively

Let G be a graph obtained from the union of $(C_n \odot K_{1,2})$ and $(P_m \odot K_3)$

Define a function f: $V(G) \rightarrow \{1,2,……..q+1\}$ defined by

$f(u_i)= 3i-2$; $1 \le i \le n$

$f(v_i)=3i-1$; $1 \le i \le n$

$f(w_i)=3i$; $1 \le i \le n$

$f(x_j)=3n+(4j-3)$; $1 \le j \le m$

$f(y_j)= 3n+(4j-2)$; $1 \le j \le m$

$f(z_j)= 3n+(4j-1)$; $1 \le j \le m$

Thus we get distinct edge labeling

Hence $(C_n \odot K_{1,2}) \cup (P_m \odot K_3)$ is a Lehmer- 3 mean graph.

Example: 2.24

$(C_5 \odot K_{1,2}) \cup (P_4 \odot K_3)$ is a Lehmer- 3 mean graph

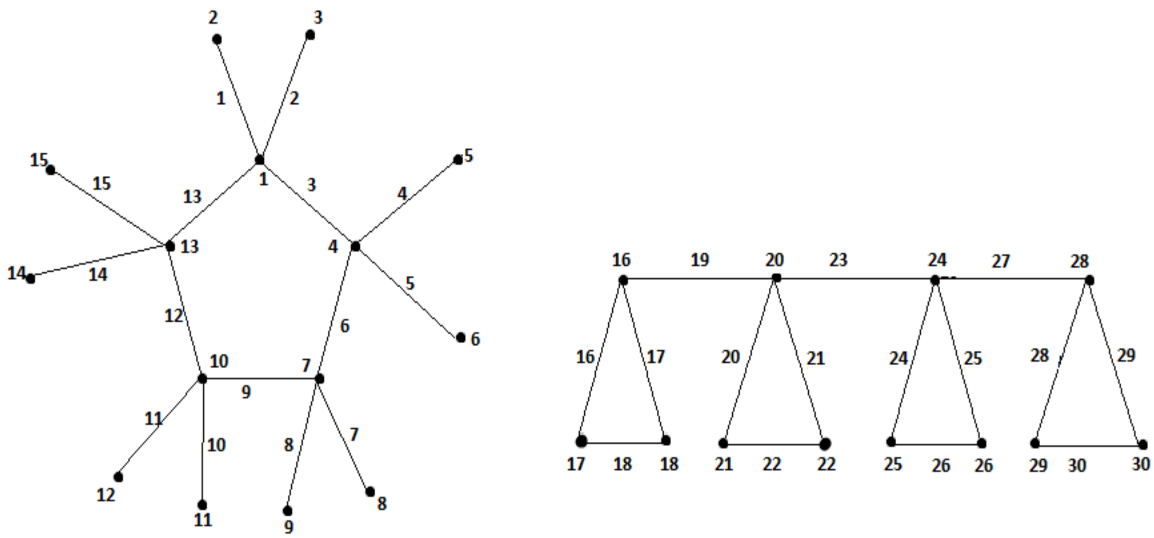

Figure-12

Theorem:2.25

$(C_n \odot K_{1,2}) \cup C_m$ is a Lehmer -3 mean graph.

Proof :

Let G be a graph obtained from the union of $(C_n \odot K_{1,2})$ and C_m

Let $(C_n \odot K_{1,2})$ is a graph with vertices $u_1, u_2 \ldots u_n$; $v_1, v_2 \ldots\ldots v_n$ and $w_1, w_2, \ldots\ldots w_n$ respectively.

Let C_m be a path with m vertices $x_1, x_2 \ldots\ldots x_m$.

Define a function f: $V(G) \rightarrow \{1,2,\ldots\ldots q+1\}$ by

$f(u_i)=3i-2$; $1 \leq i \leq n$

$f(v_i)=3i-1$; $1 \leq i \leq n$

$f(w_i)=3i$; $1 \leq i \leq n$

$f(x_j)= 3n+j$; $1 \leq j \leq m$

Thus we obtain distinct edge labels.

Hence $(C_n \odot K_{1,2}) \cup C_m$ is a lehmer -3 mean graph.

Example:2.26

Lehmer-3 mean labeling of $(C_4 \odot K_{1,2}) \cup C_5$ isgiven below

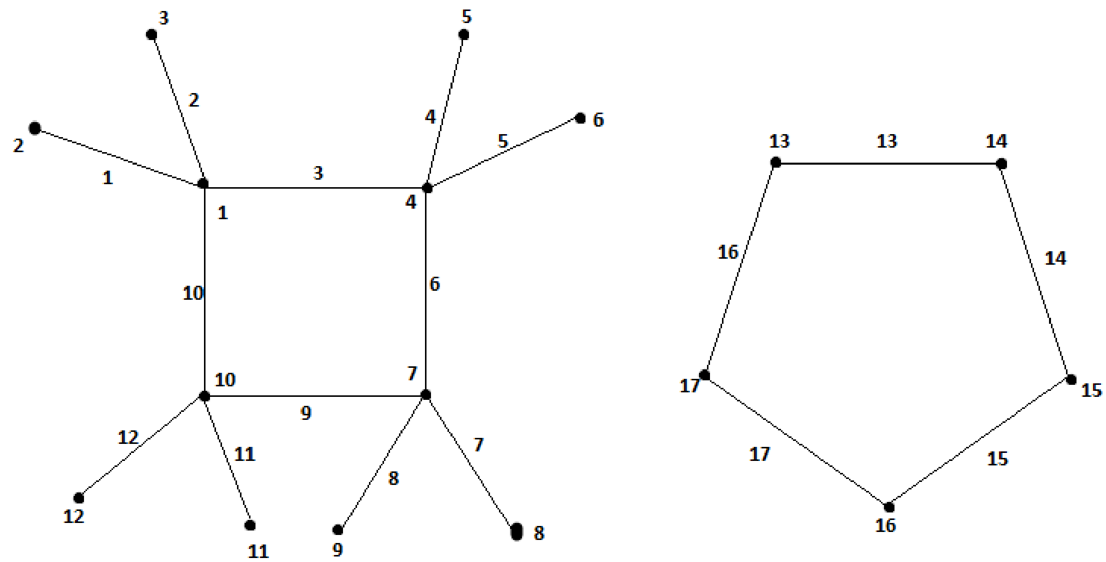

Figure-13

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